Alberto Pedro Calderón (1920–1998)

Michael Christ, Carlos E. Kenig, Cora Sadosky, and Guido Weiss

Alberto Pedro Calderón, who died April 16, 1998, was one of this century's leading mathematicians. His work (mainly in the field of mathematical analysis) was characterized by its tremendous originality and depth and its remarkable power. His contributions have been of extremely wide scope and have changed the way researchers approach, and think of, a wide variety of areas in both pure mathematics and its applications to science. His fundamental influence is felt strongly in abstract fields, such as harmonic analysis, partial differential equations, complex analysis, and geometry, as well as in more concrete areas, such as signal processing, geophysics, and tomography.

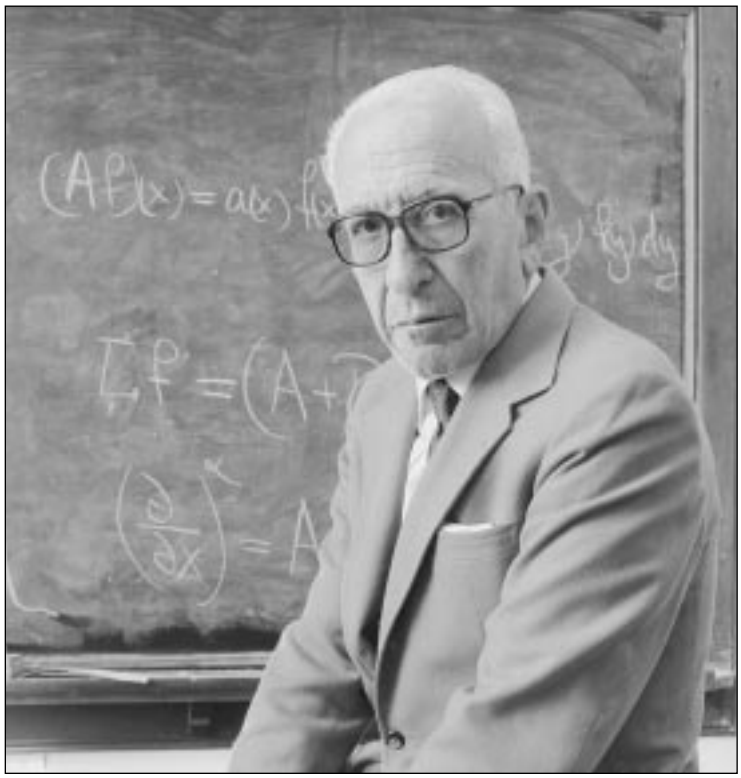
Calderón was born in Mendoza, Argentina, on September 14, 1920. He received his early education there and in Switzerland. His initial professional training was as a civil engineer at the University of Buenos Aires (graduated 1947), and he worked as an engineer for a few years. He simultaneously nurtured his passion for mathematics, partly under the guidance of Dr. Alberto Gonzalez Dominguez. Two events changed his future: His supervisor at YPF (the state-owned petroleum company) made his life very difficult, and around the same time, Antoni Zygmund, one of the world's leading mathematical analysts of the time and a professor at the University of Chicago, visited Argentina in 1948 at the invitation of Dr. Gonzalez Dominguez. Zygmund immediately recognized Calderón's brilliance, and he invited Calderón to come to Chicago to work with him. Calderón arrived in Chicago in 1949, as a Rockefeller Fellow, and by 1950 he had obtained his Ph.D. in mathematics under Zygmund's supervision. Calderón's dissertation was marvelous. In it he solved three separate and long-standing problems. From this point on, Calderón and Zygmund started one of the most successful collaborations in mathematical history. Together they created the modern theory of singular integrals, which has had enormous consequences for many areas of mathematics. They developed what has become known as the "Chicago school of analysis", one of the most influential forces in pure mathematics, which has also had a great impact on applications to science and engineering. Calderón went on to apply systematically the theory of singular integrals (and the important refinements that he obtained) to the study of partial differential equations. Calderón's contributions to their study have completely changed the landscape of that field. He not only solved fundamental specific problems but, in addition, developed a host of techniques that are now basic to the subject. Among his influential achievements were works on the boundary behavior of harmonic functions, ergodic theory, the Calderon-Zygmund decomposition, the real-variable theory of singular integral operators, complex interpolation, uniqueness in the Cauchy problem, boundary value problems for elliptic equations, commutators of operators having minimally regular coefficients, L^2 boundedness of pseudodifferential operators, real variable Hardy space theory, the Cauchy integral, and an inverse boundary problem in electrical prospection.

The biographical segment is adapted from an article written by Carlos Kenig for the British newspaper The Independent *(April 27, 1998).*

Besides his remarkable research accomplishments, Calderón was also a gifted teacher. During his career he taught at Ohio State University, MIT, the University of Buenos Aires, and the University of Chicago. He had many Ph.D. students, both in the U.S. and in Argentina. In Argentina he also served for several years as director of the Instituto Argentino de Matematica (IAM).

Calderón was recognized all over the world for his outstanding contributions to mathematics. He was a member of the National Academy of Sciences of the U.S., Argentina, Spain, and France; of the Latin American Academy of Sciences; of the Academy of Sciences of the Third World; and of the American Academy of Arts and Sciences. He received honorary doctorates from the University of Buenos Aires, the Technion (Israel), the Ohio State University, and the Universidad Autonoma de Madrid. He gave many invited addresses to universities and to learned societies, and he was awarded many prizes. Among these are the Bôcher Prize (1979) and the Steele Prize (1989) from the American Mathematical Society, and the Wolf Prize in Mathematics (1989) from Israel. In 1992 President Bush awarded him the National Medal of Science, the U.S.'s highest award for scientific achievement.

—Carlos E. Kenig



Photograph courtesy of the University of Chicago.

Alberto P. Calderón, September 1991.

Michael Christ

On a warm Chicago afternoon in the late summer of 1977 a class of new graduate students awaited their first lecture on real analysis. With naive curiosity I awaited the appearance of Professor Zygmund, author of the fattest mathematics book I had yet encountered. Instead, a distinguished-looking gentleman entered and quietly announced, "I am Alberto Calderón," substituting for Zygmund. The simple greeting still resonates in memory; its tone was not that of a teacher addressing a class, but of a man addressing colleagues.

Later, having demonstrated a disinclination towards algebra and disintuition towards geometry, I gravitated towards analysis and was urged by R. Fefferman to attend Professor Calderón's lectures. These treated primarily his own work: complex interpolation, the Cauchy integral, the commutators, the real variable theory of parabolic Hardy spaces, boundary value problems for elliptic PDE, algebras of pseudodifferential/singular integral operators with low regularity coefficients. Related work of others, such as R. Coifman and Y. Meyer, was also presented. Theorems and full details of proofs were given, with only occasional motivation and no editorializing. While Calderón was both architect and bricklayer, his lectures emphasized the bricks. The pace was decidedly slow; the thoughts of a young student wandered.

Rarely had he visible lecture notes. During one memorable long stretch the notes consisted solely of his four-page paper on the Cauchy integral, carried in an inside coat pocket and seldom consulted. The lectures were clear yet unpolished, with occasional retreats and emendations. Once in a great while the argument would founder. An irked but calm Calderón, along with the audience, would seek to bridge the gap. When one such breakdown led to a spirited discussion among Calderón, W. Beckner, and P. W. Jones, I finally understood: the lectures were planned in barest outline. Calderón was rethinking the theorems on the blackboard before us; we were expected to think along with him. Much later he confirmed this, explaining that meticulous preparation early in his career had produced lectures too rapid for his audience; he had resolved to be understood despite the occasional blow to his pride.

Those were glorious days for analysis in Chicago. Calderón's magnetism had attracted a remarkable group of young visitors, postdocs, and junior faculty, including Beckner, R. Fefferman, D. Geller, S. Janson, D. Jerison, Jones, R. Latter, P. Tomas, A. Uchiyama, D. Ullrich, M. Wilson, and T. Wolff. There were exciting lectures by Coifman, C. Fefferman, J. Garnett, Meyer, J. J. Kohn, and others and a lecture course by Zygmund. Spectacular developments included the decisive work of Coifman,

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A. McIntosh, and Meyer on the Cauchy integral; the arrival of a photocopied letter from A. B. Aleksandrov on inner functions; Uchiyama's constructive decomposition of BMO; G. David's dissertation; and the work of S. Bell and E. Ligocka on biholomorphic mappings. Calderón rarely spoke out in seminars, but in private conversation afterwards he sometimes revealed thoughts which went well beyond the lecture.

After auditioning in an oral examination, I asked for a dissertation problem. Calderón suggested the boundedness of the Cauchy integral on Lipschitz curves of large constant—long the main focus of his own research. While I half-listened in shock, he generously shared an idea for a line of attack, offered encouragement, and promised an alternative problem if I made no headway, as indeed I did not. The shock was unwarranted, for this merely illustrated both the attitude of genuine respect with which Professor Calderón invariably treated others and his concentration on the most fundamental problems. To me he later mentioned two other potential research topics: the restriction of the Fourier transform to curved submanifolds of $\mathbf{R}^n$, and L^p estimates for solutions of subelliptic partial differential equations. Today both remain major, fundamental open problems, despite the fascinating results obtained by many investigators.

Calderón's high standards for himself prevented some of his work from ever seeing the light of day. He once asked about the $\bar{\partial}$-Neumann problem, explaining that he had obtained results different from those he had found in the literature. His influential paper on an inverse boundary problem in electrostatics apparently languished for decades before finally being published.

Calderón rarely offered advice. Perhaps he considered it a presumption, feeling that others should be left free to attack problems on their own terms, just as he himself wished to be. Once, unable to supply even a single background reference for a problem he suggested, he apologetically explained that studying the literature could be confining; he felt more likely to find original ideas by working independently in complete freedom.

Despite his rigorous personal standards, Professor Calderón encouraged a young student struggling to find himself. After suggesting a problem and offering an initial suggestion, he allowed me to work in complete independence, but was genuinely pleased to hear reports of even minor progress. A long and sometimes chaotic presentation of a dissertation was endured with unflagging courtesy.

I was too overawed and too naturally reticent to glean more than rare glimpses of his personal life. Introducing his son, Pablo, he glowed with simple pride. The death of his first wife was, in his words, a terrible blow; for a time it was difficult to continue to work. Years later, resurrected in the company of Alexandra Bellow, Calderón was relaxed and full of good humor.

In his lectures Professor Calderón taught one to work with the bricks and mortar and to appreciate their beauty. But in his quiet way, by example through his own life, he taught deeper lessons.

Carlos E. Kenig

I was one of Alberto Calderón's graduate students at the University of Chicago from 1975 to 1978. This was a period of intense mathematical activity. During the 1976 Christmas break Calderón obtained his remarkable result on the boundedness of the Cauchy integral for Lipschitz curves with small constant. (The general case was obtained by Coifman-McIntosh-Meyer in 1981.) As soon as classes started in the winter quarter, I went to see Calderón in his office, where he was explaining his proof to Bill Beckner. There was real excitement in the air, which even I, a mere graduate student, could feel. Soon after, the annual meeting of the AMS took place in St. Louis in the midst of a terrifying cold spell and a terrible winter storm. In connection with the AMS meeting there was a conference in harmonic analysis at Washington University, the very first conference I attended. At this conference Calderón explained his proof with his usual elegance. One could also sense his pleasure in having finally made a dent in this problem, which he had thought about for so long. This work opened up entire new vistas of research, which are still being explored.

Shortly after our return from St. Louis, I asked Calderón for a thesis problem. His response was this: Find a problem yourself, and let's discuss it afterwards. Fortunately for me, the recent work on the Cauchy integral had opened up many new possibilities. I chose to explore the theory of Hardy spaces on Lipschitz domains and went on to obtain my degree in 1978.

Calderón was a mathematician of deep and original insights and also of great generosity with both his ideas and his time. I count myself as extremely fortunate to have been his student, especially at such a highly significant moment—an experience that greatly influenced much of my ensuing research.

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Cora Sadosky

Professor Antoni Zygmund visited Buenos Aires twice, in 1948 and in 1959. The first visit, when he met Calderón, shaped the development of real analysis for the following fifty years; the second one shaped my life. In 1948 Zygmund and Calderón started what was to become one of the most influential partnerships in mathematical analysis. Ten years later Zygmund returned to Argentina to help build a mathematical school in a land where he knew mathematical talent flourished. Calderón also started to make periodic visits to the University of Buenos Aires, where I was then an undergraduate. It was at that time that I became one of the first students of both Calderón and Zygmund. Two years later I arrived at the University of Chicago to pursue a doctorate, with Calderón as dissertation advisor and under the close supervision of Professor Zygmund.

What a privilege it was! Although I had missed Calderón's seminal course on singular integrals and its applications to hyperbolic PDE in Buenos Aires, by the time I arrived in Chicago I had already been the sole beneficiary of a course on his new theory of interpolation of operators, later published in *Studia Mathematica*. The final articles were difficult and dense, but his lectures and the notes I took from his course were crystalline. The extraordinary opportunity of discussing ideas in the making with such a profoundly original mathematician was a unique gift. At the time I did not understand, and therefore failed to appreciate fully, how unusual Calderón's openness was, and I marvel now in retrospect. I think this was one of his most remarkable traits of character: he would talk mathematics openly, sharing freely all of his thoughts, ideas, and insights.

During my years at Chicago we had long mathematical talks. Unfortunately I was too stubborn and inexperienced to pay as much attention as I should have. For instance, when Atiyah and Singer proved the index theorem, Calderón was quite excited, but told me he did not grasp the proof. His usual way to grasp a proof was to work another one for himself, so he told me he was interrupting research to study algebraic topology and advised me to join him. I did not, giving priority to my exams and losing a great opportunity to study alongside him. After a few months he announced happily that he could resume work, having understood the index theorem!

When I started on my thesis project, I met weekly with Professor Zygmund to report on my work, but I also talked with Calderón almost daily on our way home from Eckhart Hall. Many a time I was invited to stay at his home for dinner, and while I helped his wife, Mabel, to set the table, he played the piano. We shared a delight in Mozart, and after dinner sometimes he played some more for me. Other times I joined his children, Pachita and Pablo, in the basement to watch Calderón work very seriously with a large setup of electric trains he had given Pablo. He was an eager engineer and became totally absorbed in the task of constructing and managing the intricate model railway.

I was not tempted by the dissertation problem proposed by my great teachers (quite foolish of me, since it was interesting enough to be developed by themselves later) because I was obsessed with parabolic singular integrals, which seemed the natural object to study after Calderón's success with elliptic and hyperbolic PDEs. Calderón encouraged me in that interest, and, as the problem was in the air, very soon afterwards a first paper on the subject appeared by B. Frank Jones. This did not discourage me, since I came up with a notion of principal value for the integral through a nonisotropic distance, an idea which Calderón thought was "the right one". In 1963–64 he left Chicago for a sabbatical year, partly spent in Argentina, and I joined him for a three-month period at the Instituto Balseiro in Bariloche. I completed there the research for my thesis, while in the evenings Calderón, his lifelong friend Alberto Gonzalez Dominguez, and

Ph.D. Students of Alberto Calderón

Robert T. Seeley, MIT (1958)
Irwin S. Bernstein, MIT (1959)
I. Norman Katz, MIT (1959)
Jerome H. Neuwirth, MIT (1959)
Earl Robert Berkson, Chicago (1960)
Evelio Oklander, Chicago (1964)
Cora S. Sadosky, Chicago (1965)
Stephen Vagi, Chicago (1965)
Umberto Neri, Chicago (1966)
John C. Polking, Chicago (1966)
Nestor Marcelo Rivière, Chicago (1966)
Carlos Segovia-Fernandez, Chicago (1967)
Miguel S. J. de Guzman, Chicago (1968)
Daniel Fife, Chicago (1968)
Alberto Torchinsky, Chicago (1971)
Keith W. Powls, Chicago (1972)
Josefina Dolores Alvarez Alonso, Buenos Aires (1976)
Telma Caputti, Buenos Aires (1976)
Robert Richard Reitano, MIT (1976)
Carlos E. Kenig, Chicago (1978)
Angel Bartolome Gatto, Buenos Aires (1979)
Cristian Enrique Gutierrez, Buenos Aires (1979)
Kent G. Merryfield, Chicago (1980)
F. Michael Christ, Chicago (1982)
Gerald M. Cohen, Chicago (1982)
Maria Amelia Muschietti, Buenos Aires (1984)
Marta Susana Urciolo, Buenos Aires (1985)

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François Treves tried, mostly in vain, to teach me how to play billiards. There, through C-Z correspondence, we found out that Zygmund had assigned one of his students, Eugene Fabes, a problem close to mine and that we had both proved the pointwise convergence of parabolic singular integrals (by different methods)! Panic struck; Calderón defended my priority on the problem, but all was solved amicably, and upon my return Gene and I wrote our first result as a joint paper. Shortly afterwards I defended my thesis and left for Argentina, while Gene started a fruitful collaboration with Nestor Rivière, another Argentine student of Calderón, who had been an eager listener of our first results and who later became key to the development of the subject.

What a happy time that had been! I returned to Buenos Aires, leaving behind an ambience I cherished and some very interesting problems on parabolic maximal functions Calderón had suggested for working together. A loss to me, but not to mathematics, since those problems were successfully solved by Calderón and Alberto Torchinsky, another Argentine student of Calderón, who came to Chicago later. In the meantime, Calderón had directed the thesis of Carlos Segovia, one of the students selected by Zygmund in Buenos Aires, who is now a professor there. While Calderón also had, in later years, several doctoral students in Buenos Aires, the majority of his Argentine students received their Ph.D.s from the University of Chicago. Although both Calderón and Zygmund devoted themselves to strengthening analysis in Argentina and later in Spain, the results of their efforts, due to political and other circumstances, were very different in the two countries. Nowadays only one of Calderón's Argentine students is on the faculty of the University of Buenos Aires.

After graduation I did not hesitate to go back home, since the opportunities for research and teaching in Argentina were good. The flourishing of intellectual life under democracy, however, lasted only one more year. In 1966, after a bloodless military coup, the School of Sciences of the University of Buenos Aires was brutally attacked by the police, four hundred faculty members left, and our scientific dreams were shattered. In the following years tolerance decreased as military repression increased. Unable to find another academic job in Argentina, I was forced out of mathematics for some years, and to return to it I had to leave the country. In the meanwhile, Calderón's family had settled in Buenos Aires, where he stayed for longer periods after the onset of his wife's eventually fatal illness. For several years he was director of the IAM (Argentine Institute of Mathematics). In these years we had hardly any contact.

The circumstances of Argentina changed for the better in the mid-1980s, and we met again in Buenos Aires, but for a time we did not know how to renew our friendship. Then Calderón found a way in the understated mode so typical of him. One day at his IAM office he handed me a cassette, saying, "This is some of the Mozart you used to love when I played it years ago, only better played. I copied it for you." We were friends again. That gave me the joy of sharing some of the wonderful moments of his last years, when he basked in the happiness of being with Alexandra, his second wife.

Alberto Calderón was a very unassuming man of natural charm, a person of great elegance and restraint, and wonderful company. Mathematically Calderón was exceptional not only for the strength of his talent but for his peculiar way of grasping mathematics. He redid whole theories by himself, got to the core of what he wanted to know by himself, found always his own way. His ideas and the methods he developed were always extremely original and powerful. Although he was an individualist to the core, he influenced profoundly the work of others, who developed what is known as the "Calderón program". He shared his knowledge freely with his students, yet did not closely follow their careers. Calderón was modest, sure of himself, and quite indifferent to competition. He was always happy to have been an engineer and conserved a real interest in applications. In one of our last conversations he told me how intrigued he was that his work was perceived to be in the foundation of wavelet theory. I think this pleased Calderón very much.

Guido Weiss

I began my graduate studies in mathematics in the early 1950s and wrote my Ph.D. thesis under the direction of Antoni Zygmund. The graduate program offered by the University of Chicago was excellent. But I really learned most by attending and participating in the legendary "Zygmund seminar". It was there that I learned the various topics in harmonic analysis that formed the basis of what is now known as the "Calderón-Zygmund School". Alberto Calderón had received his Ph.D. at the University of Chicago before I started my studies with Zygmund and had left to join the faculty at Ohio State University and, a short time later, at MIT. I felt his presence at the University of Chicago, however, in practically every session of the Zygmund seminar. He had made important contributions in each topic we discussed: interpolation of operators, potential theory and the boundary behavior of harmonic functions, ergodic theory, and, of course, singular integrals. I was surrounded by not only a remarkable faculty but also by a large number of

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very talented graduate student colleagues who have by now made important contributions in mathematics. I could not help, however, having a feeling of great awe for this individual who was capable of making so many important contributions in such a large number of topics. As I stated above, he was an ever present participant of the Zygmund seminar.

I first met Alberto shortly before I finished my thesis in the mid-1950s. He visited the University of Chicago, and after presenting a beautiful talk on singular integrals and their connection with partial differential equations, Zygmund, the young group that was working with him, and Alberto met in Zygmund's office and discussed various topics in harmonic analysis. I remember vividly the impressions Alberto made on me. He had unique insights and ways of looking into the various subjects we discussed and in a very friendly, open, and generous way shared this knowledge with us. I began to realize what was his main strength: he made special efforts to reduce each concept he considered to clear, simple components, and from this understanding he was able to arrive at methods for solving a problem that often were applicable to many other fields in mathematics. His considerable contributions in analysis are being described elsewhere in these *Notices*; I will not try to give such a description. I do want to emphasize, however, this important feature that is ever present in his research: The methods he discovered often go way beyond the results he obtained. Ideas he used in his study of the interpolation of operators have had an important impact in fields that seem totally different. One of the important equations that characterize wavelets, for example, is really a discrete version of what is known as the "Calderón Reproducing Formula".

Since this first meeting I have had several opportunities to be with Alberto, and on each occasion all the feelings I described above were strengthened. I was asked by the mathematics department of the University of Chicago to give the "Zygmund lectures", and I presented them in March of this year. Recently I became interested in the mathematical theory of wavelets and chose this topic for the three lectures I gave. Alberto attended these lectures and expressed an interest in the subject; in particular, he asked me to send him what I had written. I was very excited at the prospect of establishing an area of common interest with him. I obtained a copy of a book on wavelets I had recently coauthored with Eugenio Hernández and put it together with a collection of papers I thought might interest Alberto, but then I heard the sad news of his passing away. Along with many others, I will miss him very much.

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Mathematical Physics

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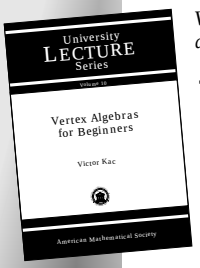
Victor Kac, *Massachusetts Institute of Technology, Cambridge*



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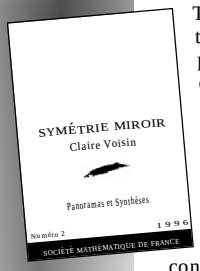
Claire Voisin



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