Book Review

The Jungles of Randomness: A Mathematical Safari

Reviewed by Rick Durrett

The Jungles of Randomness:
A Mathematical Safari
Ivars Peterson
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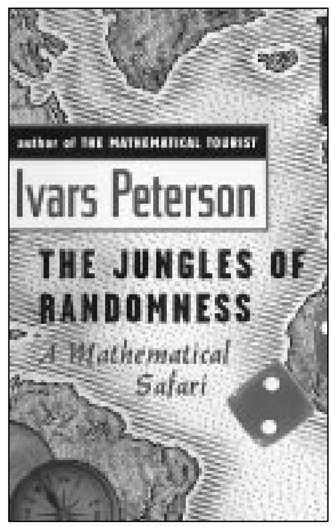
Ivars Peterson is the mathematics and physics editor for *Science News* and the author of four previous books, including the *The Mathematical Tourist: Snapshots of Modern Mathematics*, where he had the good taste to mention some of my work. Thus I approached his most recent book, *The Jungles of Randomness,* expecting to like it. However, two hundred pages later when my mathematical safari was completed, I had mixed emotions about my trip.

The writing is choppy. Thirty-second sound bites may be good for the evening news, but they make for tiring reading, and in a number of cases they completely miss the point. Peterson's discussion of self-avoiding walks on pages 156–7 concentrates on the problem of enumerating random walks and how difficult this becomes as the number of steps n increases. However, even though Peterson has clearly spoken to Gordon Slade, the book makes no mention of the spectacular achievement of Hara and Slade showing that in dimension $d > 4$ end-to-end displacements scale like $n^{1/2}$, as ordinary random walks do. Nor does he mention the interesting and still unresolved problem of computing the scaling behavior of self-avoiding walks in dimensions $1 < d < 4$, where the self-avoiding constraint makes them wander a distance of order n^a with $a > 1/2$.



A second problem with sound bites is that they make the underlying mathematics sound strange: "Ramsey theory implies that complete disorder is impossible." Physicists' notion of self-organized criticality is reduced to the triviality "Big events occur less frequently than small events, a relationship that can in many cases be expressed by a simple mathematical formula." The remarkable insight in the Black-Scholes formula becomes the enigmatic remark "The direction of price change doesn't matter. Instead, the only thing that matters is how much the stock price is likely to vary."

I guess in a popular math book one is not allowed to write down a formula like $X_t = X_0 \exp(\mu t + \sigma B_t)$ for the Black-Scholes stock price model in order to be able to say precisely that the price of an option depends only on the volatility σ but not on the drift μ. However, one can easily explain the essence of this surprising conclusion to undergraduate mathematics majors, or even MBA students, by considering a simple example. Suppose

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that a stock starts with a price of 100, one has an option to buy it in three months at 110, and at that time the price of the stock will be either 90 or 120. To be precise, our profits in the two situations when the option sells for c are

	profit from stock	profit from option
up	20	$10 - c$
down	-10	$-c$

If we buy x units of the stock and $-3x$ of the option, then our payoff in either case is $(3c - 10)x$. If $c > 10/3$, we would make a lot of money with no possibility of loss by taking x large. If $c < 10/3$, we could do the same by selling the stock and buying the option in a one-to-three ratio. The remarkable thing about this conclusion is that it does not depend on the probabilities of the two events.

In addition to errors of omission, there are a few mistakes in the mathematical explanations given. Peterson correctly states the fact that in a sequence of $n^2 + 1$ distinct numbers there is either an increasing subsequence or a decreasing subsequence of length $n + 1$. However, he bungles the well-known "proof by solitaire". Write the numbers on cards. Then put each card on top of the leftmost stack where it is larger than the top card. Each stack is an increasing sequence, while the cards you can see will always be a decreasing sequence (or otherwise you put a card in the wrong place). Since there are $n^2 + 1$ cards, there must either be a pile of size $> n$ or $> n$ piles. Peterson makes a simpler error in the definition of Lévy's random flight: steps larger than ℓ should be taken proportionally to a power of $1/\ell$ if one wants to have a process flexible enough to cover the applications indicated in the book.

At this point, Peterson and some of his fans are probably ready to scream, "The objections just raised are not important," and they are right. The book presents a lot of interesting material. For instance, in connection with Lévy's flight it mentions data on flights of the wandering albatross collected by the British Antarctic survey. Furthermore, when Peterson takes the time to actually explain things, the book is very interesting. A good example is the work of Diaconis and Mosteller on coincidences. Here the sound bites work: "If something happens to only one in a million people per day and the population of the U.S. is 250 million, then you expect 250 amazing coincidences a day." A concrete example of an event that can be explained by this type of reasoning is the fact that in 1986 a New Jersey woman won a million-dollar lottery prize twice in four months. The chance that some person specified in advance will do this in a given four-month period is a zillion-to-one shot, but leaving the person and the time interval unspecified drops the probability to about 1 in 30, according to calculations of Diaconis and Mosteller.

A more technical situation that is also nicely explained is Mark Kac's problem "Can you hear the shape of a drum?" Peterson spends more than seven pages developing the story from the initial positive results that motivated the question to Milnor's 16-dimensional drums, Buser's 3-dimensional sound-alike bells, and finally Gordon and Webb's 2-dimensional domains. One of eight color plates in the center of the book shows the results of an experiment by physicist Srinivas Sridhar at Northwestern, who made copper boxes with the Gordon-Webb shapes and introduced microwaves through tiny holes to make pictures of the first three eigenfunctions. The striking differences in the patterns make the mathematical result seem an even more remarkable achievement.

Peterson's book contains a wealth of information, and even experts will find within their subjects some new tidbits of knowledge. In the first chapter, "The Die Is Cast", among the old chestnuts that include Chevalier de Mere's sucker bet (can you roll a six in four tries?) is the new nutty fact that the sums of the faces of two dice (one with sides 1,2,2,3,3,4 and one with sides 1,3,4,5,6,8) have the same probabilities as ordinary dice, and this is the only alternative numbering that works. In the "Sea of Life" we start with the well-known and easy-to-prove fact that in a group of six people there is either a triangle of friends or a triangle of strangers, but we soon come to the less famous Budapest café problem, "Given five points positioned on a flat surface so that no three lie on a line, show that four of the points define a convex quadrilateral."

One of the things that makes Peterson's treatment of the last problem appealing is that he takes the time to paint a picture of Paul Erdős, George Szekeres, and Esther Klein sitting around a table at a café "talking politics and feeding their passion for mathematics," and he describes the solution of the problem. Some more details would have been welcome later in the chapter when he mentions Erdős's use of probabilistic reasoning to get lower bounds on Ramsey numbers. Peterson explains only that Erdős flipped coins to see who was related and then showed that "the probability of getting a party with a desired mix of strangers and acquaintances is practically certain beyond a certain size of the group," an observation that seems to me to be going in the wrong direction.

Almost half of the book concerns areas that are far from my experience, so there I just went with the flow, nodding my head as if I understood what he was saying and generally enjoying myself. The "Call of the Firefly" concerns synchronization of oscillators. The "Noise Police" concerns error correction and encryption. I am interested in biology, so I struggled to understand the "Shell Game", a chapter about viruses folding themselves into structures that resemble geodesic domes, but all that I came away with is the notion that these are interesting problems that mathematics has something to say about.

The reader can probably guess the subject of "Complete Chaos". There one finds an interesting story about pinball games in the 1930s near the Massachusetts Institute of Technology. One finds a strange claim that the amusement ride Tilt-A-Whirl is exciting because its dynamics are chaotic and hence is sensitive to its initial conditions. From there the chapter makes an excursion into the cardiac rhythms where chaos = death and relates this to an incredible analysis (in the literal sense of the words) by Pincus and Singer of Standard and Poor's index of 500 stocks. Their "calculations show that fluctuations in the index's value are generally quite far from being completely irregular or random. One striking exception occurred during the two-week period immediately preceding the stock market crash of 1987, when the approximate entropy indicated nearly complete irregularity. That change flagged the incipient collapse." Thus, I guess we should refer to the "defibrillation" rather than the crash of October 1987. Of course, such catastrophic events are much easier to predict after they happen.

The random samples of Peterson's book above should give you an idea of its content. I agree with Robert Osserman's quote on the book jacket that "Every reader, regardless of background, is bound to find something new and interesting in this book." Personally, I would have enjoyed the book much more if it had spent more time on fewer topics. However, in this case other readers might have enjoyed it less. Hyperbolic prose seems inevitable in books of this type, but in Peterson's book it is not as abundant or as annoying as in other books I have read. Last but not least, as a $15 paperback this book is one-fifth as expensive and three times as entertaining as your typical technical math book, so if you are intrigued by the topics, then give it a try. The book comes with a four-page appendix and fifteen pages of references that will help you start to figure out the underlying math if you are so inclined.