Book Review

Strength in Numbers

Reviewed by Giuliana Davidoff

Strength in Numbers
Sherman Stein
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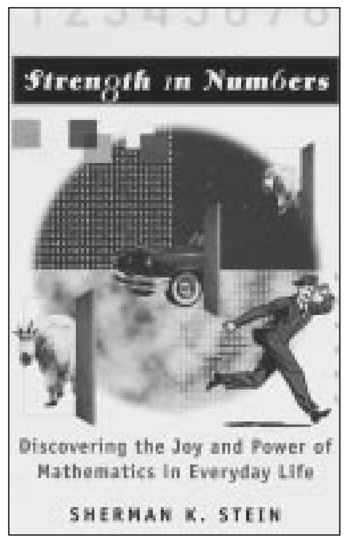
We are told these days by Madison Avenue that the cult status of mathematics is on the rise. Having sat through a popular summer movie titled "π", mathematicians and nonmathematicians alike now await the appearance of Givenchy's new perfume by the same name. Let us hope that before this trend wafts away, the public gets wind of more substantial mathematics in the form of Sherman Stein's delightful book, *Strength in Numbers*, where, in fact, one will find an engaging ode to the wonders of that very number. Writing in a fast-paced, down-to-earth style, with a dry wit that enlivens even the most abstract topics, Stein has given us a book that can hold the attention of a wide range of readers, from the experienced professional to the interested generalist. Aimed primarily at nonmathematicians, the book covers a variety of subjects, including examples of the appropriation (or misappropriation) of mathematics for political purposes, then moving on to remarkably lucid introductions to the calculus and other specific mathematical topics. Sprinkled throughout are demonstrations of the continuing relevance of mathematics to the issues and activities of our daily lives. Underlying the entire book and surfacing in a series of anecdotes, asides, and beautifully drawn connections is Stein's deep respect and affection for the elegance and power of mathematics. Even the professional mathematician, already familiar with the particulars of the mathematics, will benefit from reading this book, whether it be to learn something new about his or her own field, to be reminded of public misconceptions promulgated by misuse of tools of our trade, or simply to join Stein in the great pleasure he takes in doing, teaching, and experiencing mathematics.

Giuliana Davidoff is professor of mathematics at Mt. Holyoke College. Her e-mail address is `gdavidof@mhc.mtholyoke.edu`.



The book is divided into three parts, each written in short, often self-contained chapters. The subjects of Section I, the longest of the three, are diverse, though perhaps unified by an aim to dispel some of the myths associated with numbers, mathematics and computers, and mathematicians. These are the chapters that may hold the most interest for the professional mathematician, though one wishes that they could be made required reading for every elected official. For example, the author discusses "cool" and "hot" numbers. The first are those noncontroversial numbers that, nonetheless, carry emotional import. A 9000-point stock market or a thirteenth floor in a building come to mind as examples of numbers whose associations to certain conditions are commonly understood. Though a stock market at 8999 is virtually indistinguishable from one at 9000, the latter certainly resonates with the public in a way that the former does not. On the other hand, "hot" numbers are those thrown around in debates of major issues. Like the cost of health care in the year 2000, the number of existing HIV cases, or the number of people at a demonstration, these numbers cannot be known precisely, yet they play an enormous role in shaping the political discussion and manipulating public opinion. Stein also issues wise warnings against the harm that can result when single numbers, such as IQ scores, are used to measure quantities that, like intelligence, are surely multidimensional. Citing pertinent examples in each of these cases, he reminds his readers of our obligation to sustain the skepticism necessary for a sophisticated consumption of statistical information.

Taking a different tack, the author points out how anecdotes and extreme-case scenarios often substitute in the political discourse for a reasoned, possibly scientific, assessment of issues. One need only listen to current discussions of the effect on poor families of our recent welfare reform to witness a critical situation that cries out for some systematic examination. Like "hot" numbers, the true measure of this effect may be almost impossible to know, but in the meantime we are led to believe by advocates on both sides of the question that the anecdotal evidence of one or two families should lead us to one conclusion or the other. One can only nod in emphatic agreement when Stein asserts, "The sheer size and complexity of the issues that face our nation ought to demand an appropriate updating of the way we settle them."

In another chapter with an underlying political message the book addresses the impossibility of deciding in the present which seemingly abstract mathematical ideas will have important applications in the future. It presents knot theory, the Radon transform, and a number theoretic result of Euler as examples of pure mathematics that ultimately became powerful tools for modern science. Through these three ideas, all apparently far removed from the "real" world, we now better understand the DNA molecule, reap the medical benefits of CT-scans, and enjoy the privacy afforded by security systems based on the RSA public-key code. Stein's argument is an important one: namely, that curiosity, rather than necessity, is often the best motivation for invention, that mathematics moves forward because "the questions [are] intriguing and the discoveries surprising, profound, eternal, and beautiful." We live in an era when the public is goaded all too often to measure the value of investment in terms of immediate, often shallowly defined returns. Stein reminds us of the importance of supporting what may appear to be impossibly abstract directions in mathematical research, not only because of their intellectual value, but because many of them in the end will provide unforeseen practical rewards.

In Section I the reader will also find a detailed presentation of the relevance of mathematics to the workplace. The author describes the result of his extensive research into the level of mathematics required for a large variety of jobs, presenting his findings in both tabular and narrative form. While he does not presume to suggest that mathematics courses are crucial to success, one is struck by the differences in career paths available to those with and without exposure to high school and college mathematics.

The first section finishes with a highly interesting discussion of the history of curricular reform in this century, a topic that clearly engages Stein deeply. In addition to listing the changing sequence of goals in the evolutionary arc of reform, he examines four particular experiments in some detail. The first took place in 1929, implemented by a highly imaginative school superintendent, L. P. Benezet, in Manchester, New Hampshire. Benezet hypothesizes, "It is nonsense to take eight years to get children thru [sic] ordinary arithmetic. The whole subject could be postponed until the seventh grade and could be mastered in two years by any normal student." The story of his effort is absorbing. The second experiment was spearheaded by Stein himself, along with one of his colleagues, when, in 1968, they devised an innovative high school curriculum based on small-group learning. Though their initiative survives, it does so, according to Stein, in a form no longer recognizable as deriving from the original. We would do well to pay attention to his comments on the problems that arise when even the worthiest approach is left in the hands of not-so-competent teachers. The last two of the reforms he discusses at length will be familiar to mathematicians: they are the "New Math", begun in 1958 and largely invisible by 1970; and the most recent, the NCTM *Standards*.

Stein is obviously passionate about teaching and dedicated to doing it well. On the other hand, he is clearly weary of the endless promises of reform that "spring forth even though there is no agreement on the cause of the problem." Before we jump enthusiastically onto the bandwagon of the next popular idea, we might consider his admonition that "It is as though a doctor keeps plying patients with a variety of pills without ever figuring out what ails them. Looking back at the old reforms, I get the impression that the prophets are constantly reinventing the flat tire."

The second section of the book is devoted to revisiting some of the mathematics encountered in grade school and high school. However, while it does indeed go into some basics, such as properties of arithmetic with integers and fractions and graphing functions, there is much that will be new to nonmathematicians. Beginning with a useful chapter on how to read mathematics, it moves on to review the subtle issues involved with mathematical statements that must hold for all numbers, including those too large to examine experimentally on even the fastest computers. In a seamless segue from a simple definition of prime numbers to a crystal-clear exposition on Merten's conjecture to a brief mention of the Riemann hypothesis and RSA codes, Stein gives the reader a look at the impossibility of deciding mathematical truth from any finite number of examples, no matter how large. In this section we also find an explanation of how to sum a geometric series and why such a series explains how banks "create" money. Here, too, is a presentation of Cantor's diagonal proof that the continuum is uncountable, as well as the discussion of π mentioned in the first paragraph of this review, both topics untouched by the normal high school curriculum. The chapter on the ubiquitous π is particularly entertaining, exploring as it does the diverse settings in which the number can appear. It is in this chapter that the reader senses most acutely Stein's genuine wonder at the "astonishing connections" through which "mathematics reveals its unique and beguiling charm," a wonder that is summarized in a lovely poem on the subject of π written by the author's wife.

The third and shortest section of the book addresses some mathematics usually encountered for the first time in the beginning semester of a calculus course. It starts with a clever discussion of the indeterminate form 0/0, illustrating some of the problems involved in evaluating such an expression, and then uses geometric series to handle a class of special examples. From there the text moves into a discussion of slope and steepness, leading to an explanation of the derivative, though that technical term is never used. Naturally, the next step is to find the area of a two-dimensional shape with a curved boundary. Here Stein follows a proof by Fermat that determines the area between the curve $y = x^2$ and the x-axis in the interval $[0, 1]$ and again relies on summing geometric series. To finish off, he returns to yet another interesting setting for the number π and, finally, gives two proofs that reveal his own mathematical aesthetic.

This book will hold the interest of many, but for professional mathematicians it will hold fewer surprises than for the nonmathematician. Given the importance of many of its points and the clarity of its exposition, it provides the interested amateur an unusual opportunity to gain an insight into some important ideas that before might have seemed unattainable. One hopes that it will find the broad general readership it deserves.