Einstein's Static Universe: An Idea Whose Time Has Come Back?

Aubert Daigneault and Arturo Sangalli

A Tribute to Irving Ezra Segal (1918–1998)

What is the shape of space? Is the universe finite or infinite? Did the world have a beginning, or has it always existed? These fundamental questions have intrigued and baffled humans since the most ancient times. But it was only in the twentieth century, with the development of powerful tools for probing the immensity of the skies, that it became possible to explore the cosmos beyond our galaxy's neighborhood. Concurrently, advances in physics and mathematics provided the conceptual framework—the language, so to speak—in which to formulate comprehensive theories whose validity could be objectively put to the test. Scientific answers to the above questions finally appeared to be at hand.

Space-time is defined as the totality of pointlike events—past, present, and future—in the history of the world. A "pointlike event" means an "instantaneous event taking place at a point of space" as opposed to an "event" that is extended in either time or space, such as World War II. In 1917 Albert Einstein proposed a model for space-time known as the Einstein Universe (EU), in which the totality of physical space is finite and curved [3]. "Nothing in general relativity has intrigued the lay public more than Einstein's possibility of a closed, finite spatial universe," observed Theodore Frankel in his 1979 introductory book to Einstein's theory.

In EU, space may be mathematically described as a three-sphere S^3 of fixed radius r, i.e., as the boundary of a four-dimensional ball, by the equation $u_1^2 + u_2^2 + u_3^2 + u_4^2 = r^2$. In Einstein's model, time had no beginning: it is infinite in both directions, and so the universe has always existed and will always exist. Hence EU may be presented as the cartesian product $R \times S^3$ where R is the whole real timeline.

In 1929 Edwin Hubble interpreted the galactic redshift phenomenon detected by Vespo Slipher starting in 1912—the change in the observed frequency of light waves—as a Doppler effect, that is, as caused by the motion of a luminous source away from the observer. The Doppler interpretation became known as the Expanding Universe theory, whose most developed form is the cosmology of Friedmann and Lemaître and according to which galaxies are moving away from each other. As told in 1997 by David Dewhirst and Michael Hoskin in *The Cambridge Illustrated History of Astronomy*, "There is no doubt that the nearly simultaneous detection of the redshifts and the derivation of solutions of Einstein's equations that suggested that the universe would be expected to expand greatly encouraged this interpretation."

The Expanding Universe theory later begot the Big Bang theory, which maintains, in addition to universal expansion, that time and the universe had a beginning, the "Big Bang".

Hubble also stated his famous law: The galaxies recede from each other with a velocity

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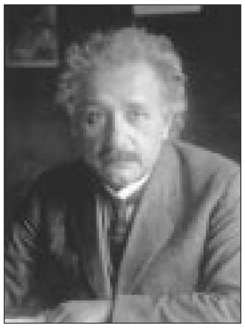
Albert Einstein



proportional to the distance separating them. But he did not rule out that, by virtue of some unknown mechanism, the redshift might result from space being curved. His reservations led him to write in 1936 that "the assumption that redshifts are not velocity shifts is more economical and less vulnerable, except for the fact that, at the moment, no other satisfactory explanation [i.e., apart from the Doppler effect] is known." Once Hubble's expansion seemed to have been established as an empirical fact, Einstein was forced to abandon his static universe model.

The observation in 1964 of the so-called cosmic background radiation (CBR) is often considered to be the smoking gun supporting the assumption that there was a Big Bang. But in fact the CBR might have other causes. Erwin Finlay-Freundlich and Max Born had already predicted the existence of such radiation in 1953, eleven years before its detection, on the basis of a stationary universe. The interpretation of the CBR as a faint echo of the birth of the universe gave greater credence to the Big Bang theory, which became widely accepted as the standard astronomical gospel.

A Two-Time Cosmology

By the early 1970s the rapidly increasing mass of statistical data on quasars and galaxies provided a substantial basis for questioning the Big Bang cosmology. In 1972 Irving Ezra Segal, a professor of mathematics at the Massachusetts Institute of Technology, picked up where Einstein had left off and proposed a variant of special relativity [13]: chronometric cosmology (CC), so called because it is based on the analysis of time. His 1976 book *Mathematical Cosmology and Extragalactic Astronomy* [14] contains a detailed presentation of the theory.

According to CC, Einstein's model is the correct one to understand the universe as a whole (i.e., global space-time), except that there are two kinds of time: a cosmic or Einstein's time t, and a local or Minkowski's time x_0, which is (perhaps!) the time measured by existing techniques. In Segal's words, "The key point is that time and its conjugate variable, energy, are fundamentally different in the EU from the conventional time and energy in the local flat Minkowski space M that approximates the EU at the point of observation." Simply put, Einstein's cosmic time t is the "real" one, whereas Minkowski's time is only an approximation of t.

Which coordinates, those of EU or those of M, are actually measured in observations is empirically immaterial, as the two differ by unobservably small amounts. Indeed, assuming "as is commonly believed that $r \geq 10^8$ lightyears," the two "deviate by less than one part in 10^{15} out to distances of 1 lightyear, or of less than 1 part in 10^6 out to galactic distances. There is no apparent means to detect such differences in classical observations" [13, p. 766].

This two-time situation arises from the essentially uniquely defined conformal immersion of Minkowski space $M = R \times R^3$, in which space is ordinary three-dimensional Euclidean space, into $\mathrm{EU} = R \times S^3$. This immersion is unique up to the group of causality-preserving motions of EU and is a relativistic variant of stereographic projection. Minkowski space M can be thought of as being tangent to EU just as the complex plane is tangent to the Riemann sphere. In spite of this, the immersion of M into EU preserves neither the time coordinate nor the space coordinate in the factorizations of these space-times as "time × space". When units are chosen to make the speed of light c equal to 1, Einstein's and Minkowski's time coordinates are related by the equation

$$x_0 = 2r\tan(t/2r), \tag{1}$$

from which the relation

$$z = \tan^2(t/2r) \tag{2}$$

of an observed redshift z to time of propagation t or, equivalently, geodesic distance on the 3-sphere, may be derived essentially by simple differentiation.

Equation (1) implies that as t varies from $-\pi r$ to $+\pi r$, ordinary time goes from minus infinity to plus infinity. Hence eternity (past and future) in the ordinary sense corresponds to a finite interval of cosmic time, which cosmic time t, nevertheless, varies over the whole real line. As for equation (2), it reveals that for small values of t (or, equivalently, of the distance), the redshift varies as the square of t, in contradiction with Hubble's law, which is linear. From (2) we also see that as r tends to infinity, z tends to 0. Hence, as envisaged as a possibility by Hubble himself, the curvature of space is the reason for the cosmic redshift in CC.

An application of l'Hospital's rule to equation (1) shows that, as expected, x_0 tends to t as r tends to infinity. The above-mentioned unobservably small differences between the two times can easily be established from the series expansion of x_0 in powers of t:

$$x_0 \sim t + t^3/(12r^2) + t^5/(120r^4) + \dots. \tag{3}$$

Because the radius r of the universe is very large, it follows from (3) that only extragalactic observations can be relevant for telling the two times apart. As Segal writes, "…in the absence of precise observations of masses and distances outside the solar system, only the redshift and related observations appear to fall in the category required to distinguish, potentially at least, between the two clocks" [10, p. 857].

Equation (2) may also be derived [12, 17] in (and above) the observable frequency range by a rigorous analysis based on Maxwell's equations governing light and, more generally, all electromagnetic radiation. These equations are defined primarily in M, but extend uniquely, along with their solutions of finite Minkowskian energy, to the larger universe EU. It follows from this analysis that a free photon will experience a redshift when propagated over a very long period according to Einstein time.

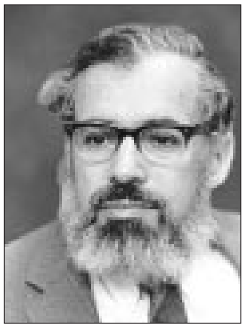
In CC the Einstein universe EU is first thought of as empty of matter just like Minkowski space-time M, which is the ordinary space-time of special relativity, and appears as a natural alternative to the latter. In that sense EU belongs in the first place to special relativity rather than to general relativity (Einstein's theory of gravitation) in whose framework it was presented by its initial proponent. EU is uniquely arrived at in CC by means unrelated to gravitation.

Thus a distinction is introduced in CC between a space-time thought of as empty and a universe which is inhabited with matter and which enjoys only approximately the symmetries of the underlying space-time. This distinction has no counterpart in general relativity, where there is no a priori geometry and where it is precisely the introduction of matter that determines the geometry.

Causality

Space-time defined as the totality of all events—past, present and future—is, before anything else, a partially ordered set; the relation $p < q$ amongst two events means that p precedes q. This relation, known as the relation of causality, of precedence, or of anteriority, is the most immediate observational data. It conceptually and psychologically precedes the measurement of distances and duration and is independent of any observer.

The models of space-time generally considered plausible are known as Friedmann-Robertson-Walker (FRW) space-times. These FRW space-times are all endowed with such a causality relation, which derives from their pseudo-Riemannian metrics, and they can all be immersed by essentially unique causality-preserving maps into the Einstein universe, though the metric of time or space is not preserved by such immersions. EU thus appears as the maximal FRW space-time. Such a space-time is defined as a product $I \times S$ where I is an interval (which may or may not be bounded from below or from above) on the real timeline and S, the space, is a three-dimensional Riemannian manifold of constant curvature.



Photograph courtesy family of I. E. Segal.

Irving Ezra Segal

If $d\sigma^2$ denotes the metric of S, the FRW space-time is endowed with the hyperbolic pseudo-Riemannian metric also known as the Lorentz metric

$$ds^2 = -c^2 dt^2 + r^2(t) d\sigma^2,$$

where c is the speed of light and $r(t)$ is a function of time t in I known as the "radius of the universe at time t". This is a mathematically unrestricted scale factor; in particular, its time derivative $r'(t)$ may very well exceed the speed of light without special relativity being violated. Indeed, in an FRW space the distance between two stationary points x and y of space increases at the rate $r'(t)$. The popular inflation theory of Alan H. Guth [4], an MIT physicist, claims that "…during the earliest moments of time all space expanded far faster than the speed of light."

In Big Bang models, $r(t)$ tends to 0 as t tends towards the lower limit of I, the instant of the Big Bang (excluded from I), whereas in EU, $r(t)$ is a constant and I is the whole real line. Any such FRW space-time is at each point (= event) locally approximated by its tangent space, which is essentially Minkowski space-time.

A vector $x = (x_0, x_1, x_2, x_3)$ in Minkowski space M is called a time vector if $x_0^2 - x_1^2 - x_2^2 - x_3^2 > 0$; it is said to be oriented towards the future if $x_0 > 0$. The causality relation in M is defined by saying that $x < y$ if and only if $y - x$ is a time vector oriented towards the future. Special relativity's essential prescription is that the worldline of any material point m, i.e., the set of all (t, x) such that m is at spatial point x at time t, be a timelike curve, i.e., one whose tangent at any point is generated by a time vector of the tangent Minkowski space at that point of space-time.

A 1953 theorem of the Russian mathematicians A. D. Alexandrov and V. V. Ovchinnikova, rediscovered a decade later by E. C. Zeeman, turned out to be fundamental for CC (that was to be developed later) as it entails that causality-preserving maps between FRW spaces are the same as conformal maps. This theorem is well expounded by Gregory L. Naber in his 1992 book *The Geometry of Minkowski Spacetime*.

In FRW spaces, in particular in EU, the causality relation is defined by declaring that an event p precedes an event q if p can be joined to q

by a directed curve such that at each point the forward-looking tangent is a time vector oriented towards the future.

Conservation of Energy

The Einstein universe and Minkowski space-time are the only two space-times satisfying general conditions embodying three fundamental physical principles: first, the cosmological principle, also known as the Copernican principle, which states that there is no preferred direction or point in space; second, the "principle of inertia", the statement that there is no preferred timelike direction, which implies the equivalence between observers in relative motion at the same point; and third, the statement that there is no preferred moment on the time axis, which is a precondition for the principle of conservation of energy to hold. Such principles should be abandoned only in the face of compelling evidence to the contrary. Segal contended that not a shred of such evidence exists and that, on the contrary, the verifiable consequences of these principles are confirmed by all available astronomical data.

Segal's explanation of the origin of the cosmological redshift is free from the adjustable parameters that intervene in the Expanding Universe theory. Moreover, unlike the Big Bang cosmology, CC does not need to resort to ad hoc scenarios—such as "evolution" for quasars and radio sources, and "inflation" at the hypothetical beginning of the universe—reminiscent of the epicycles introduced by ancient astronomers in their futile efforts to reconcile the observed planetary orbits with a geocentric universe. Guth's still very fashionable "inflationary universe" [4] was invented in the early 1980s to resolve two outstanding problems of Big Bang theory, namely, the so-called "flatness" and "horizon" problems, which simply do not arise in CC.

A major merit of CC is to reestablish the principle of global energy conservation, which supporters of Big Bang cosmology are willing to relinquish. "The explicit time dependence of the Friedmann-Lemaître [expansionary] models break Lorentz invariance as well as conservation of energy-momentum. The loss of such theoretically fundamental as well as experimentally well-established laws must be weighed in the balance against the simplicity of a Doppler explanation for the redshift" [11, p. 4805]. Just as there are two times in CC, there are correspondingly two concepts of energy. The photon frequency is a measure of the Minkowski energy, and the redshift reduces that energy, which is therefore not conserved. But in CC, thanks to its cosmic time, this energy is not lost.

Freshly emitted photons have small spatial support, and the energy of light composed of such photons is conventionally inferred from laboratory measurements of the wavelength or the frequency, in accordance with the Minkowski energy. However, photons of extragalactic origin are necessarily of cosmic spatial support when observed. And "a direct measurement of the Einstein energy of an extragalactic photon may depend on more than the pointwise oscillation of the photon wave. Indeed mathematical analysis shows that the Einstein energy of a photon depends not only on the oscillation frequency of the photon but also on the number of oscillations, which appears beyond the scope of the present methods to observe" [17, p. 316].

As Roger Penrose says, "The utility of the concept of energy, in general, arises from the fact that it is conserved." In Einstein's theory of general relativity, as opposed to what is the case in the special theory of relativity, there is only a differential (or infinitesimal) conservation law and not an integral conservation law.[1] The former implies the latter only in the presence of a group of temporal translations that are isometries of the space-time model. Energy is integrally conserved if the flux of energy through the boundary of any compact 4-dimensional domain of space-time vanishes. An example of such a compact domain is a finite portion of space during a finite interval of time [5, pp. 61, 62].

Aside from Minkowski space-time, the Einstein universe is the only one amongst the FRW spaces that has such a temporal group of isometries and consequently the only one in which the principle of integral conservation of energy is valid.

The energy that is conserved is, in particular, that of electromagnetic radiation in vacuum "which is basically all that is observable in large scale astronomy." Both electromagnetic energies, the Minkowskian and the Einsteinian, are conserved in their respective cosmos M and EU. "They are both representable as the integral over space in the respective cosmos of the square of the electromagnetic field, in terms of spatially natural coordinates,…and the latter integral is always larger" [17, p. 317]. The free electromagnetic field can be construed as a Hamiltonian system in infinitely many dimensions whose total energy is as just said, and the dynamical group is induced by the isometry group of temporal translations of space-time, both in M and in EU.

Challenging Hubble's Law

In 1929 Edwin Hubble made the sensational announcement that the galaxies were receding from each other. This phenomenon had already been detected more than a decade earlier by Vespo Slipher, an astronomer at the Lowell Observatory, who had reported that at least some of the faint

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[1] *We purposely refrain from using the word "local" in this connection, as it sometimes means "differential" and sometimes "integral".*

patches of light then called nebulae appeared to be receding from the Milky Way. When Hubble plotted the observed redshift (or rather, in his interpretation, the recession velocity v) against the estimated distance d for a small number of galaxies, he claimed to have detected a "roughly linear" pattern, corresponding to the relation $v = Hd$, where the constant of proportionality H is known as Hubble's constant.

As Hubble himself admitted, the estimated distances from which his law was derived were in part uncertain, but the best then available. In the ensuing decades other astronomers followed up with their own verifications, and the linear redshift-distance relation is by now generally accepted.

But Segal denounced the Hubble school for having based its confirmation of the law on observations of a relatively small class of objects, the bright cluster galaxies, and without specifying a selection criterion: "The central source list for such galaxies has been the catalog of Abell, who stated: 'In determining whether a cluster meets this criterion [for inclusion in the catalog], it was assumed that the redshifts of clusters are proportional to their distances.' […] For this [circular argument] and other reasons, the observational case for the Hubble Law on the basis of cluster galaxy samples is at best inconclusive" [16, p. 295].

Since the proposal of CC (chronometric cosmology), Segal and his collaborators, notably J. F. Nicoll of the Institute for Defense Analyses, have carried out an extensive analysis of all published and reliable astronomical data on galaxies and quasars in accordance with modern statistical principles and in a reproducible manner. Their conclusions have invariably been consistent with the predictions of CC and at variance with those of the expansionary theory and Hubble's law. As Segal points out, "Older statistical methods were dependent on parametrizations that developed from plausibility and convenience more than direct observation. Rapid computer simulations now make possible efficient nonparametric methods that provide objective probabilities of deviations of specified observations from theoretical prediction, in circumstances far more complex and realistic than would be possible by theoretical estimation of probabilities."

Using a sophisticated bootstrap statistical technique that they call ROBUST, which takes into account the so-called "observational cutoff bias" making faraway celestial objects less likely to be observed than closer ones, Segal and Nicoll demonstrated that the quadratic redshift-distance law predicted by CC fits the available experimental data on the more than 2,000 quasars of a reputed comprehensive 1989 catalogue much better than does Hubble's law [16].

Because Hubble's law is incompatible with chronometric cosmology, its status has become a crucial test for this theory. In 1992 the statistician Bradley Efron, a creator of the bootstrap technique, and Vahé Petrosian, a physicist, both at Stanford University, published a paper [2] which appears to disprove Segal's claims. However, the data on which they based their conclusions are not readily available and have been "criticized for being too inaccurate," as Petrosian himself acknowledges. In addition, the criteria for selecting a subsample of 492 redshifts used to confirm the Hubble law was not specified.

In a 1997 article [6] Daniel Koranyi of the Harvard-Smithsonian Center for Astrophysics and Michael Strauss of Princeton University claim that since its original announcement by Hubble, "observational evidence has mounted steadily" in favor of the linear law. This opinion conflicts with Segal's: "Careful investigation shows that the reported confirmation of Hubble's Law has been based on large components of wishful thinking…." At any rate, discussing an earlier paper [15] by Segal and his collaborators, the authors point to what they perceive as a methodological pitfall affecting Segal's analysis against Hubble's law: the sensitivity of the estimation of the luminosity function (i.e., the probability distribution of the intrinsic luminosity of galaxies) to the number of bins into which the sample used for that purpose is decomposed. This, they contend, "could lead one to mistakenly favor the quadratic law." However, Koranyi and Strauss's statistics suffer from other shortcomings, some already pointed out by Segal in [15].

Such challenges to Segal's work are rare. Over the years there has been very little response to his claims, and he has refuted all published criticism of CC.

The Three Empirical Pillars of Big Bang Theory

The tenants of the Big Bang theory interpret the redshift as a Doppler effect indicating the expansion of the universe. They interpret the cosmic background radiation as an echo of the initial explosion. The third empirical pillar of the theory is called "Big Bang nucleosynthesis", which claims to predict the abundances of the four light elements: deuterium, helium-3, helium-4, and lithium-7.

The much-trumpeted alleged concordance of Big Bang predictions with observations goes essentially as follows. What is called the abundance of a chemical element is the ratio of the amount of this element in the universe to that of hydrogen, which is the most prevalent. In his 1989 review of Big Bang nucleosynthesis, Gary Steigman of Ohio State University [18] explains that the only parameter on which the predicted abundances depend is η, the universal ratio of nucleons to photons. For a narrow interval of values of η tightly constrained by the observed abundances, "the standard hot Big Bang model predicts the individual abundances to lie precisely in this range."

But, as Steigman says, "Abundances are not *observed*. Abundances are *derived* from the observational data, often following a long and tortuous path involving theory.…Errors (or uncertainties), often systematic, may be introduced at many steps in the overall process of deriving abundances from observational data. Furthermore we are here concerned with primordial abundances. Even if present day universal abundances were known to arbitrary accuracy (which they are not!), we still would have to employ theory and observation to extrapolate back to obtain primordial (or at least pregalactic) abundances. Additional errors (uncertainties) are surely introduced here too."

N. Yu Gnedin and Jeremiah Ostriker of Princeton University, who are also Big Bang supporters, declared in 1992: "Light element nucleosynthesis has been a central pillar supporting the standard FRW hot Big Bang cosmological model.…But there are several confusing and apparently inconsistent elements in the canonical picture which have led to 'patches' which are quite ad hoc and are accepted only because of our familiarity with them and our basic belief that the underlying standard model is accurate."

Steigman still expresses some reservations in a 1996 paper: "Indeed, the success of standard Big Bang nucleosynthesis in predicting the abundances of the light nuclides with only one adjustable parameter η restricted to a narrow range while the abundances range over some 9 orders of magnitude is impressive indeed. However, as with the nineteenth century standard model [Newtonian gravity], some clouds have now emerged on the horizon. Recent analyses…point to a crisis unless the data are in error, or the extrapolation of the data are in error, or there is new physics." In more recent articles valiant efforts are made to alleviate the crisis, which nevertheless perdures.

These current difficulties of Big Bang cosmology may ultimately lead to an "agonizing reappraisal" of its basic tenet, the Doppler interpretation of the redshift. But there is no sign of this yet, as the notion of an expanding universe is by now such an entrenched belief in the scientific community and the population at large.

In any case, chronometric cosmology has responses to these contentions often presented as definite "proofs" of the Big Bang theory. In CC the cosmic background radiation is interpreted as the equilibrium state of free photons that have been scattered, reemitted, or absorbed many times in the course of possibly thousands of circuits of the 3-sphere. In Segal's words: "The observed blackbody form of the cosmic microwave background is simply the most likely disposition of remnants of light on a purely random basis, assuming the classic principle of the conservation of energy, and is not at all uniquely indicative of a big bang."

Back in 1926 A. E. Eddington had already calculated that the "temperature of space" produced by the radiation of starlight would be found to be 3 degrees Kelvin, as is the case of the CBR.

In a private communication to one of the authors, Bertram Kostant, a mathematician at MIT, summed up some recent joint work of his and Nolan Wallach's of the University of California at San Diego as follows: "One of the major props for the Big Bang theory is the black body radiation profile of the background radiation. Cosmologists say that only an enormous heat source could account for that profile. One of the things that we have shown is that there exist solutions of Maxwell's equations in the Segal universe which have that profile and have been around for all times. In particular they did not require a heat source to exist. In fact we have classified all such solutions."

According to Segal and Zhengfang Zhou, a mathematician at Michigan State University, the treatment of nucleosynthesis is similar in CC and in Big Bang theory to first order. "The difference is only that a stochastic sequence of mini Bangs, associated with, e.g., the formation of galaxy clusters, replaces the unique Big Bang. Cluster formation would be expected to be accompanied by extremely high temperatures, which, as in Big Bang theory, would be productive of light elements" [17, p. 323].

The discovery in 1999 of apparently abundant molecular hydrogen in the universe, a possibility already envisaged by physicist Paul Marmet in 1989, may complicate matters even further for Big Bang nucleosynthesis while at the same time rendering unnecessary the concept of (to this day undetected) nonbaryonic dark matter except for maintaining Big Bang theory.

Which Model for the Universe?

Can Segal be right against almost the entire astronomical community? Isn't there an experiment that would settle the question? The problem is that a direct, empirical confirmation of CC—or of Hubble's law for that matter—is not feasible given the current technological means, the greatest difficulty being to determine the distances to astronomical objects. These distances are not observed but are estimated in a complex manner that depends on the model chosen. What can be observed, apart from the redshift, are the apparent brightness and the angular diameter. Consequently, any comparison of the merits of conflicting cosmological theories must hinge not on direct observation but on a statistical analysis of experimental data which would hardly be tractable without high-speed computers.

Irving Ezra Segal died suddenly on August 30, 1998, a few days before his eightieth birthday. He was still actively engaged in research, pursuing, amongst other mathematical endeavors, his lifetime passion: to understand the universe. A 1940 Yale graduate, he taught at Harvard, Princeton, and Chicago before becoming an MIT professor in 1960. He is the author of several monographs and countless scientific articles in prestigious journals and was a member of the American and the Royal Danish Academies of Science. The width and depth of his knowledge of physics and mathematics, and of the interaction between the two, was truly astonishing. His work may be summarized as the search for the right mathematical models to describe certain physical theories, such as cosmology and quantum field theory.

"Why has this work not received an adequate evaluation?" asks Edward Nelson, a Princeton University mathematician and former Ph.D. student of Segal's, in a memorial article [1]. And, answering his own question: "Part of the reason lies in Segal's style of scientific exchange—at times it resembles that of Giordano Bruno (later burned at the stake), who very shortly after his arrival in Geneva issued a pamphlet on *Twenty Errors Committed by Professor De la Faye in a Single Lesson*. But part of the fault lies with cosmologists and particle physicists intent on defending turf. The time for polemics is past. Segal's work on the Einstein universe as the arena for cosmology and particle physics is a vast unfinished edifice, constructed with a handful of collaborators. It is rare for a mathematician to produce a life work that at the time can be fully and confidently evaluated by no one, but the full impact of the work of Irving Ezra Segal will become known only to future generations."

Segal never let up in his crusade against the expansionary theory, alternating sound scientific arguments with emotional tirades. In a 1992 letter to *The New York Times*, he reacted to the interpretation of detected "ripples" in the CBR as support for the Big Bang: "In this day of budding mass movements that appear dangerously irrational, it is deplorable that some scientists, as well as the media, indirectly support detachment from rationality by effectively (if with the best of intentions) catering to the popular fondness for the Big Bang."

A major objection often put forward against EU as a realistic model of the universe, aside from its alleged inability to explain the observed redshift, is its purported and much decried instability: if EU were somewhat perturbed, it would either expand or collapse. However, the proof of such instability is based on the widespread and rather naive notion that the energy-momentum tensor of Einstein's gravitational equation can realistically be represented by that of a perfect fluid, a notion that is only a natural starting point and that ignores, in particular, any possible electromagnetic part in the tensor.

Segal expressed considerable skepticism [14, p. 189]: "…Probably still less justified physically is the application of general relativistic hydrodynamics to extragalactic questions such as the mass density and the stability of the entire Cosmos. The approximation of the distribution of galaxies by a fluid is quite uncontrolled and open-ended; at best, conclusions drawn in this way are merely suggestive."

Nobel laureate Hannes Alfvén, the founder of modern plasma physics, i.e., the physics of electrically conducting gases, advocates what he calls a "plasma universe". Eric Lerner has championed and popularized this idea in *The Big Bang Never Happened*: "The plasma universe…is formed and controlled by electricity and magnetism, not just gravitation—it is, in fact, incomprehensible without electrical currents and magnetic fields."

"Hannes Alfvén and his colleagues have shown that a nonexpanding universe can be stabilized by electric and magnetic forces," writes Gerald S. Hawkins of the Harvard-Smithsonian Observatories.

Already in 1920 Hermann Weyl had speculated on the consequences of a spatially closed universe, observing that if such were the case, then "it would become possible for an observer to see several pictures of one and the same star. These depict the star at epochs separated by enormous intervals of time (during which light travels once entirely round the world)."

In the December 1998 issue of this journal, Neil J. Cornish, a research fellow at the University of Cambridge, UK, and Jeffrey R. Weeks, a geometer working as an independent scholar, took up Weyl's idea in the context of the Big Bang theory: "When we look out into the night sky, we may be seeing multiple images of the same finite set of galaxies." They even went one big step further and, referring to an artist's impression drawn on the cover of that issue of the *Notices*, they wrote: "It is possible that one of the galaxies seen in this image is our own Milky Way, and the light we receive from it has made a complete trip around the universe."

This possibility can be contemplated in CC as well: "The transparency of cosmic space implies that photons in EU will typically make many circuits of S^3 before being absorbed or undergoing interaction. A free photon will be infinitely redshifted at the antipode of S^3 to its point P of emission, but on returning to P it will be in its original state, as a consequence of the periodicity of free photon wave function in EU" [17, p. 324].

We believe that Segal's chronometric model for cosmology merits serious consideration. It is true that Albert Einstein, the most famous advocate of a static, unevolving universe, later changed his mind when he disavowed the "cosmological constant" Λ (lambda), a kind of antigravity factor that he had introduced in his equations, just as C. Neumann and H. Seelinger had done in a Newtonian context at the end of the nineteenth century, to prevent the universe from collapsing under gravity [9]. Einstein is said to have referred to the introduction of lambda as his "greatest blunder," and yet...

General relativity reduces gravitation to curvature. Einstein's original equation implies that space is flat, in the absence of matter, whereas the modified equation that includes the Λ-term allows empty space to have a curvature of its own in addition to that resulting from the presence of matter. The modified equation is the most general obeying some minimal conditions and which allows empty space to be curved or flat [8].

Saying that Λ is necessarily zero in the absence of matter can be compared to saying that if it were not for mountains, rivers, and valleys which define local curvature on a piece of land, the earth would be flat. The curvature defined by local topography masks the global curvature of our planet, but, of course, it is the other way around if one looks at it from a sufficiently large distance.

"Theorists are already scrambling to understand what 20 years ago would have been unthinkable: a cosmological constant greater than zero yet much smaller than current quantum theories predict," observes Case Western Reserve University physicist Lawrence M. Krauss in recent articles [7, for instance]. It is not unthinkable that a rehabilitation of lambda, originally related to the radius r of S^3 by the equation $r = \Lambda^{-1/2}$, could give Einstein's static universe a second chance—and help Irving Ezra Segal's chronometric cosmology get the attention it deserves.

After demonstrating the well-known Poincaré theorem of Hamiltonian mechanics, the mathematician V. I. Arnold writes in his excellent treatise *Mathematical Methods of Classical Mechanics*: "The following prediction is a paradoxical conclusion from the theorems of Poincaré and Liouville: if you open a partition separating a chamber containing gas and a chamber with a vacuum, then after a while the gas molecules will again collect in the first chamber. The resolution of the paradox lies in the fact that 'a while' may be longer than the duration of the solar system's existence."

Hence we may conclude on a rather speculative but perhaps conciliatory note. Assuming CC, if the concept of the totality of the matter dispersed in space S^3 as a "gas of galaxies", i.e., a gas the molecules of which are the galaxies, is valid, as is generally taught, then this gas is almost always in equilibrium. However, if Poincaré's theorem is applicable to it, on rare scattered occasions, yet infinitely many times in past cosmic eternity, it has taken very implausible configurations giving rise to fairly big "bangs", and the same would be in stock for the future.

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