Anneli Cahn Lax (1922–1999)

Mark Saul

Anneli Lax

Anneli Lax's life ended on September 25, 1999—a life filled with service and friendship to the mathematical community. Lax was a gifted mathematician, a master of language, a remarkable teacher. Yet the defining characteristic of her life was perhaps not any of these gifts, but rather how she used them. She seemed to have an inner drive to share with others what she could do, and this drive led her from one endeavor to another in the service of mathematics and mathematicians.

Lax the Mathematician

Anneli Cahn was born in Katowice, then a German city, but now part of Poland, on February 23, 1922. Her family fled Hitler's regime in 1935 and settled in New York. She married Peter Lax, a fellow mathematician, in 1948. Their lives together included a shared love for mathematics.

Anneli Lax earned a bachelor's degree from Adelphi University in 1942 and moved on to graduate work at New York University (NYU). She received a Ph.D. in 1955 with a thesis done under the supervision of Richard Courant. The title was "On Cauchy's problem for partial differential equations with multiple characteristics", and it was published in *Communications on Pure and Applied Mathematics* in 1956. She rose through the faculty ranks at NYU to become a professor in 1961; she retired in 1992. (See the sidebar for a synopsis of her mathematics research.)

Lax the Editor

Lax's gift for language showed early in her career. Among other projects, she helped translate into English Courant and Hilbert's book *Mathematical Methods of Physics*. In a 1992 interview in *Focus* she remarked: "Courant often asked me to edit things that other people had written. In fact, he claimed that he hired me because I seemed more literate than most people. In the fifties, publishers didn't have people who could do mathematical copy editing. …I ended up doing everything. I even made page dummies. That was kind of fun: it was like playing with paper dolls."

Lax's greatest contribution to mathematical literature was triggered by a very different sort of event. The launch of the Soviet satellite *Sputnik* in 1957 was a shock to the American scientific community, a shock felt on every level. Much thought was devoted to the education of a new generation who would accelerate the pace of American scientific productivity. In mathematics education, a major contribution was made by the School Mathematics Study Group, a consortium of mathematicians and educators who scrutinized the

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mathematics curriculum. Their work continues to influence the field.

Out of this endeavor grew the New Mathematical Library. The notion was to make accessible to interested high school students, and to a more general public, deep results in mathematics described by research mathematicians. (This sort of work had long been going on in Eastern Europe.) Lax was asked to take over as general editor for this series, and under her guidance it grew to be the foremost mathematical expository series in the language. Upon her death it was renamed in her honor.

The New Mathematical Library, now published by the Mathematical Association of America, grew to 39 volumes and is still growing. Two generations of mathematicians found early sustenance in its contents, and numerous prominent members of the mathematical community found in it a vehicle to pass their knowledge on to a new public (see sidebar).

Lax was a skilled editor and strove to bring out the best work of the mathematicians who wrote for the series. Perhaps the most interesting of these interactions was one that in fact did not occur at all. In her own words: "The last chapter of *[An Introduction to] Inequalities*, by Beckenbach and Bellman is an interesting story. I wrote the last chapter and inserted it into the manuscript. Each of them thought that the other had written it and never said boo."

Lax the Educator

Lax's interest in communication did not stop with the written word. She became involved in education even before she got her Ph.D. She was always thinking about the lectures she was giving as well as those she was listening to, and was always looking to improve her teaching. As she remarked in her 1992 interview, "I started teaching at NYU in the mid-forties, before I had the degree…. [I]n all of the many years I've taught, I now, in retrospect, think that I didn't really understand teaching until the last ten years or so." Joanne V. Creighton, president of Mount Holyoke College, remarked in a ceremony awarding Lax an honorary degree in 1997 that she "led the way in changing mathematical pedagogy, in exploring the connections of mathematics to the larger curriculum, in understanding the interplay between language and mathematics."

This interplay very early became the focus of Lax's attention. When incoming groups of NYU freshmen found difficulty in learning mathematics, she designed and helped teach a course in mathematics and writing for which students got double credit.

This proved successful, but not successful enough for Lax. She was determined to follow the problem to its roots in the high schools. She teamed up with John Devine, a professor of education with significant experience working with teachers

Photographs courtesy of Peter Lax.

Anneli and Peter Lax.

in inner-city New York schools. Together they got funding from the Ford Foundation to train teachers from these schools in the methods Lax had pioneered at NYU. Devine recalls, "We brought the math teachers and English teachers together for joint sessions after school. This was unheard of. They didn't know each other. Anneli ran these sessions like a mathematical psychoanalyst. She was able to get the English teachers to lose their fear of introducing mathematical terms and concepts and procedures into their English classes. On the mathematics side, she was able to get the math teachers less afraid of word problems."

This work led Lax further into the details of teaching and learning, and she soon found herself tutoring students in these inner-city high schools, using her experience to understand how people outside the mathematical community think about our subject. Devine recalls,

> Anneli would come into the tutoring rooms and work with the kids themselves. This was beautiful to behold. She would sit at a tutoring table with some ninth-grade girl who had poor reading and writing skills. Although she was capable in higher realms of mathematics, Anneli would always begin where the student was. Her interest was in knowing the student's thought processes. She would do everything she could to try to get at the way kids were thinking, not the way she herself was thinking. She would get them talking, and suddenly the kid would be saying, "I went to the store this morning and helped Grandma figure out her food stamp budget." So

Anneli Lax's Research Mathematics

The mid 1950s was a period of intense interest in a basic existence-uniqueness condition for linear partial differential equations (PDEs) known as the "Cauchy problem". Lars Gårding had proved a fundamental result for constant-coefficient linear PDEs, and Jean Leray was just beginning his study of global solutions for linear PDEs with holomorphic coefficients.

The general *Cauchy problem* concerns a linear partial differential operator L of order m in n variables. The equation under study is $L(u) = 0$. Some submanifold S of dimension $n - 1$ is given, and initial values of the unknown function and its first $m - 1$ outgoing normal derivatives are specified on S. Some hypotheses are imposed on L, S, and the initial values. The question is whether there exists locally a unique solution of the equation on one side of the surface so that the initial conditions are satisfied.

For the situation of interest, Anneli Lax, with Richard Courant, had already proved a theorem that reduced one direction for the question in n variables to the question in 2 variables. It involved a parametrized family of 2-dimensional problems and gave a sufficient condition for the n-dimensional existence-uniqueness in terms of the sufficiency in 2-variables; the sufficiency in 2 variables had already been proved by E. E. Levi in 1909.

Lax's thesis dealt with the necessity in the 2-variable case. Let us state the result precisely when L has constant coefficients. Take the variables to be (x, t), and let $a_{i,j}$ be the coefficient of $\partial^{i+j}u/\partial^i x\partial^j t$. By a linear change of variables if necessary, we may assume that $a_{0,m} \neq 0$. Group the terms according to their order, and define

$$p_k(z) = a_{0,k}z^k + a_{1,k-1}z^{k-1} + \cdots + a_{k,0}.$$

If the roots of $p_m(z)$ are $\lambda_1, \ldots, \lambda_m$, then the top-order terms of L may be written

$$a_{0,m}\left(\frac{\partial}{\partial t} - \lambda_1\frac{\partial}{\partial x}\right)\cdots\left(\frac{\partial}{\partial t} - \lambda_m\frac{\partial}{\partial x}\right).$$

We assume that the λ_i are real but not necessarily distinct.

The lines $x = -\lambda_i t + c$ for each i and c are called *characteristics*. These have long been known to play a special role. This role can already be seen for the special case $(\partial/\partial t - \lambda(\partial/\partial x))u = 0$, whose general solution is $f(x + \lambda t)$ for any function f of one variable. Specifying initial data on a noncharacteristic line determines f everywhere, but specifying data on a characteristic line $x = -\lambda t + c$ determines f only at the one point c.

A curve S in the (x, t) plane is called *noncharacteristic* for L if it is nowhere tangent to a characteristic. The equation $L(u) = 0$ is said to be *properly solvable* relative to S if, for some k, all sets of k times differentiable initial data determine a unique solution of the Cauchy problem in a one-sided neighborhood of S.

It was known that the Cauchy problem is properly solvable for any noncharacteristic curve if the real numbers λ_i are distinct. Lax's theorem allows repetitions among the λ_i:

Theorem. *The Cauchy problem for the constant-coefficient equation $L(u) = 0$ in 2 variables and a noncharacteristic curve is properly solvable if and only if the greatest common divisor of the polynomials*

$$p_m(z), \frac{dp_m(z)}{dz}, \ldots, \frac{d^k p_m(z)}{dz^k}$$

divides $p_{m-k}(z)$ for $k = 1, \ldots, m - 1$.

In the previously known case in which the λ_i are distinct, the greatest common divisor in the theorem is 1 and therefore divides all $p_{m-k}(z)$.

Gårding's earlier result gave a different necessary and sufficient condition in the 2-variable constant-coefficient case, but it did not permit verification of the condition by inspection of the coefficients. Lax's thesis went on to consider the 2-variable variable-coefficient case. She proved that a certain condition generalizing the one in the above theorem was necessary and sufficient for proper solvability when the curve is noncharacteristic. The condition is now sometimes called the Levi-Lax condition. L. Svensson extended the theorem to n variables in 1968.

—*Anthony W. Knapp*

> Anneli would become interested in the food stamp budget. She would get around to the textbook, but only after understanding the kid's view.

Lax the Friend

Anneli Lax's accomplishments in mathematics, in writing, and in teaching are perhaps the easiest to document. Less concrete but perhaps more lasting are her contributions to the support of others in the field. For Anneli Lax was a steadfast and valued friend to many, offering support in countless tangible and intangible ways. One beneficiary of this support, Louise Raphael, writes:

> During my sabbatical year at Courant, I had the deep pleasure of living with the Lax family. Anneli had a genius for friendship. She was most loyal to all of her friends and accepted and loved them as they are.
>
> Anneli was a "mathematical egalitarian". She respected brilliance and could hold her own. She was mathematically

curious and demanded explanation of a theorem in terms she could understand. She encouraged women mathematicians and affected each of us deeply. She was ecstatic for us when our research went well. On the other hand, she never let us forget the need of engaging our students in the mathematical process and curriculum reform. She also made us realize the importance of mathematicians being involved in kindergarten through high school math education, an activity most researchers eschew. She volunteered her services to the New York City public schools, and on occasion she even volunteered my services. …It was Anneli who gave me the courage to become involved in elementary math education.

It is said that when we give of our possessions we have less, but when we give of our talents we have more. Throughout her life, Anneli Lax gave wholeheartedly of her many talents. We can only hope that Anneli felt rewarded by her own generosity. But we know for sure how much richer we all are for it.

References

[1] E. Pace, Anneli Cahn Lax, 77, a leader in the publishing of mathematics, obituary in the *New York Times*, 29 September 1999.

[2] D. Albers, Once upon a time: Anneli Lax and the New Mathematical Library, interview in *Focus*, June 1992.

New Mathematical Library

1. *Numbers: Rational and Irrational* by Ivan Niven
2. *What Is Calculus About?* by W. W. Sawyer
3. *An Introduction to Inequalities* by E. F. Beckenbach and R. Bellman
4. *Geometric Inequalities* by N. D. Kazarinoff
5. *The Contest Problem Book I: Annual High School Mathematics Examinations 1950–1960*. Compiled and with solutions by Charles T. Salkind
6. *The Lore of Large Numbers* by P. J. Davis
7. *Uses of Infinity* by Leo Zippin
8. *Geometric Transformations I* by I. M. Yaglom, translated by A. Shields
9. *Continued Fractions* by Carl D. Olds
10. *Replaced by NML-34*
11. *Hungarian Problem Books I and II*. Based on the Eötvös Competitions
12. *1894–1905 and 1906–1928*, translated by E. Rapaport
13. *Episodes from the Early History of Mathematics* by A. Aaboe
14. *Groups and Their Graphs* by E. Grossman and W. Magnus
15. *The Mathematics of Choice* by Ivan Niven
16. *From Pythagoras to Einstein* by K. O. Friedrichs
17. *The Contest Problem Book II: Annual High School Mathematics Examinations 1961–1965*. Compiled and with solutions by Charles T. Salkind
18. *First Concepts of Topology* by W. G. Chinn and N. E. Steenrod
19. *Geometry Revisited* by H. S. M. Coxeter and S. L. Greitzer
20. *Invitation to Number Theory* by Oystein Ore
21. *Geometric Transformations* II by I. M. Yaglom, translated by A. Shields
22. *Elementary Cryptanalysis—A Mathematical Approach* by A. Sinkov
23. *Ingenuity in Mathematics* by Ross Honsberger
24. *Geometric Transformations III* by I. M. Yaglom, translated by A. Shenitzer
25. *The Contest Problem Book III: Annual High School Mathematics Examinations 1966–1972*. Compiled and with solutions by C. T. Salkind and J. M. Earl
26. *Mathematical Methods in Science* by George Pólya
27. *International Mathematical Olympiads—1959–1977*. Compiled and with solutions by S. L. Greitzer
28. *The Mathematics of Games and Gambling* by Edward W. Packel
29. *The Contest Problem Book IV: Annual High School Mathematics Examinations 1973–1982*. Compiled and with solutions by R. A. Artino, A. M. Gaglione, and N. Shell.
30. *The Role of Mathematics in Science* by M. M. Schiffer and L. Bowden
31. *International Mathematical Olympiads 1978–1985* and forty supplementary problems. Compiled and with solutions by Murray S. Klamkin
32. *Riddles of the Sphinx* by Martin Gardner
33. *U.S.A. Mathematical Olympiads 1972–1986*. Compiled and with solutions by Murray S. Klamkin
34. *Graphs and Their Uses* by Oystein Ore. Revised and updated by Robin J. Wilson
35. *Exploring Mathematics with Your Computer* by Arthur Engel
36. *Game Theory and Strategy* by Philip D. Straffin Jr.
37. *Episodes in Nineteenth and Twentieth Century Euclidean Geometry* by Ross Honsberger
38. *The Contest Problem Book V: American High School Mathematics Examinations and American Invitational Mathematics Examinations 1983–1988*. Compiled and augmented by George Berzsenyi and Stephen B. Maurer
39. *Over and Over Again* by Gengzhe Chang and Thomas W. Sederberg

Other titles in preparation.