

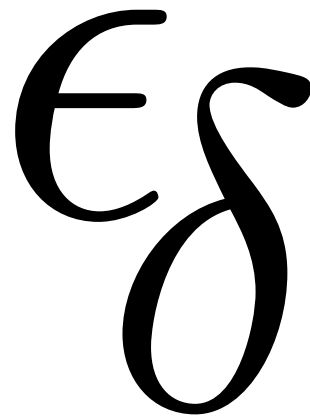
Eventually

Marianne Freundlich

Rick Norwood's letter (December 1997), which I read with enthusiastic agreement, prompts me to enlarge on his comments about δ and ϵ to talk about my personal experiences with them. In a sense I have been reflecting on the matter since at least my junior year in college, when I had a first course in analysis from S. S. Cairns, then at Queens College. I just simply didn't get it! I probably went to all his office hours and recall how patiently he said, over and over, what Norwood now says "...is crystal clear, and in plain language says this: a function has a limit at an (input)¹ number x if and only if, etc. etc."—and I, too, now see it with crystal clarity. But in my junior year I didn't, though even then I had the sophistication to know that this "it" was crucial, while I also knew that I didn't get "it" or have any idea how to use "it". Professor Cairns kindly gave me a C, and on I went to grad school at Illinois, where an Intro to Analysis was again required. It was "déjà vu all over again," this time with a wonderful instructor named Ralph Fox, who had red hair, worked in Knotentheorie, and told a funny story about how he married his Quaker wife. (He also put me in touch with a chess player named Marseille, who was investigating baking ragweed

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¹This word "input" is not in Cauchy's classic formulation and makes the definition of limit unnecessarily a whole word longer, without actually adding meaning.



pollen into bread for people like me with hayfever.)

I still had no idea and turned in a bluebook final in which I wrote only my name. Fox gave me an A, so I went to see him and complained bitterly: "How could you do that? I didn't even pretend. I don't get it. How could you give me an A?" He said that he hadn't wanted to spoil my record and anyone with the ___ (none of the words now coming to the reader's mind were then current, so I truly don't recall how he put it!) would be sure to get it "eventually." I was more baffled than ever but gamely said, "Well, what do I do next?" And he said, "Take the graduate course!" And he sent me down the hall to David Bourgin. [I think Laurie Snell may recall the class also. Also Jewell Schubert, Frankie Morfoot (Mrs. M. M. Day), and Roger Livesay were there.]

So now I'm in the grad course in analysis with Bourgin, and that man *made* me a mathematician, I guess. Or else I was just finally "ready"? The numbers ϵ and δ actually never came up, but there were gorgeous theorems. And as he never had any notes (just odd bits of old envelopes), I took on the job of writing up his lectures for the class, including complete proofs. Somehow I learned to *do*

math. With a *lot* of talking to him.² Anyway, one day, maybe halfway through what was now my second year of grad school, I said, “Wow, I get it! Eureka, aha,” or words like that. I’m sure I never found it either obvious or trivial, though, yes, crystal clear finally!

There are several interesting sequels here. Perhaps the reader should be informed that in that second year I switched from physics to math; and, who knows, “getting it” about δ and ϵ may well have been a part of that switch, though that behavior too was, like most important life choices, overdetermined. Another couple of years later Professor Cairns was invited to become the Illinois math department’s chairman; I was by then the apple of a lot of people’s eyes, and when I went to the party given for him, he saw me walking in and reacted with “What are *you* doing here?” I think I explained that I now understood about δ and ϵ !

But I never stopped wondering about it and eventually decided, while clinging to “ontogeny recapitulates phylogeny”, that sometime near the middle of the nineteenth century Cauchy figured out a way to tame infinity (years later, for my students, I worked this into a metaphor of a lion tamer handling a large and dangerous animal using nothing but a small wooden stool while inside the lion’s cage) and that this simple, crystal-clear definition in fact represents an enormous step in intellectual history: humanity’s victory over difficulties that baffled the Greeks, as we learn from Plato, not to mention church fathers puzzling over angels dancing on pinheads and infinite sums that could be added up and others that couldn’t. I’ve always felt that we, all of us Thinking Reeds, really held our collective breath from the time Plato’s Socrates had a ten-year-old slave boy prove $\sqrt{2}$ irrational and Zeno’s turtle beat Achilles until Cauchy, armed only with that wooden stool, tamed infinity. An intellectual development incubated for over two thousand years could well take me three.

Eventually I came to think my slowness was “developmental”: after I had a lot of children and watched who did what when, I saw that the “when” seemed to matter less and less compared to the “what”. (Maybe Ralph Fox knew something like that.) For a while I had wondered if perhaps my slowness might have to do with sex-linked traits, but two of my sons had similar experiences in junior-level analysis.

As a calculus teacher I was, with all this personal experience, always on the side of “wait and see”, though I never gave an A for a blank final, maybe because no such paper was ever turned in to me! A lot of students have experiences similar to mine in junior-level analysis, and of course many sail

right on through there and never find it hard. Them I’ve never really understood! What got me through and out the other side was *desire*—I cared and stayed in there, and one day I “got it”. I doubt whether anyone who hasn’t had to wait for that clear light really appreciates those two little words the way I do, but of course that may be envy speaking! I don’t think we can *teach* students such mysteries; time and, alas, patience and some will to learn (to power?) are what are needed. The New Math had to fail because it jumped students into logical simplicities, skipping over at least five hundred years of phylogeny.

This is becoming too long. Let me end with an example from my proverb collection: “If it isn’t logical, then it’s psychological!”

²Fifty years later, hindsight whispers that Bourgin may have been using the Moore method. (The writer regrets the mixing of metaphor.)