Book Review

Zero: The Biography of a Dangerous Idea

Reviewed by Jeremy Gray

Zero: The Biography of a Dangerous Idea
Charles Seife
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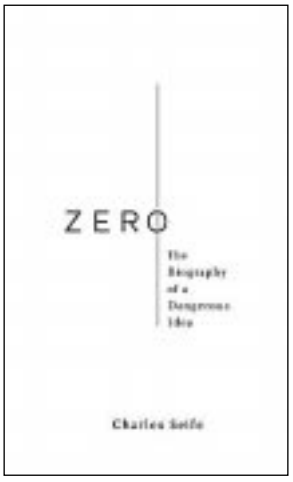
Popularising mathematics is an important task, and, done well, the history of mathematics offers as good a way as any. Some simplification to get the message across is in order, and authors can be forgiven if their enthusiasm carries them away. But it is hard to see what can justify this overblown and incoherent book.

Consider the opening paragraph. "The story of zero," we read, "has its roots in the time thousands of years before the first civilization…. It is an Eastern concept, born in the Fertile Crescent a few centuries before the birth of Christ." Well, which? Thousands of years before Egypt got going or well after? No matter; we are about to learn on this same opening page that "within zero there is the power to shatter the framework of logic." Was it shattered? When? Who repaired it and how? Or is it still completely broken? Fortunately, all that is meant is that multiplying and dividing by zero can cause problems until one knows what to do. Logic hasn't been affected at all, unless one actually believes, as is asserted on page 23, that dividing by zero allows for the destruction of the entire foundation of logic and mathematics. It would be hard to say whether the assertion stems from misunderstanding or simply constitutes absurd hyperbole, but division by zero won't be demystified here. Seife hasn't written that kind of book.

Inconsistencies abound. The Mayan calendar—which starts with 0, not 1—makes more sense, apparently, than the Western one, because it would have celebrated the recent millennium after 2,000 years and not after 1,999. Our calendar is troublesome. But we also learn on the same page (page 18) that the Maya had three calendars on the go the whole time, generating a different name for every day in a 52-year cycle. Trouble, anyone?



The hyperbolic tone persists right through the book. Zero and infinity are here found at the centre of the Renaissance, first at the vanishing point of Brunelleschi's painting of the Baptistery in Florence, where "everything sufficiently distant…is squashed into a zero-dimensional point and disappears." This happens, apparently, because "the vanishing point has caused most of the universe to sit in a tiny dot" (pages 86, 87). It would be too tedious, presumably, to explain that in single-focussed perspective a large amount of the domain plane is mapped arbitrarily close to the vanishing point, when instead the author can claim that "Zero had transformed the art world."

Not content with that, zero and infinity (no need to speculate very hard on how infinity gets into this book) move on to reach "the very center of the philosophical war taking place during the sixteenth and seventeenth centuries" (page 93). Here we learn that Descartes "rejected the void but put it at the center of his world." This brings up, not for the first time, the author's confusion between nothingness and zero that runs through the book. Problems with understanding the vacuum, whether medieval or quantum mechanical, are often very difficult, but conflating concepts of the vacuum with zero just muddles things. It's not

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that the transition from writing about zero to writing about the void is illegitimate, but it must be done with care. This book ends with the claim that "zero is behind all the big puzzles of physics" (page 214), whereas what is nearer the truth is that our intuition fails us in the very small and the very large, so the models we create to represent those realms sometimes give us misleading answers, such as zero and infinity.

Precision is not, however, Seife's strong point, and nowhere does his inattention to detail let him down more than in his discussion of the calculus. Almost inevitably we must read of 0/0. Plagiarism is apparently widespread in this electronic age, but at least if students plagiarise this book, their guilt will show clearly. The potted history of Newton and Leibniz is woefully out of date (Newton's laws were not cast as differential equations until Euler), but let that pass. To read that Leibniz's differentials still had the same 0/0 nature that plagued Newton's fluxions (page 123) is bad enough. To read that "L'Hôpital's rule took the first crack at the troubling 0/0 expressions that were popping up throughout calculus" (page 124) is almost completely wrong (grammar included). The calculus is not about giving a meaning, even a context-dependent one, to 0/0. The relevant concept here is that of the limit, which is not easy to explain but worth the effort. L'Hôpital's rule is a particular example of how limiting arguments can be used, but it has nothing to do with explaining how the core of the calculus works.

Next up, imaginary numbers. What can be said about an exposition that begins "Zero and infinity always looked suspiciously alike" (page 131)? We read on to discover that it is "possible to understand the infinite by studying zero." How? Via the Riemann sphere, of course, because "i held the key to zero's strange properties." And so on, and so on. Cantorian set theory somehow emerges from the discussion of the Riemann sphere, because, it seems, infinity has become an ordinary number ready for study (page 145). Although, "most appalling of all, infinity can be a zero" (page 147). There follows a quick trip through the distinction between countable and uncountable sets and the all-too-familiar parody of the relations between Cantor and Kronecker that E. T. Bell created (and for which Dauben's book [1] should have been the antidote). But how can an infinity be zero? Because the set of rationals on the line can be covered with a carpet of arbitrarily small size. It's not clear from this exposition whether the carpet is made of two-dimensional pieces, in which case the conclusion is trivial, or if they are one-dimensional strips, in which case a more interesting statement is being made. But in any case the confusion of cardinal properties and measure-theoretic properties is not helpful, nor does it tell us anything about the disagreements between Cantor and Kronecker.

Einstein's theories of space and time turn up a while later, and we learn that the speed of light is the ultimate speed limit, from which it follows, apparently, that "Nature has defended itself from an unruly zero" (page 178). But don't relax your guard, for in the next line we read that "However, zero is too powerful even for nature," all because Einstein's theory of general relativity permits black holes. Bizarrely, it is the general theory that Seife credits with the concept of space-time, and so space-time is the springboard for his account of rubber-sheet geometry. From curvature we get to black holes, which Seife mistakenly believes are point-size: "A black hole is a point. It takes up zero space, so there is no outer edge, no space where space begins to flatten out again" (page 182). This fundamentally confuses his account of the nature of black holes, event horizons, and singularities.

There are passages of accurate writing in this book, but far too much of it is way off. The bibliography shows one reason why: very little of the historical references are up to date, leaving the reader stranded in the nowhere land of Bell's *Men of Mathematics*. But a more fundamental reason is that Seife does not want to instruct. He wants the reader to marvel. Sloppy thinking is conducive to uncritical gawping at the wonder of everything, and inconsistency helps. Did Hippasus die for revealing the discovery of incommensurability (page 26), or is it all legend (page 37)? It doesn't matter if all one wants to do is make people say gosh. But if in this difficult, precious world of ours one of the minor wonders is that we have finally learned to talk a certain amount of sense about numbers (including zero) and to use them skillfully in geometry and physics, not least through the calculus, then the worthwhile aim is to convey some of that understanding. The things we understand are truly exciting, and the understanding is far better than the befuddled wonderment on offer here. One might even think that some of this understanding has become so well organised that we can teach to large audiences what even Newton struggled to apprehend. Careful focus on the task at hand, separating out topics that are easily confused, and confidence in the material are essential. Seife's book has too little of these. But if you thrill to hear (page 145) that "zero and infinity are eternally locked in a struggle to engulf all the numbers", then, hey, this is the book for you.

Reference

[1] J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite*, Princeton University Press, Princeton, NJ, 1990.