Ralph Phillips (1913–1998)

Peter Sarnak

Ralph Phillips

Ralph Phillips died on November 23, 1998, a year after his wife of fifty-five years, Jean, had died. To his colleagues, many friends, and collaborators around the world, Ralph will be remembered as the upbeat, tough-but-fair, straightforward, modest, and delightful person that he was.

Born in Oakland on June 23, 1913, he received his bachelor's degree from the University of California at Los Angeles in 1935 and his Ph.D. from the University of Michigan in 1939. His thesis advisor was T. H. Hildebrandt. There is no doubt that parts of Ralph's personality, particularly his toughness and his ability to work hard steadily, were rooted in his living through the Great Depression. From 1939 until 1942 he was a member of the Institute for Advanced Study in Princeton, an instructor at the University of Washington (where he met his future wife, Jean), and an instructor at Harvard. In a recent article in *The Mathematical Intelligencer* [8], Ralph described the prevalence of anti-Semitism in the academic culture of that period and how it affected his life. During the war he led a research group at MIT's Radiation Laboratory, the facility where much of the theoretical and practical work on radar technology was done. This work led to his book *Theory of Servomechanisms* [2], which for many years was the standard text in the subject. After the war he returned to mathematics, taking an assistant professorship at the Courant Institute of Mathematical Sciences. He moved to the University of Southern California the next year and moved again to the University of California at Los Angeles in 1958. In 1960 he moved for good to Stanford.

Ralph's research covered many areas, primarily in analysis, establishing him as one of the leading analysts of his time. He wrote over 125 research articles and 3 books over a period spanning more than sixty years. He was actively engaged in research almost to the very end. Moreover, the quality of his work remained consistently at the highest level. He was a prominent counterexample to the common view that mathematicians do their best work in their youth.

I will try to describe briefly the impact of some of Ralph's work. In his response [9] to receiving the 1997 AMS Steele Prize for Lifetime Achievement, Ralph recorded a list of his papers that he considered his most insightful. I have used this list as the basis for choosing which of his works to highlight here. There are three periods in his mathematical works, the first being the period before 1957, in which he was mainly concerned with functional analysis and particularly with semigroups of linear operators. In a series of influential papers on the foundations of semigroup theory, he developed new approaches that led to refinements and extensions of the theory. For example, his paper [7] introduced Banach algebra techniques into the subject. These works led to a coauthored revision of Hille's book [1] *Functional*

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Ph.D. Students of Ralph Phillips

Julius S. Bendat, USC (1953)
Arun V. Balakrishnan, USC (1954)
H. Potter Kerfoot, USC (1954)
Allen R. Sims, USC (1954)
Dale W. Thoe, Stanford (1964)
James W. Daniel, Stanford (1965)
Norman Alexander Shenk II, Stanford (1965)
Gerd Grubb, Stanford (1966)
E. J. P. Georg Schmidt, Stanford (1966)
James Paul Fink, Stanford (1967)
Michael Charles Reed, Stanford (1968)
Stanly Steinberg, Stanford (1968)
James V. Ralston, Stanford (1969)
James Thomas Beale, Stanford (1973)
Andrew J. Majda, Stanford (1973)
Chong Kiu Chan, Stanford (1974)
Daniel Alan Bondy, Stanford (1975)
John Nelson Palmer, Stanford (1975)
Peter Lawrence Trudinger, Stanford (1980)
Alex Christopher Woo, Stanford (1981)
Bettina Erika Christina Wiskott, Stanford (1982)

Analysis and Semi-Groups, which contains many of Ralph's new results and which is still a standard text today. After this he wrote very little on the theory of semigroups, though they play a prominent role throughout his later works.

The second period spans the years 1957 to 1977, during which time Ralph turned to partial differential equations and in particular to dissipative and hyperbolic systems. He also began studying the wave equation, which I think was his greatest mathematical love and which was from then on always at the center of his research. This period also marked the beginning of his long and very fruitful collaboration with Peter Lax. (When I once compared this collaboration with that of Littlewood and Hardy, Ralph's immediate reaction was the question, Which was he—Littlewood or Hardy?) Together Lax and Phillips developed a novel and powerful geometric approach to scattering theory for the wave equation in the exterior of a compact obstacle in $\mathbb{R}^n$. Their method, which is time dependent, allowed them to resolve many problems associated with solutions to the wave equation in such a domain (such as exponential decay of solutions [3]).

One finds in their work the key insights into the relations between poles of the scattering matrix and dynamics of rays, a relation that in related and analogous settings became a very active area of investigation. A key role in their axiomatic theory is played by a semigroup and its infinitesimal generator. Their theory is described in detail in the second edition of their book [4], which also contains a survey of the solution by others of some of the fundamental conjectures by Lax and Phillips about the poles of the scattering matrix.

In the mid 1970s Ralph was diagnosed with cancer, but successful treatment gave him a new lease on life and a new mathematical journey. In this, the third period of his work, he turned to automorphic forms, especially scattering theory on locally symmetric spaces, and related geometry and discrete groups. It is amazing that at age sixty-five his research branched out into completely new areas. He was always open to learning new things. He and Lax pursued their geometric time-dependent approach to scattering theory in the setting of finite- and infinite-volume hyperbolic manifolds. Their work, appearing in the book [5], contained new proofs of the spectral analysis of the Laplacian for these manifolds and, in particular, the analytic continuation of Eisenstein series and a derivation of the Selberg Trace Formula. An important and novel feature in their method is the introduction of a nonselfadjoint operator (naturally enough, the generator of the appropriate semigroup) whose spectrum consists of the eigenvalues of the Laplacian together with the poles of the Eisenstein series. With this setup one can study what happens to the spectrum of this operator when the hyperbolic surface is deformed. In his paper [10] this technique was used to show that the discrete spectrum (that is, the so-called Maass cusp-form spectrum) is very unstable under deformation. In particular, in today's understanding these elusive cusp forms, which are the building blocks in modern automorphic-form theory, exist in abundance only for reasons related to arithmetic. Ralph's interests in spectral problems for the Laplacian broadened, and in the paper [6] it is shown (in connection with the problem of hearing the shape of a drum) that the set of planar drums that sound the same is compact in a suitable C^∞ topology. During the last years of his life Ralph worked on generalizing his approach to scattering theory to higher-rank locally symmetric spaces. Unfortunately his health failed him, and he could not see this interesting project through.

I would like to end with some personal memories of Ralph. They all come from the period during which I knew him (1977–98) at Stanford. From the point of view of students and young postdocs, he was a model professor. He always welcomed people into his office to discuss mathematics or to do joint readings. Naturally he directed the theses of many students, many of whom are leaders in their fields today (see sidebar). In seminars and colloquia he never hesitated to ask a basic question, even though it might well show some ignorance. His were the questions that many in the audience were wondering about but were afraid to ask. In fact, Ralph was never interested in dazzling; he never put on airs. He was happy and satisfied with what he was doing, and it was

contagious. He much preferred to work together with others, and as is clear from his many successful collaborations, he was very good at it. He would listen to what a person had to say and come back the next day with a critical and insightful analysis of the previous day's discussion. He was a great colleague, and his contributions to the Stanford mathematics department, administratively and otherwise, were substantial. He had many passions besides mathematics. These included playing classical music. (Jean played the violin, and Ralph the clarinet.) He was an avid sailor and was always looking for some young hands to help man his boat "Wave Equation". He was a diehard 49-er football fan (not unique among Stanford faculty), watching and analyzing each game. He told me not long before he died that he had had a good life and was lucky to enjoy what he did. I also feel very lucky to have enjoyed what he did, and no doubt others feel the same way.

References

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