The Number Devil

Reviewed by Deborah Loewenberg Ball and Hyman Bass

The Number Devil
Hans Magnus Enzensberger
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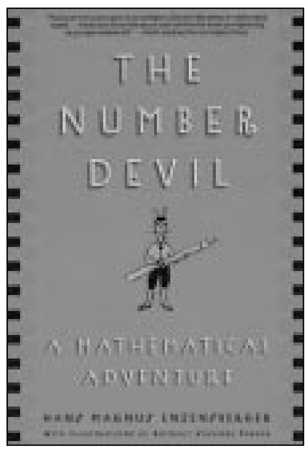
Engaging A Mathematically Disengaged Public

In 1959 C. P. Snow poignantly described the chasm between the sciences and the humanities in his provocative Rede lecture, *The Two Cultures*. Straddling this chasm himself, he noted stereotypes which are held by members of each culture and which shape profound misconceptions about the work and endeavors of the other. Forty years later the divide which Snow lamented is as wide as ever. John Allen Paulos's popular books illustrate ways in which many otherwise well-educated Americans lack fundamental quantitative sensibilities and are alienated from mathematics [1]. The same person who would never dream of saying, "I can barely read," remarks lightheartedly, "I was never good at math." The same person who can be fascinated with a linguistic detail or a fine point about sociology or politics or music is frequently uninterested in matters of chance, insensitive to orders of magnitude, and impatient with even a modest use of mathematical terminology or symbolic notation. Mathematical ability is widely believed to be innate, mathematics a pure "hard" science. Other fields are seen as "soft".



One result of this chasm is that fewer and fewer American students voluntarily enroll in mathematics courses past high school, and the number of American mathematics majors in our colleges and universities has dropped steadily. Another equally serious result is that the public is unengaged with mathematics, and matters that require quantitative reasoning or sense are poorly understood. A natural response to this problem has been widespread and often ambitious efforts to improve mathematics instruction at all levels. One wave after another of reforms has swept over our nation's schools over the past hundred years, leaving a mixed residue of changes and some improvement. These efforts, usually focused on curricular change, have been the subject of controversy and debate and have confronted reformers with the sprawling nonsystem of American schooling and the independent complexity of creating educational change which permeates practice as intended.

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Another path to changing America's ambivalent relationship with mathematics, less often pursued than educational reform, is to produce mathematical exposition for public consumption. At their best such efforts can make good mathematics accessible to the public and engender fascination with the subject. Consider, for instance, the writings of John Allen Paulos, Reuben Hersh, Philip Davis, Martin Gardner, Keith Devlin, Ian Stewart, and Brian Rotman, among others. Some authors—Lewis Carroll, Anno, Marilyn Burns—have written books for children. In *The Phantom Tollbooth*, for example, Norton Juster tucked fascinating mathematical excursions and lessons in and around the adventures of Milo, a bored young child who has "*nothing* better to do." [2]. Writing books intended for children can produce an interesting twist, for adults often read such books to children. Parents and teachers, grandparents and babysitters thereby also engage with the mathematics of these books. *Anno's Mysterious Multiplying Jar* offers a glorious visual and textual portrait of the mathematics of factorials. Traveling with Milo *(The Phantom Tollbooth)* to Digitopolis, meeting the Dodecahedron, and tracing his encounters with infinity and precision, readers are drawn into fascination with and understanding of fundamental mathematical ideas.

One common-sense fallacy is that interest is innate: some people are interested in mathematics; others are not. However, interests, as John Dewey argued, are made, not found. A central task of education is to engender interest, to help build connections between a learner's present and the invitational potential of the subject. One thing then that popular texts involving mathematical ideas can do is to open mathematical worlds to the public, interesting people in ideas of number and space; pattern and relationships; and the language, tools, and ideas that mathematicians use to explore these worlds. A central responsibility of the mathematical writer is to create these texts in ways that can connect with their readers, foreign travelers to these mathematical worlds. If successful, these books should send readers away eager for more and surprised by what they have encountered.

We take a closer look here at one such book recently published: *The Number Devil* by Hans Magnus Enzensberger. We begin by providing a snapshot of the book and then trace its interior, following the adventures of Robert and his new friend, the Number Devil. We stand back then and comment on the book's efforts to open mathematics to children and their adult companions. Does it engender interest in mathematics? Does it present mathematics as something in which anyone can engage? Are some significant mathematical ideas made accessible? What would it take for a child or a teacher or parent to read and profit from this book?

The Number Devil: Enzensberger's Mathematical Proselyte

Hans Magnus Enzensberger is a well-known and respected European intellectual and author with wide-ranging interests. He gave a speech on mathematics and culture, "Zugbrücke außer Betrieb, oder die Mathematik im Jenseits der Kultur—eine Außenansicht" ("Drawbridge out of order, or mathematics outside of culture—a view from the outside"), in the program for the general public at the International Congress of Mathematicians in Berlin in 1998. The speech was published under joint sponsorship of the AMS and the Deutsche Mathematiker Vereinigung as a pamphlet in German with facing English translation under the title *Drawbridge Up: Mathematics—A Cultural Anathema*, with an introduction by David Mumford. Enzensberger's knowledge of mathematics is discerning and appreciative; he is an adventurous amateur whose treatment of mathematical topics stays within reach of ordinary language and yet is surefooted without fundamental confusions or conceptual dissonance. In this book he has created a character called the Number Devil who diabolically hounds a young boy, Robert, taunting and tantalizing him with mathematical mysteries and wonders.

The Number Devil is a book for older children. The book jacket announces it as "a gift to readers of all ages: a cross between *Alice in Wonderland* and *Flatland*, in which we discover exciting secrets about math, without ever having to do a single math problem." In fact, the mathematical ideas are substantial and could easily engage even adult readers. The book's illustrations by Rotraut Susanne Berner are nicely and simply done and supportive of the narrative. Even the mathematical notation and expressions are rendered as illustrations, in soft thick lines and pleasing colors.

The story unwinds as a narrative of dreams in fantastic settings in which Robert has encounters across twelve nights with the Number Devil. Robert and the Number Devil are the only active characters of the book, though Robert's mathematics teacher, Mr. Bockel, is often mentioned, usually with derision. The dream environments, one for each of the twelve nights, include such settings as a forest of trees shaped like 1's, a cave with wall paintings, and an overturned boat on an empty seashore. These environments are incidental to the story, except as colorful conversational props. They take the place for Robert of some nightmarish experiences—being swallowed up by a slimy fish, falling down an endless slide, for instance—which tormented his nights before he met the Number Devil. Avoiding these distinctly unpleasant perils serves to keep Robert engaged with the sometimes taunting and aggravating Number Devil. The descriptions of the dream environments are brief and not particularly gripping. They are somewhat arid as descriptive prose, sketches of surrealistic landscapes. It is hard to imagine their being highly captivating to children.

In keeping with the fanciful style of the narrative, many mathematical terms and characters are colorfully renamed: Fibonacci numbers are now "Bonacci numbers", primes are "prima donnas", and irrational numbers are "unreasonable". "Hopping" is taking powers of a number, and hopping backwards leads to taking "rutabagas", i.e., square roots. Felix Klein gets the pseudonym "Dr. Happy Little", but we have not figured out why Gauss is called "Professor Horrors". Although there is no table of contents, there is an index, called the "Seek-and-Ye-Shall-Find List", which locates and explains these imaginative terms. A warning there explains that when Robert and the Number Devil talk about mathematics, they use some "unusual expressions." The reader is cautioned to use standard terms when not in the fanciful world explored by the Number Devil and Robert.

We turn next to a closer look at the adventures of Robert and the Number Devil and the mathematical territories of their explorations.

Devilish Mathematics

The book opens with Robert, who sleeps fitfully, plagued by strange puzzle-filled dreams. Determined to banish the tricks his mind plays on him in the darkness of night, one night he dreams of a meadow. Suddenly he notices an elderly man the size of a grasshopper perched animatedly on a leaf, staring at him with bright eyes. "I am the Number Devil!" asserts the small creature. Dubious, Robert dismisses him, saying there is no such thing as a Number Devil and, "Besides, I hate everything that has to do with numbers!" When the Number Devil inquires why, Robert replies that he must never have gone to school. "Or perhaps you are a teacher yourself?"

After some tussling back and forth, the Number Devil endeavors to show Robert that what he thinks of as mathematics from what he has seen in school is a far cry from the mathematics that the Number Devil could show him: "The thing that makes numbers so devilish is precisely that they are simple. And you don't need a calculator to prove it." He starts out: $1+1, 1+1+1$, $1+1+1+1$. "Any idiot" can see that these go on forever, infinitely. "Have you tried it?" asks Robert, curious. "It would be a waste of time," says the Number Devil dismissively. Here begins their characteristic pattern of interaction: the Number Devil triumphantly shows Robert interesting things, but when Robert challenges or asks questions, the Number Devil is derisive. When Robert asks more questions about the infinity of numbers, for example, the Number Devil shouts at him impatiently and proceeds to show there are infinitely small numbers too. Robert does not seem to understand what the Number Devil shows him, which makes the Number Devil impatient again, accusing him of being afraid of numbers. He proposes that he can make all numbers out of ones and excitedly presents Robert with the palindromic pattern, $1 \times 1 = 1$, $11 \times 11 = 121$, $111 \times 111 = 12321$, $1111 \times 1111 = 1234321$, and so on. Robert, following along, becomes skeptical and, suspecting that the Number Devil is bluffing, asks about

$$11,111,111,111 \quad \times \quad 11,111,111,111.$$

This flusters the Number Devil, who tries it. "You were right. It doesn't work. How did you know?" He is annoyed with Robert for embarrassing him. When Robert admits he was guessing, the Number Devil huffs that "mathematics is an exact science" where no guessing is allowed.

Their explorations continue in this vein, spanning a range of mathematical fascinations. On the third night the Number Devil takes up factoring, introduces "prima donnas" (primes), and cautions, more or less dogmatically, that division by zero is strictly forbidden. The sieve of Eratosthenes is illustrated with the numbers 1–50, inviting the reader to complete the task. Then, gladly quelling Robert's initial satisfaction, the Number Devil asks, "What if you have a number like 10,000,019, or 141,421,356,237,307? Is it prima donna or isn't it?" He notes that "even the greatest number devils have come to grief over it." And later, "The more simple-minded number devils use giant computers," a curious characterization.

To Robert's plea, "What's the point of it all?" the Number Devil replies, "You *do* ask stupid questions! Luckily you've got me to initiate you into some of the secrets." He then asserts (without discussion) that between any number larger than one and twice that number there is a prima donna. "And when I say always, I mean always!" Then the Number Devil enthusiastically goes on to announce that any even number larger than 2 is the sum of two prima donnas. Robert tries a few examples and sees what the Number Devil is saying. But when Robert asks why, the Number Devil confesses that "no one knows why [this Goldbach Conjecture is true]."

On the fifth night, after an encounter with the infinity between 0.0 and 1.0, the Number Devil warns of the existence of "unreasonable" (irrational) numbers, which "refuse to play by the rules." By way of motivation he recalls the "hopping" numbers—2, 4, 8, 16, 32, etc.—and proposes the problem of "hopping backwards", which he calls "taking the rutabaga" (square root). This leads to the square root of 2, which the Number Devil asserts is unreasonable. He does give a rare and beautiful geometric proof (sans Pythagoras) that the diagonal of the unit square has length $\sqrt{2}$. On

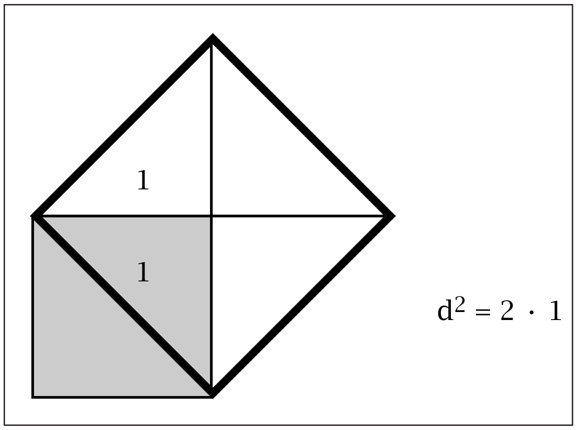

Figure 1. Geometric proof that the diagonal of the unit square has length $\sqrt{2}$.

the diagonal the Number Devil erects another square, which he illustrates is composed of four triangles, two of which can form the unit square (see Figure 1). Thus the larger square has twice the area of the unit square, whence the claim.

Next the Number Devil notes that the unreasonable numbers are not rare, in fact there are "more" of them than there are reasonable numbers. Robert protests, since he already knows well that there are infinitely many reasonable numbers, so how can there be "more than infinity?" No answer is offered.

Robert's dreams continue, with encounters with triangular numbers, Gauss's argument for the sum of the first 100 (or N) numbers, Pascal's triangle, Fibonacci numbers, permutations, the golden mean, and more. Never are these ideas named as such. Sometimes they are given fanciful names (e.g., Bonacci numbers) and sometimes left unnamed. Rarely are explanations given. One exception is on the ninth night, which deals with some infinite phenomena. The Number Devil seeks to convince Robert that all sequences (whole numbers, odd numbers, triangular numbers, primes, etc.) have the same number of members. Next he gives a geometric proof that $1/2 + 1/4 + 1/8 + 1/16 + \cdots = 1$, followed by a proof that the harmonic series diverges, by forming groups of terms of length 2^n for $n = 2, 3, 4, \ldots$. This is one of the rare instances where some substantial mathematical arguments are executed with a fair degree of completeness.

The eleventh night is particularly interesting, since Robert himself begins to ask the Number Devil *why* all the things shown to him are true: "You've *shown* me things, but never *proved* them." The Number Devil answers, "So you want to know the rules of the game, what a mathematician wants to know." He continues, "Showing is easy and fun, guessing and testing guesses is better, but proof is all." He explains how important, and how hard, proving is for mathematicians, often taking centuries and plagued with mistakes along the way. When Robert asks for an example, the Number Devil chooses a curious one: "$1^0, 8^0, 100^0$, all = 1," which is a (well-rationalized) convention, not a theorem. The Number Devil says, "I could prove it to you, but you'd go mad in the process." He then goes on metaphorically to describe proving as similar to crossing a stream in which one cannot swim: one must step from one stone to another sufficiently nearby. Previous results established by mathematicians furnish sturdy dry rocks, and the shore is built of axiomatic types of propositions (such as that any (whole) number has a unique successor, or two points determine a unique straight line). The Number Devil explains that even though patterns may persist for very large computations, this does not constitute a proof, and exceptions (which are fatal) may always show up when the computations are further extended. The example from the first night is mentioned. Robert reminds him that he became angry when Robert raised questions about the extension of the pattern. "You had a good nose," admits the Number Devil. "Even though the formula worked well enough for a while, it collapsed in the end, without proof. In other words, even a Number Devil can fall on his face."

The Devil Is in the Details: How Well Does *The Number Devil* Work?

The Number Devil is an attractive and imaginative fantasy. Robert and his devilish guide visit some of the loveliest sites of mathematical discovery and achievement. Their banter is entertaining, the ideas attractive. Robert is bright in a common sense way, and gradually over the course of the book he becomes more and more engaged with and expectant of his encounter with his unusual mentor. The Number Devil is sprightly, a bit mischievous, temperamental, and cunningly seductive with Robert. The book is aesthetically appealing, with attractive illustrations and clear and colorful fonts and line drawings. But how well would Enzensberger's book work to attract the mathematically hesitant or uninterested reader? What is the book saying about good mathematics and about who can do it? What is it offering the reader mathematically, and what would it take for readers to profit from it?

We worry on several counts. First, real mathematics is repeatedly said to be different from what goes on in school. Good enough. This could be an encouraging message for those who have been turned off by what they encounter in math classes. Robert's teacher, Mr. Bockel, who never actually appears, is a scapegoat symbolizing oppressive practices (e.g., long, boring arithmetic drills) and backward attitudes (e.g., forbidden use of calculators). School, it appears, is *not* where one learns real mathematics. It is really not Mr. Bockel's (or other teachers') fault, for he cannot go "rock leaping" whenever he wants, as the Number Devil and Robert do, because he has to correct homework.

The implication is that math teachers do not have the mathematical appetite or experience to engage students in "real mathematics". Mr. Bockel is a caricature, who, beyond his intellectual narrowness, is fat and hungry and secretly wolfs pretzels during class. Implicitly, schoolteachers do not know much mathematics, and school mathematics is trivial. Of a typical word problem, the Number Devil says, "That kind of problem has nothing whatever to do with what I'm interested in. Most genuine mathematicians are bad at sums. They have no time to waste on them. That's what calculators are for."

A second concern is that the book may reinforce an image of mathematics as dependent on inside expertise and restricted to privileged access. Although this may be intended to be funny to those who have endured deadening mathematics classes or to caricature what passes for mathematics in school, an insidious message is that teachers are mathematically incompetent and, worse, that real mathematics is found in an insider's world for a select and privileged few—the world into which the Number Devil entices Robert. The topics explored are interesting, and many are substantial. Appreciating Enzensberger's mathematical adventure depends, however, almost entirely on the reader's own mathematical experience and knowledge. How much a mathematical newcomer might be able to glean is far from clear. Virtually all the mathematical ideas are rendered, with due theatrics, by the Number Devil. Robert rarely raises questions, speculates, or uncovers mathematics on his own. Thus, mathematics appears as something delivered by inside experts to a privileged audience.

A third concern has to do with the images implied by the book's characters—who they are and what that may say about who does mathematics. Robert and the Number Devil, both good at mathematics, are male. Of course, so is Mr. Bockel, an unsavory character. But the fact that neither of the main characters is female is significant. Could a bright young girl not have met up with the Number Devil instead of, or along with, Robert? Or, more boldly, could the Number Devil have been a brilliant and imaginative woman instead of an bright, impatient man? The choices of gender in the book fall unfortunately in line with cultural stereotypes which are unlikely to foster change or appeal invitingly to girls. Neither does the characterization help break stereotypes of mathematicians. The Number Devil is disdainful and impatient and thinks others are stupid and slow. He sees mathematics as simple and bears little responsibility for explanation, often disparaging Robert's questions or confusions. Is this the kind of being that children should see as typical of a good mathematician? Funny as he may be, does he help attract people to mathematics?

A final issue centers on mathematical warrants and what is implied about the basis for learning and knowing mathematics. The mathematical visitor will likely come away from the book aware of certain facts and truths, but still learning mathematics by acceptance rather than by conviction. The mathematics is well and colorfully staged. The scope of the book ranges widely, some fascinating doors are opened, and some surprising connections are made, such as between the multiple incarnations of the Fibonacci sequence and the golden mean. Ironically, however, mathematics is presented a bit like magic—intriguing and slightly mystifying, without explanation or development. Hardly any credible approximations to a proof of anything are offered. The ideas are more like invitations to something to be pursued beyond the book itself, like a preview of a good film, with highlights of dramatic scenes intended to lure the reader into pursuing the full version. The notion of proof is actually one of the topics dealt with in the book, on the eleventh night, but even that is left in metaphorical form rather than offering a substantial example. The book presents only a couple of illustrations of significant reasoning. We found the exposition of the mathematics somewhat frustrating—enticing but never consummated. One could get the feeling that mathematics was akin to a bunch of magic acts, a message surely not desirable to entice broader interest in the subject.

Given these concerns, we hesitate to recommend *The Number Devil* as a book that could be profitably read by most upper elementary and middle school children without adult guidance—that is, read for acquiring new mathematical interests, curiosities, and understandings. So we ask instead: Could children and their adult companions—teachers, parents, babysitters, grandparents—learn some mathematics from *The Number Devil*? What would it take for them to come away from their encounters with Robert and his friend with a new appreciation for the subject and their appetites whetted for more? Our answer is, unavoidably, a great deal. One thing it would take is knowledgeable guidance and interpretation. Reading this book aloud to a class or to one's child, one would have to be able to explain ideas on which the book only briefly alights. What, for example, went wrong when the Number Devil tried to multiply

$$11,111,111,111 \quad \times \quad 11,111,111,111,$$

and why? Why does the drawing of the two squares show that the diagonal of the first square is the square root of 2, why is this called "unreasonable", and what is important about the idea of an irrational number? Simply reading the book as is, children will (and should) have questions. A second thing it would take is good sense about how to respond to such questions. Which questions are worth pursuing? Which might simply be answered on the spot? A third thing is that the adult reader would need to understand the imaginative names given to mathematical terms and ideas, see the humor in them, and be able to introduce children to the conventional names. Most of the conventional names have their own interesting histories and bases, something that is left to be discovered in the wake of Enzensberger's choice to use a fanciful invented lexicon, suitable for dreams but not for mathematical real life.

An adult reader would need to have good mathematical sense and taste to help children use and profit from *The Number Devil*. The chances are slim that most readers who use this book with children will have such sensibility. This reduces the chances of their learning along with their junior companions and reduces what they can help children glean from this fanciful story. But fortunately mathematical sensibility and experience are not the only resources that could support children learning from this book. Good sense about what will intrigue children would help, as would sensitivity to what is, to the young reader, especially complicated or especially accessible. A mathematically curious and adventurous adult might be able to guide children's experience of *The Number Devil* in fruitful ways. Similarly, a mathematically sophisticated adult who is fascinated with young children's thinking might also be able to use this book to open fascinating doors. Without some modicum of both, however—interest in children and sensibilities about significant mathematics and its pursuit—*The Number Devil* may fall short of its ambition and its potential.

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[1] JOHN ALLEN PAULOS, *Innumeracy*, New York, 1988; *A Mathematician Reads the Newspaper*, New York, 1995.

[2] NORTON JUSTER, *The Phantom Tollbooth*, New York, 1961.