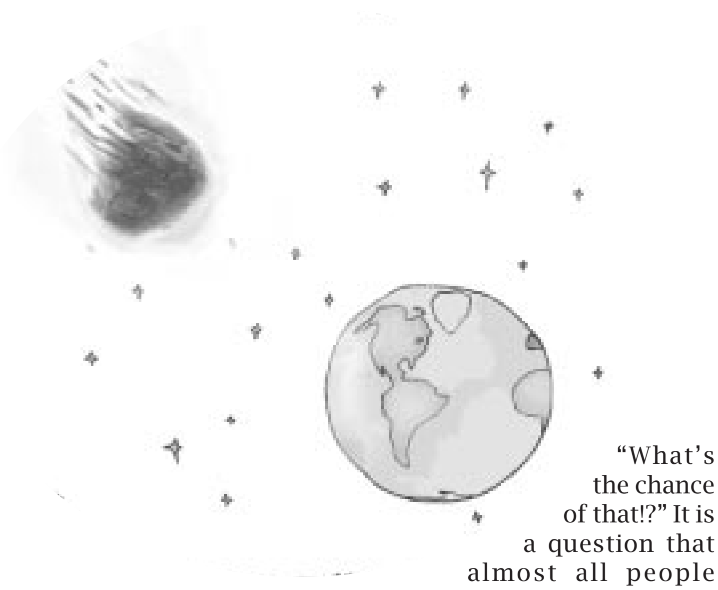
To Teach, per Chance, To Dream

Daniel Rockmore and J. Laurie Snell



"What's the chance of that!?" It is a question that almost all people ask—sometimes after the fact—in trying to make sense of a seemingly improbable event and, at other times, in preparation for action, as an attempt to foresee and plan for all the possibilities that lie ahead. In either case, it is mathematics in general, and probability and statistics in particular, that the public looks to for a final answer to this question. One out of one hundred, 4 to 1 odds, an expected lifetime of 75 years—these are the sorts of answers people want. When used honestly and correctly, numbers can help clarify the essence of a confusing situation by decoupling it from prejudicial assumptions or emotional conclusions. When used incorrectly—or even worse, deceitfully—they can lend a false sense of scientific objectivity to an assertion, misleading those who are not careful enough or knowledgeable enough to look into the reasoning underlying the numerical conclusions.

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Illustration above by Erica Wood.

It is important to be able to distinguish between these two scenarios. Chance is a part of life, and its quantification is the stuff of science, social policy, and individual decision making. As such, it is a place where it is easy to show that the techniques and problem-solving approach of mathematics make a real difference. In this way, the study of chance can provide a real pedagogical opportunity for mathematics departments.

It is in this spirit of pedagogy and quantitative reasoning that nine years ago, a Chance course was first conceived by Laurie Snell (Dartmouth College) and Bill Peterson (Middlebury College) and was funded by the Pew Foundation. Subsequently, the project was funded by the National Science Foundation and grew into a project encompassing a Web site, newsletter, lecture series, and workshops.

Originally, the intent was to create a case study course based on the kind of articles that appear in *Chance Magazine*. *Chance Magazine* is a magazine of the American Statistical Association, whose aim is to show the applications of probability and statistics in everyday life to a broad audience. This first version of the Chance course was to be restricted to the study of four or five significant probability and statistics applications chosen from topics typically discussed in *Chance Magazine* and the media in general, such as political polls, DNA fingerprinting in the courts, the census, streaks in sports, coincidences, and clinical trials. Peterson developed such a Chance course at Middlebury as part of the freshman writing seminar program.

Soon after, some firsthand experience with the freewheeling Geometry and the Imagination course—created at Princeton University by John H.

Conway, Peter Doyle, and Bill Thurston—led to the creation of a more dynamic version of Chance, as well as to a successful proposal to the NSF for funding the Chance Project as a consortium consisting of Dartmouth College (John Finn and Laurie Snell), Grinnell College (Tom Moore), Middlebury College (Bill Peterson), Spelman College (Nagambal Shah), University of California at San Diego (Peter Doyle), and University of Minnesota (Joan Garfield).

The full version of Chance is now taught at Dartmouth to about one hundred students per year, as well as at several other institutions around the country. Based on current events, the course uses all possible sources of statistical or numerical claims—such as advertising, the media, and scientific journals—as the material for classroom discussion and analysis. This style of course was first taught in the spring of 1992 at Princeton University by Doyle and Snell.

In a typical Chance class students are given a current news article and three or four discussion questions related to it. Examples of news topics used recently are the year 2000 census, depression related to Internet use, genetic testing, and coaching for the SATs. Students are asked to form groups of three or four, read the article, and spend fifteen or twenty minutes answering the discussion questions. The group's answers form the basis for a general discussion of the article and the probability or statistical issues that it raises.

Here is one of the discussion questions from the first day of the first-ever Chance class, taught at Princeton:

The recipe for pizza from Laurel's Kitchen says: Let the dough rise only once, about 1 and 1/2 hours. How long should you let the dough rise if you use Fleischmann's Rapid Rise Yeast, whose package states that it "rises 50% faster"?

The students gave what we thought were reasonable interpretations, but when we called the Fleischmann Company to ask what they meant by it, we were told: If your recipe says to let the dough rise twice, with Fleischmann's Rapid Rise Yeast, once is enough. Our first day's example showed that mathematics and the real world are often quite different.

Here is a set of discussion questions from that first course based on a newspaper article:

Read the New York Times *article on China "Stark data on women: 100 million are missing" and answer the following questions:*

(a) Suppose that a certain society values sons more than daughters. In this society, a couple will continue bearing children until they produce a son, at which point they will retire from the child-bearing business. Would this family-planning scheme tend to produce more boys or more girls?

(b) Using coin tosses, simulate the generation of twenty families. Make histograms for the number of sons and the number of daughters. Find the average and the standard deviation of the number of sons and the number of daughters. In light of this data, are you inclined to change your answer to the previous question?

(c) What do you think accounts for the missing daughters in China?

The *New York Times* article was based on research by a Princeton demographer, who provided us with more detailed information about the study. This exchange of information is not uncommon, and local colleagues in other disciplines are often happy to give guest lectures to provide background information for chance issues in the news. A nice example of this collaboration is provided by the researchers from the Center for Disease Control whom Nagambal Shah regularly invites to her Chance course at Spelman College.

In general, we have found that following the news leads naturally to the need to understand various topics, including study design, descriptive statistics (including graphics), probability, correlation, polls and surveys, estimation, and test of hypotheses.

These fundamental statistical ideas and techniques arise in the news in a variety of familiar and meaningful settings. Lotteries are always in the news and provide an interesting introduction to simple counting problems. Medical stories occur regularly and lead naturally to discussions of the difference between randomized clinical trials and epidemiological experiments. Issues of relative and absolute risk arise here as well. Another good source of elementary probability is the daily weather prediction in the newspaper: The weather forecaster predicts "a 30 percent chance of rain," and it rains. What does this mean? Was she right? How does one "score" a weather forecaster? In recent years we have also been able to get a lot of mileage out of El Niño, hurricane and tornado prediction, and global warming (or not!).

Classroom activities provide a dynamic way to elucidate statistical concepts. To illustrate the concept of correlation, we often use a cookie-tasting activity. We provide the students with about ten brands of chocolate chip cookies of varying prices and ask them to design an experiment to determine if there is a correlation between the taste and the price of the cookie.

Classroom activities can also be used to help students understand the concept of a test of hypothesis. For example, we might ask the students in their groups to develop and carry out an experiment to determine if one of their members can tell the difference between Pepsi and Coke. Students often come up with a variety of experiments.

For example, suppose that in one group Mary claims that she has some ability to tell the difference between Pepsi and Coke and she is given 10 trials, with each trial randomized to be Coke or Pepsi. Skeptic John says that Mary must get 8 or more correct to convince him that she is not just guessing. If Mary is guessing, getting 8 or more correct out of 10 trials would be like getting 8 or more heads in 10 tosses of a coin. A simple counting argument shows that this is $(1+10+45)/1024 \approx 5.4$ percent. So John is happy with this experiment, since if his skepticism is well founded, then with high probabilty (almost 95 percent) he will be shown to be right.

Illustration by Peter Sykes.

How does Mary feel about this experiment? She wants an equally high probability of establishing her claim. To be precise, Mary feels that she can tell the difference about three-fourths of the time. Thus her successes should be like the number of heads that turn up when we toss a biased coin with a 75 percent chance of turning up heads. With the biased coin, 8 heads is now about what we would expect, and the probability of 8 or more heads turns out be 53 percent. So Mary is not very happy with the proposed experiment, because if she does have the ability that she claims, she will have only a 53 percent chance of convincing John.

We then talk about the need to increase the number of trials to make both Mary and John satisfied so that the truth will prevail. For a historical perspective, we tell the students about R. A. Fisher's famous lady-testing-tea experiment.

It is surprising how often the simple coin-tossing model can explain what is going on. Here is an example from a study reported in the *New York Times.* A large study was carried out to see which of the two drugs t-PA or streptokinase is more effective in preventing death in the period immediately following treatment for a heart attack. In a randomized study, 10,000 patients being treated after a heart attack were given t-PA, and 10,000 were given streptokinase. In the six-month period following the operation, there were 630 deaths for the t-PA group and 740 for the streptokinase group. It was important to know if the differences could be due to chance, since t-PA costs $2,400, while streptokinase costs $240. Also, while the difference between 7.4 percent and 6.3 percent is small, it would save about 2,000 patients a year in the United States if it is real.

To see if this difference is significant, we assume that the two drugs are equally effective. Then since the patients were randomly assigned to the two groups, the 1,370 deaths would be equally likely to have come from each of the two groups. Thus the number of deaths from t-PA can be modeled as the number of heads in 1,370 tosses of a fair coin. The expected number of heads in 1,370 tosses is 685, and the study resulted in a deviation of 55 from this expected value. How likely is it to get a deviation as large or larger than this? The standard deviation for the number of heads is the square root of $(1,370 \times 1/2 \times 1/2) \approx 18.5$. Thus a deviation of 55 represents more than 3 standard deviations from the expected number, so this outcome would be highly unlikely if the two drugs had the same effect. Thus we conclude that the difference is significant.

Outside of class, students do more traditional homework assignments from a textbook such as *Statistics* by Freedman, Pisani, and Purvis. They also keep "journals". The journals contain commentaries and answers to further questions related to the articles discussed in class. The course concludes with the "Chance Fair", a one-day extravaganza at which the students give poster presentations of final projects and exercise their newfound statistical skills at the "Chance Casino" by playing blackjack and roulette.

Part of the goal of the Chance Project is to help initiate Chance-like courses at other institutions. One way to facilitate this is to prepare and distribute background material ("profiles") of topics that occur regularly in the news. For example, our lottery profile discusses how large the jackpot needs to be in the well-known "Powerball" lottery in order to make this a favorable bet. Incredibly, the answer turns out to be $270 million, which happens to be the largest jackpot to date. Our analysis takes into account the effects of differing payoff schedules, taxes, and the possibility of sharing the prize.

Another way to help teachers is to identify interesting news articles that they can use in their classes. Since both Chance News and the profiles depend on timely information, the Internet is a natural way to distribute the work. Thus, we developed a Chance Web site, `http://www.dartmouth.edu/~chance`, to provide profiles of topics based on our teaching experiences, data sets related to articles we have discussed, activities that we have used, and links to other resources for a Chance course.

Included there are issues of an electronic newsletter, "Chance News", which reviews current articles in the news based on probability or statistical concepts and provides discussion questions for them. It is sent out monthly by e-mail to about 1,500 teachers of probability and statistics. While intended for a Chance course, "Chance News" is also used extensively by teachers wishing to introduce current events in more traditional probability and statistics courses. For example, Joan Garfield does so regularly in her courses, and she was led to call these "Chance-enhanced" courses. The preparation of this newsletter has become a joint effort of Laurie Snell at Dartmouth, Bill Peterson at Middlebury, and Charles Grinstead at Swarthmore. Readers also send suggestions and comments on previous articles.

Most "Chance News" items are based on articles from national newspapers such as the *New York Times*, the *Washington Post*, the *Boston Globe*, the *Wall Street Journal*, and the *Los Angeles Times*. It has been estimated that about 80 percent of science articles covered in the news come from a very few journals, including *Science*, *Nature*, the *New England Journal of Medicine*, the *Journal of the American Medical Association*, and *Lancet*. Most colleges have these journals and newspapers in their libraries. In addition, all these newspapers and journals have electronic versions. Many libraries have student access to Lexis-Nexis, which provides a much wider variety of newspapers and magazines than listed above. Thus it is easy to find the source of most of the articles discussed in "Chance News".

The most recent addition to the growing treasury of Chance materials are the "Chance Lectures", a video archive of the Chance Lecture Series that we have run the past two years at Dartmouth. Each year we have invited a group of experts in a variety of subjects, ranging from casino gambling to insurance policy redemption, to give one-hour lectures on the way in which probability and/or statistics impacts their work. The lectures are intended for a scientifically literate and interested audience with no more than a "newspaper knowledge" of the subject at hand.

The lectures are videotaped and made publicly available on the Web via streamed video for viewing using the "RealAudio" application. The format of the videos is such that they are accompanied by displays of the speaker's overheads that update automatically as the video proceeds. The Web site contains twenty-six lectures in all, seventeen of which are from the two lecture series. Viewing these requires at least a 58kbs connection and a computer with a clock speed of at least 150 megahertz. To make them easier to use in the classroom, we have made these lectures available on a CD-ROM that requires a browser but not a network connection.

Each lecture is long enough that it would be possible to devote an entire class to the analysis of a single one. When the speaker makes a particularly difficult or controversial remark, the teacher can stop and discuss it with the class before going on. An accompanying indexing scheme also makes it easy to use only segments of a video.

The subject matter runs the gamut, from playing fields to planetary science. For instance, in our first lecture series, Hal Stern, professor of statistics at Iowa State University (and editor of *Chance Magazine*), gives a beautiful lecture on the probability and statistics which can be found every day in the sports pages. He considers three quite different examples to show how probability and statistical theory, when applied to real sports data, can enhance the understanding of a sport and help determine optimal game strategies. These examples are: the use of Markov chain theory to determine baseball strategies, the use of regression to rate college football teams, and the use of data and the normal distribution to estimate, at a given point during the game, the probability that a particular team wins. This last example sheds light on the folk theory that the final outcome of a basketball game is determined by what happens in the last quarter, and by the first seven innings for a baseball game.

In our second lecture series, Clark Chapman of the Southwest Research Institute speaks on "The risk to civilization from extraterrestrial objects". Chapman, a leading researcher on planetary cratering, discusses the science behind the determination of the rate at which extraterrestrial objects strike the earth, as well as the different types of collisions which can and do occur. He explains the computation of the odds that any individual on Earth will die due to such an event and justifies his claim that "It's as likely that your epitaph will read that you died of an asteroid collision, as by an airplane crash." If the odds of these disasters are the same, then should we devote similar amounts of energy and money to preparing for asteroidal collisions as we do for airline safety? This leads naturally to discussions of the larger topic of risk and resource management.

So, like any good Chance topic or Chance course, Chapman's lecture is about more than mathematical analysis. It is about framing a question clearly and giving a precise answer; it is about distilling a general principle from a particular example. In short, it is about what we, as mathematicians, try to do every day.