Mathematical Challenges of the 21st Century: A Panorama of Mathematics

Reception outside Royce Hall.

On August 7–12, 2000, the AMS held the meeting Mathematical Challenges of the 21st Century, the Society's major event in celebration of World Mathematical Year 2000. The meeting took place on the campus of the University of California, Los Angeles, and drew nearly 1,000 participants, who enjoyed the balmy coastal weather as well as the panorama of contemporary mathematics provided in lectures by thirty internationally renowned mathematicians.

This meeting was very different from other national meetings organized by the AMS, with their complicated schedules of lectures and sessions running in parallel. Mathematical Challenges was by comparison a streamlined affair, the main part of which comprised thirty plenary lectures, five per day over six days (there were also daily contributed paper sessions). Another difference was that the Mathematical Challenges speakers were encouraged to discuss the broad themes and major outstanding problems in their areas rather than their own research. Many of the speakers made serious efforts to communicate to a wide mathematical audience rather than to specialists. Taken together, the lectures provided a captivating portrait of a field with a seemingly inexhaustible appetite for intellectual challenges.

It was on the morning of August 8, 1900, that David Hilbert delivered his historic lecture at the International Congress of Mathematicians in Paris. In connection with that lecture, Hilbert posed twenty-three outstanding problems that subsequently had an important impact on mathematical research. While Mathematical Challenges was in part a celebration of Hilbert's lecture, no single individual's view of mathematics dominated the meeting. In fact, a hallmark of Mathematical Challenges was a diversity of views of mathematics and of its connections with other areas.

The Mathematical Association of America held its annual summer Mathfest on the UCLA campus just prior to the Mathematical Challenges meeting. On Sunday, August 6, a lecture by master expositor Ronald L. Graham of the University of California, San Diego, provided a bridge between the two meetings. Graham discussed a number of unsolved problems that, like those presented by Hilbert, have the intriguing combination of being simple to state while at the same time being difficult to solve. Among the problems Graham talked about were Goldbach's conjecture, the "twin prime" conjecture, factoring of large numbers, Hadwiger's conjecture about chromatic numbers of graphs, and the Riemann hypothesis.

The comfortable and elegant setting for the Mathematical Challenges lectures was UCLA's stately Royce Hall, which was built in 1929 and modeled on a basilica in Milan, Italy. Graham's lecture was followed by the Opening Ceremonies and a reception held on the terrace outside of Royce Hall. The meeting began in earnest the next day, Monday, August 7, with a lecture by Charles Fefferman of Princeton University. Speaking about the Navier-Stokes and Euler equations of fluid mechanics, Fefferman set a high standard for clarity of exposition at the meeting. A central question about these equations is whether or not solutions develop singularities in finite time. Fefferman drew on mathematical analysis of the

Michael Freedman

James Arthur

Left to right: Sergiu Klainerman, Helmut Hofer, and Peter Sarnak

equations as well as on information about simulations and physical experiments to describe the current state of knowledge.

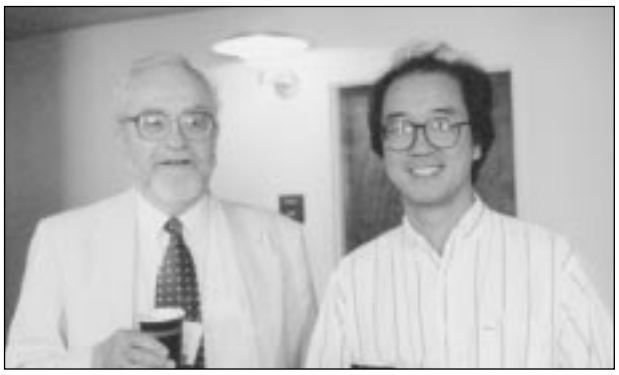
Fefferman's Princeton colleague Sergiu Klainerman also presented a lecture about nonlinear partial differential equations (PDEs), but from a completely different viewpoint. Rather than focusing on specific equations, Klainerman presented an overview of a wide swath of the field of PDEs, taking a conceptual approach to a subject that is often shrouded in technicalities. Klainerman categorized types of equations according to their origins, such as, for example, equations arising from fundamental notions in geometry or mathematical physics, or equations that emerge from taking limits or symmetry reductions of more complicated equations. Two essential goals are understanding the range of validity of approximations and developing useful notions of generalized solutions.

Geometry and geometrical ideas arose in many of the talks. William P. Thurston of the University of California, Davis, discussed his "geometrization conjecture", a generalization of the Poincaré conjecture that provides a way of classifying 3-manifolds. Although the conjecture is not proved yet, it is widely believed to be correct and to provide deep insights. Using two laptop computers, Thurston dazzled the audience with an array of computer software tools that are designed to help mathematicians develop new intuition about 3-manifolds. Clifford Taubes of Harvard University talked about the very different world of 4-manifolds. The topological classification of 4-manifolds was done by Michael Freedman in the 1980s, but the plethora of smooth structures possible on 4-manifolds makes a smooth classification extremely difficult. Taubes pointed to certain research directions that look promising.

Plenary Speakers at Mathematical Challenges of the 21st Century

James G. Arthur, University of Toronto
Alexander A. Beilinson, University of Chicago
Michael V. Berry, University of Bristol
Haim Brezis, Université de Paris VI and Rutgers University
Alain Connes, Collège de France and Institut des Hautes Études Scientifiques
David L. Donoho, Stanford University
Charles L. Fefferman, Princeton University
Michael H. Freedman, Microsoft Research
Ronald L. Graham, University of California, San Diego
Helmut H. W. Hofer, Courant Institute, New York University
Richard M. Karp, International Computer Science Institute
Sergiu Klainerman, Princeton University
Maxim Kontsevich, Institut des Hautes Études Scientifiques
Peter Lax, Courant Institute, New York University
Simon A. Levin, Princeton University
László Lovász, Microsoft Research
David Mumford, Brown University
Peter Sarnak, Princeton University
Saharon Shelah, Hebrew University and Rutgers University
Peter W. Shor, AT&T Laboratories
Yakov G. Sinai, Princeton University
Richard P. Stanley, Massachusetts Institute of Technology
Dennis Sullivan, City University of New York Graduate School
Clifford H. Taubes, Harvard University
Jean E. Taylor, Rutgers University
William P. Thurston, University of California, Davis
Karen Uhlenbeck, University of Texas, Austin
S. R. S. Varadhan, Courant Institute, New York University
Edward Witten, Institute for Advanced Study
S. T. Yau, Harvard University

Edward Witten



All photographs by Allyn Jackson.

Program Committee Chair Felix Browder (left) and Local Arrangements Committee Chair Tony Chan.

Another theme was the use of mathematics in science and technology. David Donoho of Stanford University discussed the field of data analysis, which is becoming increasingly important as humankind amasses ever more, and ever more complicated, data. He pointed out that most of the techniques now used in data analysis were developed sixty years ago. Ever more powerful computers have pressed these techniques to their limits, and now all the barriers to improvements are mathematical. The keen need for ideas from mathematics was also illustrated in the lecture by David Mumford of Brown University, who talked about the use of statistical methods in modeling visual perception, and in the lecture by Richard Karp of the International Computer Science Institute, who spoke on the use of mathematics in molecular biology, particularly genomics.

Many of the lectures provided vivid illustration of the impact of computers on mathematical research, not only in the way that they provide means for experimentation, but also in the deep mathematical problems they raise. One such problem is that of quantum computing, the subject of two lectures at the meeting. Peter Shor of AT&T Laboratories talked about his groundbreaking work on a quantum algorithm for factoring integers and on quantum error-correcting codes. He pointed out that although quantum computers with 7 "quantum bits" have been built, one needs several thousand such bits to factor large numbers. Providing a different take on quantum computing, Michael Freedman of Microsoft Research described his ideas for exploiting topology, in particular braid groups and the Jones polynomial, to model quantum computation.

Many at the meeting appreciated the lecture by James Arthur of the University of Toronto, who gave an especially accessible and clear overview of the Langlands program, an important area of research that seems destined to have an enormous impact on the subject but that is quite technical and difficult to explain to those outside the field. Touching on some of the same themes was the lecture by Peter Sarnak of Princeton University, in which he spoke of "the unreasonable effectiveness of modular forms in mathematics" and the mysteries of the Riemann hypothesis. Another highlight was the lecture by Edward Witten of the Institute for Advanced Study, who discussed the importance of quantum field theory in physics and mathematics. The last lecture of the meeting, presented by Alain Connes of the Collège de France and the Institut des Hautes Études Scientifiques, provided a marvelous ending. Connes presented a lucid description of noncommutative geometry, from its roots through its current directions. He also briefly outlined a program for proving the Riemann hypothesis using the ideas of noncommutative geometry.

Mathematical Challenges was first proposed by AMS president Felix Browder, who chaired the program committee. "The meeting provided a way of crystallizing and focusing attention on classical problems in mathematics and on the diversity of mathematical development," he said. One of the purposes of the meeting was to demonstrate the dynamism of mathematics and its connections to other areas of science, "and this came through sharply." Browder is now busy collecting papers based on the talks in preparation for publication of a proceedings volume. "It was a very unusual meeting," Browder said, "and one that may have a significant influence on the future of mathematics."

—Allyn Jackson