Book Review

Paul Dirac: The Man and His Work

Reviewed by Clifford H. Taubes

Review of Paul Dirac: The Man and His Work
Abraham Pais, Maurice Jacob, David I. Olive, and Michael F. Atiyah
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This review concerns the book *Paul Dirac: The Man and His Work*, which consists of a collection of four essays on the life and work of Paul Dirac. The book also contains a short address by Stephen Hawking on the occasion of the dedication of a plaque at Westminster Abbey in honor of Dirac. The essayists are Abraham Pais (a physicist and historian of physics), Maurice Jacob (a physicist), David Olive (a mathematical physicist), and Sir Michael Atiyah. Peter Goddard served as the book's editor. The book is short and wonderful.

The essays describe Dirac's influence in both physics and mathematics. The list of Dirac's fundamental ideas is simply stunning. One might ask which physicists of the twentieth century will be remembered in the thirtieth? Surely Dirac and Einstein and perhaps a few more. In any event, Dirac is truly one of the heroes of twentieth-century physics. The essay by Pais consists of a concise biography of Dirac, and so describes Dirac's most profound scientific achievements. Pais's essay is full of charming quotes from and about Dirac, who was an iconoclast, to say the least.

Dirac is most famous for his celebrated differential equation, which revolutionized both physics and mathematics. This equation is simplest in the case where the space in question is the circle parameterized by an angle t between 0 and π. In this context the Dirac equation is defined by the operator

Clifford H. Taubes is professor of mathematics at Harvard University. His e-mail address is chtaubes@math.harvard.edu.

$$D = i\frac{d}{dt}$$

on complex valued functions. In higher dimensions the analogous operator D acts on certain vectors of complex valued functions, the ever mysterious spinors. Here are D's salient features: This operator is first order, it is symmetric, and its square is minus the standard Laplacian. In a Lorentz signature metric, the square is the wave operator.

In physics the operator D was used by Dirac to predict the behavior of relativistic electrons, and it led directly to Dirac's prediction of the existence of antiparticles. (I will explain this below.) These days the Dirac operator and the notion of an antiparticle are fundamental to physics's description of all elementary, spin 1/2 particles. Thus, Dirac's equation profoundly changed humanity's view of physical reality. Jacob's essay describes the influence in modern physics of Dirac's equation and his ideas about antiparticles.

In mathematics Dirac's operator lies at the very heart of a great deal of late twentieth-century geometry; moreover, it still regularly delivers profound surprises (as this writer can avidly attest). And the Dirac operator promises to deliver surprises into the twenty-first century. The Dirac operator bewitches more than just analysts, because some version can be written down on any Riemannian (in fact, conformal) manifold. In this context it is completely natural yet still mysterious. The Dirac operator sees something very deep about manifolds, and the nature of its vision is still beyond our ken. Atiyah's essay sketches the mathematics behind Dirac's operator while describing some of its mathematical incarnations and applications.

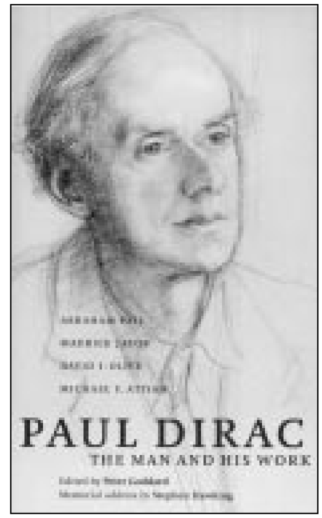
Less well known to mathematicians is Dirac's work on magnetic monopoles. These are hypothetical particles which consist of a single magnetic pole, solely "north" or solely "south". Their existence restores to Maxwell's equations for electricity and magnetism a fundamental symmetry which is lost in the presence of charges and currents. (Modulo a sign, this symmetry interchanges the electric and magnetic fields.) In spite of some serious searching (in laboratories, on earth, and in space), no monopoles have been found to date. In any event, Dirac pointed out that the fact that all electric charges are integer multiples of a single constant is a direct consequence of the existence in this universe of a single monopole. There are those who speculate that Dirac's work on magnetic monopoles will prove as central to twenty-first century geometry as the Dirac equation is to geometry of today. The essay by Olive describes Dirac's work on monopoles and its current remarkable incarnations in certain supersymmetric field theories.

The essays of both Olive and Atiyah sparkle to the eyes, and I finished each wishing for more. Meanwhile, Pais's essay was entertaining and very touching, for I still have heroes and Dirac is one of them. Finally, Jacob's lecture was fascinating, but probably dry without prior knowledge of the physics or, at minimum, the mathematics of antiparticles. For those lacking this knowledge, what follows is a brief introduction to the subject.

Roughly the antiparticle story is as follows. In the absence of interparticle forces, the dynamics of a single electron moving in space is controlled by the Dirac operator in the following sense: The possible states of the electron at a given time t are described by a spinor (which is to say a vector of functions on which the Dirac operator can act) whose absolute value squared integrates to one. Then the function which is the square of the absolute value of the spinor is meant to be the probability distribution for finding the electron at points in space.

Here is how dynamics enters the story: Having described the electron state at time t by a certain vector of functions whose norm square integrates to one, one can obtain the state at any later time from the original via a certain unitary action of the group of translations on the line. The key point here is that the generator of this unitary action is supposed to be the operator D. In particular, the eigenvalues of D are to be interpreted as the allowed energies of electrons whose probability distributions are time independent.

So far so good, except that D's spectrum is not bounded from below. For example, the eigenvalues of D for the case where space is the 1-dimensional unit radius circle have the form n where n can be any integer. The fact that D has accessible states which have arbitrarily negative eigenvalues renders D useless for the description of our universe. Indeed, such a universe would not last long, since the slightest generic perturbation would send all the electrons pathologically cascading ever deeper into the negative energy states.



Of course, Dirac was aware of the negative energy problem. But still, his D explained various major paradoxes about the behavior of electrons, and so he was not quick to abandon it. And after a time Dirac resolved the negative energy problem, and this resolution leads unavoidably to the antielectron. (This particle is now called the "positron".) To explain, understand first that the resolution invokes an extra feature of electrons that was well known to Dirac but not yet introduced here. This feature is the "exclusion principle", which was first enunciated by Pauli and which leads to the explanation of chemistry. The exclusion principle asserts that no two electrons can occupy the same quantum state. (Chemistry appears here for the following reason: The chemical properties of an atom are essentially determined by the states of its orbital electrons. Meanwhile, the exclusion principle prevents all but one of the electrons in an atom from occupying any given orbital state. Thus, atoms with different numbers of electrons must have different chemical properties.)

With the exclusion principle understood, Dirac made the audacious proposal that essentially all of the negative energy states in the universe are already filled with electrons. Then, Pauli's exclusion principle makes these states inaccessible to the remaining electrons, and the negative energy instability disappears. Of course, in making such a proposal Dirac was forced to consider the observational consequences of the occupation of essentially all of the negative energy states. What Dirac realized was that this assumption is essentially unobservable when all negative energy states are filled. Moreover, an empty negative energy state is essentially indistinguishable from a particular positive energy state which is filled by the antiparticle. The idea here is that a missing negative charge is indistinguishable from a present positive charge. (Since interparticle interactions would unavoidably knock electrons from filled negative energy states into positive energy states, Dirac was forced to consider the observational consequences of a relatively small number of negative energy states.)

The filling of the negative energy states led Dirac to the antielectron. Remarkably enough, the first observations of the particle occurred only a few years later.

As I remarked at the outset, this book is quite short; it is not a text to learn a great deal about either Dirac's physics or math, or even about his life. Rather, this book is a small gem of an introduction to all three of these subjects.