The Nature of Space and Time

Reviewed by Claude LeBrun

The Nature of Space and Time
Stephen Hawking and Roger Penrose
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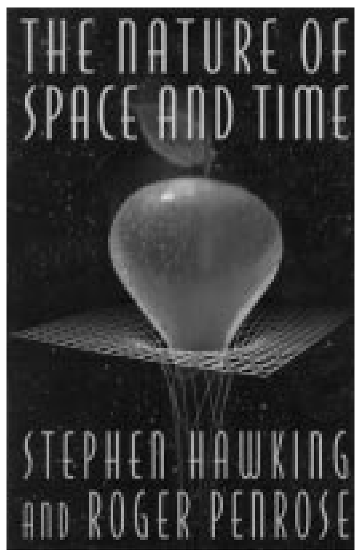
On May 9, 1994, a motley crowd packed the main lecture hall of the Newton Institute in Cambridge, England. Renowned physicists and mathematicians, often accompanied by their brightest graduate students, were sprinkled through the crowd. Around them pushed a mob of intellectual tourists—"town and gown"—spilling into the foyer and out the side doors of the Institute. Television cameramen waded through the crowd, blinding the audience with klieg lights and littering the floor with a spaghetti of cables. Somehow the culmination of a series of semitechnical lectures on the foundations of physics was beginning to look like a media circus.

The occasion was a public debate between Stephen Hawking and Roger Penrose, Fellows of the Royal Society and joint recipients of the Wolf Prize. For the tourists, of course, these credentials were as irrelevant as the identity of the moderator (Michael Atiyah). After all, both participants were bona fide celebrities: best-selling authors [5, 12, 13] and hosts of popular television documentaries. One has become a sort of pop culture icon: the wheelchair-bound genius with the synthetic voice. The other's ideas have quite literally become the intellectual wallpaper of our age.



This brief, charming book faithfully records the three lectures given by each man, as well as the debate which followed. The lectures attempted to be nominally intelligible to advanced Cambridge undergraduates and primarily sought to convey a few of the key ideas of the subject, along with some sense of the intellectual excitement inherent to the field. Intellectual tourists are by all means welcome on this journey, and indeed it is they who may find the trip most exhilarating. The style of the lectures was informal and highly personal. It is therefore high time that the lecturers be properly introduced.

Roger Penrose, an algebraic geometer by training, revolutionized general relativity in the 1960s by proving [9] the stability of black-hole singularities, using techniques of global differential geometry. Even earlier he had begun [8] to systematically develop the theory of spinors on 4-manifolds [14]. This ultimately led to a remarkable set of links, collectively known as twistor theory, between 4-dimensional geometry and complex manifold theory

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[11]. As Rouse-Ball Professor of Mathematics at Oxford, Penrose became a tireless advocate of the special nature of dimension four. Armed with results on Yang-Mills moduli spaces proved by twistor methods, he thereby helped lay the foundations for the revolutionary work of Donaldson.

Visualizable mathematics is Penrose's lifelong obsession. As the present book will attest, his geometric visions have altered the face of gravitational physics, but their influence by no means stops there. In his twenties Penrose invented the perspectival paradox exploited by M. C. Escher in *Ascending and Descending*, and in his forties he invented his eponymous nonperiodic tilings of the plane. Like those of many other great geometers, Penrose's discoveries have presented themselves as revelations rather than as syllogisms, leaving him convinced—as he argues in two engaging but controversial bestsellers [12, 13]—that human thought differs fundamentally from the algorithmic processes studied by present-day computer science.

Stephen Hawking, who is a decade younger than Penrose, first made his mark [2] by applying Penrose's differential-geometric insights to the theory of the big bang. This occurred while he was still a graduate student and not long after he had been diagnosed as suffering from amyotrophic lateral sclerosis, a rare (and typically fatal) degenerative disease of the nervous system. Walking with a cane, Hawking began a collaboration with Penrose in which the two eventually proved [7, 6, 10] that generic solutions of Einstein's equations must be geodesically incomplete. By simply surviving to see these papers published, Hawking defied all the predictions of his physicians. Astonishingly, however, his best work was still to be done—albeit in a motorized wheelchair. Intrigued by analogies between thermodynamics and the behavior of black holes, he discovered, to the shock and disbelief of his colleagues, that quantum mechanics predicts that a black hole cannot be black at all, but must emit thermal radiation at a temperature proportional to its surface gravity [3].

While this phenomenon, now known as *Hawking radiation*, was initially described in terms of conventional pseudo-Riemannian space-time, Hawking soon discovered that the temperature of a nonrotating black hole could be succinctly understood in terms of the geometry of an associated (positive-definite) Riemannian 4-manifold obtained by analytic continuation. As Lucasian Professor of Mathematics at Cambridge, Hawking has dedicated much of his time to the pursuit of this intriguing lead. By doing so, Hawking, like Penrose, has adopted Einstein's belief that ultimate explanations should be geometric in character—although this, in part, is a matter of necessity for Hawking, whose medical condition long ago deprived him of the use of pencil and paper. Whatever the physical merits of Hawking's Riemannian approach to quantum gravity [1, 4], it has, in any case, had a profound impact on pure mathematics.

The lecture series began with a pair of talks on classical general relativity, developed from the point of view of causal structures [6, 10]. Hawking's lecture was primarily aimed at the theory of black-hole event horizons, while Penrose's was primarily dedicated to the differential geometry of the big bang. The degree to which the two men's points of view have influenced each other is strikingly illustrated here; after all, it was Penrose who first used these techniques to study black holes, and Hawking who first used them to study the big bang. Already, however, their fundamental disagreements were beginning to surface. For Hawking the big bang might as well be considered a time-reversed black hole. For Penrose these two entail wildly different curvature singularities, and this is precisely what ultimately distinguishes the past from the future.

The second pair of lectures dealt with quantum theory in curved space-time. Hawking's talk began with a clear discussion of the connection between the temperature of a static black hole and the geometry of the Riemannian analog of the Schwarzschild metric and concluded with a highly speculative theory of the virtual creation and annihilation of pairs of black holes. Penrose instead chose to discuss the measurement problem in quantum mechanics and proposed that the collapse of the wave function might be an objective physical process imposed by the nonlinear nature of Einstein's equations. Both lectures make for engaging reading, but in both cases the reader is apt to have the uneasy feeling that we are no longer on solid ground.

The final pair of lectures asked how one might apply quantum theory to the universe as a whole. Hawking's proposal was that one should first build a quantum theory of Riemannian metrics on compact 4-manifolds and then analytically continue the answers to provide transition amplitudes for our approximately pseudo-Riemannian world. Penrose's point of view instead focused on the crucial role played by complex numbers in quantum mechanics and connected this with the ways in which 4-dimensional geometry may be given a complex-manifold interpretation via twistor theory.

As the lectures progressed, the level of disagreement between the participants became more pronounced, and it was all too appropriate that the series should end with a systematic airing of differences. The final debate provided a suitable forum, although what transpired was actually less a debate than an exchange of critiques, both of which were substantially on target. While the reader may thus come away with the conviction that nothing has fundamentally been settled by this exchange of views, it is nonetheless laudable that the authors have shown the courage to openly discuss foundational difficulties of modern physics, which are usually passed over in embarrassed silence. It remains to be seen, however, whether these difficulties are susceptible to direct attack or whether their resolution must await the arrival of revolutionary new ideas from some unexpected quarter.

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