

The Universe and the Teacup

Reviewed by Daniel Rockmore

**The Universe and the Teacup:
The Mathematics of Truth and Beauty**

K. C. Cole

Harcourt Brace, 1998

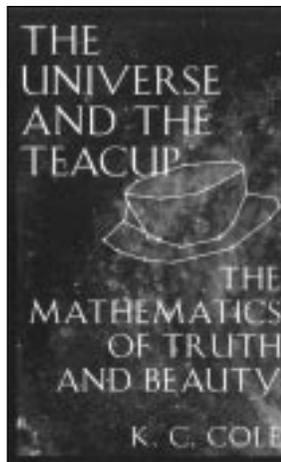
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One of my favorite parts of the Passover Seder is the retelling of a story in which a father discusses how to respond to his four sons as they each ask him in their own way about the meaning of the Seder. The sons are of four types: one wise, one wicked, one simple, and one who does not even know how to ask a question. The parable is as much about how to pose a question as it is about how to frame an answer; and as a teacher and researcher, someone whose life seems to be spent asking and answering questions, I always feel a special resonance with this story. It is not too hard to see four kinds of students in the sons: one self-motivated, one recalcitrant, one earnest but confused, and the last, the student who feels so disconnected from the subject that he or she wouldn't even know how to begin thinking about it. Each of these students benefits from a different approach, and the last of them, the one who does not even know how to ask a question, presents a special challenge, particularly for mathematics.

How do we bring such a person to the point where he or she might begin to think about asking a question? How do we tell a story of mathematics that feels relevant and universal? If our listener

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feels no connection, declarations of the beauty of the subject ring hollow; appeals to technology, while impressive, still often miss the mark, being more about the product than the mathematics that underlies it. So, in short, how do we mediate the experience of mathematics in such a way that it speaks to everyone and not just to the scientist? Analogous questions have been

asked and answered in other sciences, most notably biology and physics. Mathematicians need to do this too, for this is the first step on the road to public appreciation. We have a good story to tell, and furthermore, like any good story, it will teach as much to those who tell it as to those who listen to it.

K. C. Cole's recent book *The Universe and the Teacup: The Mathematics of Truth and Beauty* is a book for those who do not even know how to ask a question about mathematics. Ms. Cole is a science writer for the *Los Angeles Times* who often writes on mathematics. By her own admission, her point of view is that of a writer who started out interested in social issues, and these initial conditions have led her to write a book which is "an attempt to demonstrate how mathematics informs the kinds of questions people really think and worry about." She goes on to say that "If I could accomplish one thing in this book, it would be to

show that an interest in the quality of life is in no way diminished by quantitative arguments...scientists and mathematicians, as well as saints and philosophers, search for the fundamental hows and whys of existence. And although they have different standards and proof, quantitative insights do help us understand qualitative problems.”

Ms. Cole understands that “mathematics is not about numbers so much as it is a way of thinking, a way of framing questions that allows us to turn things inside out and upside down to get a better sense of their true nature...a way of thinking that helps make muddy relationships clear.” In particular, she wants to explore what happens when this approach is trained on the problem of defining things that we all care about. Sometimes these things are the fodder of everyday concerns like health and safety, other times they are larger issues like justice, truth, and beauty. In any case, mathematics both reveals and is revealed by aspects of these universal concerns and so gives a toehold to everyman, allowing the uninitiated to see what was previously hidden and giving the professional a new perspective. It is an ambitious goal, and along the way she meets with varying degrees of success and failure, but the trip is well worth taking.

The book is divided into four parts. Part I touches on “innumeracy”, the problems that so many of us have in understanding the daily barrage of numbers and statistics in such a way that we can incorporate them into our personal decision-making processes. A chapter called “Calculated Risk” gives a very interesting discussion of the gulf that can occur between mathematical estimates of risk and intelligent behavior. Included here is a summary of the famous Kahneman-Tversky experiments on risk aversion, just one of the many nuggets of science not often found in an average mathematical education.

To quantify something is to measure it, and Part II devotes itself to various aspects of the notion of measurement. Problems of measurement lead naturally to both a discussion of the misuse of statistics (which reappears at regular intervals) as well as a little quantum mechanics and the nature of uncertainty. The latter leads to a discussion of scale (where we learn why a 50-foot human is impossible and flea circuses are possible), and from scale to a discussion of emergent behavior. At what point does emergent behavior emerge? Thus are we brought to the dual problems of prediction and detection.

Throughout, technical concepts are brought to life by analogy with real experience. For example, predicting the out-

come of a single coin toss or a thousand coin tosses is juxtaposed with predicting a single individual’s actions and the behavior of a crowd. Phase transitions are compared with “tipping points”, the point in time at which an accumulation of small quantitative changes in behavior

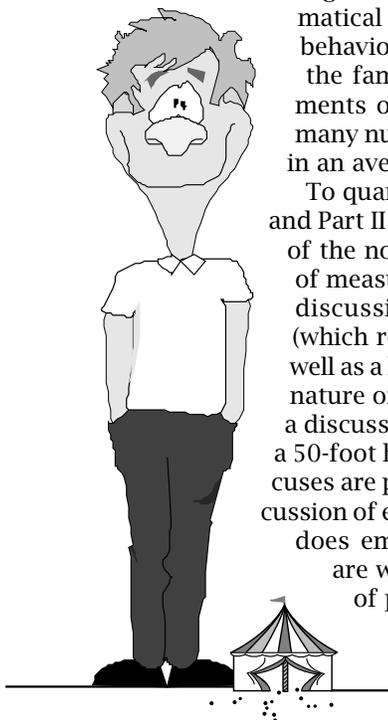


cause huge qualitative differences. Filtering describes both the search for a new particle and our ability to concentrate in an office abuzz with the hum of a computer. Cole is at her best in presenting and reflecting on connections like these. At times the example-driven nature of the writing makes these sections feel more like a discussion of physics rather than mathematics, but as mathematicians know, and as

Cole points out, “the tools of mathematics allow one to see otherwise invisible patterns and connections.” Without the examples we can’t see the connections.

Part III, “Interpreting the Social World”, is, in my eyes, the most successful part of the book. The three chapters which constitute it consider problems of fairness and justice, as reflected in the mathematics of voting theory (Arrow’s Impossibility Theorem), The Prisoner’s Dilemma, the problem of fair division, and variations on Axelrod’s famous “Tit for Tat” experiments. They all make for fascinating reading, and I would venture to say that none of these topics makes it into the usual mathematics curriculum.

Finally, in Part IV, Cole tackles the subjects of truth and beauty as understood in terms of probability and statistics, invariance and symmetry. She writes that “...maybe it is a bit pretentious to talk about the mathematics of truth. But mathematics offers some powerful ways to get at least closer to the truth—and these methods are in use almost everywhere we look, although most people are not aware of them.” These methods mainly include the role of probability and statistics in helping to understand cause and effect, coincidence, and the difference between correlation and causation. These are delicate problems, and while these techniques can bring you “closer to the truth,” often they do not furnish the whole truth. History is rife with stories of the misuse and misunderstanding of statistics, and Cole includes several cautionary examples in this regard. In a separate and very interesting chapter, mathematical truth is compared and contrasted with legal and



scientific truth, as referees are compared to juries, and juries are compared to repeated experiment.

The discussion of symmetry and invariance is left to the last chapter of the book, “Emmy and Albert: The Unvarying Nature of Truth”, which revolves around Emmy Noether’s work on general relativity. This is also Ms. Cole’s favorite chapter of the book. The connections between symmetry and beauty are a well-trodden area, with Hermann Weyl’s *Symmetry* the classic reference. Ms. Cole sees invariance and symmetry as a way to get from truth to beauty, adding that “...deep truths can be defined as invariants—things that do not change no matter what; how invariants are defined by symmetries, which in turn define which properties of nature are conserved, no matter what. These are the selfsame symmetries that appeal to the senses in art and music and natural forms like snowflakes and galaxies. The fundamental truths are based on symmetry, and there’s a deep kind of beauty in that.”

This chapter is also the most technical, and, unfortunately, the technical writing is not one of the book’s strong points. Here and in a few other places there are inaccuracies. For example, a section on geometric symmetries gives a square four symmetries instead of eight and ignores the possibility of translational symmetries for tiling patterns; some discussions include nonrotational symmetries, others don’t. Nevertheless, even here, as throughout the book, there are gems of observation: the idea of truth as that which remains invariant in an issue after consideration from every possible angle; the process of searching for invariance as a life goal; a discussion of broken symmetry and artistic appeal, which struck me as I was driving on the highway staring at an endless (and monotonous) blue sky.

As might be imagined from the above synopsis, the breadth of material presented here is both a strength and a weakness of the book. The sheer number of topics makes it difficult to investigate any single mathematical subject in much detail. At its worst this can dissolve into a sort of technical name-dropping, with the effect being much like a dinner party consisting solely of appetizers and dessert: the guests leave feeling full but still wishing for the main course. Nevertheless, the breathless manner in which these subjects appear also seems to reflect Ms. Cole’s excitement upon discovering that for the questions that “people really think and worry about,” mathematics has developed a useful set of tools. This is an excitement which I believe will lead those who did not even know how to ask a question to a place where they may begin, while those who already knew how to ask the questions will be shown ways to answer, and ask them, differently.