The Education of a Mathematician

Reviewed by David Gale

The Education of a Mathematician
Philip J. Davis
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Philip J. Davis, an applied mathematician (numerical analysis, approximation theory, computer graphics) is perhaps best known to the mathematical community as coauthor, together with Reuben Hersh, of the award-winning book *The Mathematical Experience*. The title of the present autobiographical book suggests that again we will be getting Davis's outlook on the world of mathematics, and, indeed, throughout the 350 pages we become familiar with his views of the subject, acquired, as he modestly puts it, by "just a slow and steady progression of day-to-day experiences that added up to a set of opinions."

Before talking about the content of the book, I will try to describe its format, which is rather unconventional. The book is divided into thirteen parts, some essentially chronological (Part I: "In the Beginning"; Part III: "World War II"; Part VI: "At Brown University"), some devoted to special topics (Part X: "Mathematics and the French Enlightenment"; Part XI: "Applied Mathematics of Jeffersonian Interest"). The parts vary in length from forty pages for "Mathematics in Washington" to barely four pages for "World War II".

Each part is subdivided into chapters or sections, and here the headings range from "Why I Didn't Go for Marxism" and "What Path Shall I Take in Life?" to "Pulled by Pappus beyond Zariski" and "Modesty Is Not a Virtue: Norbert Weiner". An average section is about four pages long, and some are less than a page. One might say the book consists mostly of a series of vignettes, each concerned with some experience or idea that has captured the author's fancy. It is also clear that one of his goals is to entertain. As he says himself at one point, "I'd walk a mile for a good punch line."

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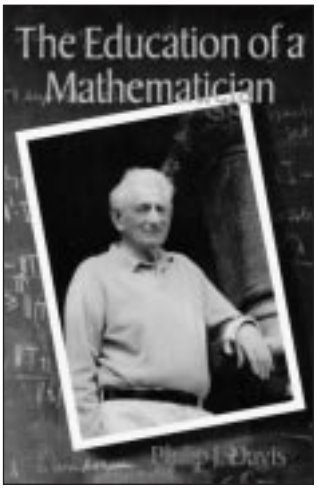

The overall style is informal, personal, almost conversational.

In order to give some structure to this collection of reminiscences, Davis has adopted a rather unusual device. It seems that Thomas Jefferson was exceptionally knowledgeable on the important mathematics of his day, a fact that I at least was unaware of (I would have thought Benjamin Franklin a likelier candidate). By way of illustration, the following passage in a letter from Jefferson to John Adams is quoted several times. Jefferson has just read some papers on celestial mechanics by the American mathematician and astronomer Nathaniel Bowditch and remarks that one of them "impairs the confidence I had reposed in LaPlace's demonstration, that the eccentricities of the planets of our system could oscillate only within narrow limits, and therefore could authorize no inference that the system must, by its own laws, come to an end." Motivated by the discovery of this and other similar documents, Davis and a Danish colleague got the idea of writing a book on the development of mathematics over the past two hundred years in order to update the Jeffersonian Ghost on what has happened to mathematics since he left the scene. Although the earlier book project did not materialize, Davis uses the device periodically throughout this volume in the form of "Dear Ghost" letters.

The chapters intended to update Jefferson, however, occupy only a small fraction of the book. Part of the rest is taken up with the author's education in the conventional sense: Lawrence High School, where he discovers Euclidean geometry (probably the initial turn-on to mathematics for

many of us); then "Undergraduate Years" at Harvard where he finds out that he enjoys writing, is turned off by formalism after a course given by W. V. O. Quine, gets back to geometry at the higher level with Oscar Zariski, but is most influenced by the lectures and informal meetings with the philosopher of science, Philipp Frank. World War II is a very brief interruption for Davis. He is drafted into the army in 1944 and sent to Langley Field, Virginia, where he analyzes aerodynamic loading of wings of fighter aircraft. A year later he is back at Harvard in graduate school, and his education continues.

Most of the book is taken up with the author's thoughts and experiences or with people, famous or otherwise, whom he has met. Among memorable experiences was a meeting with Earl Warren, at which, even though there was barely time for a handshake, the Chief Justice was able to get in the remark (groan) "Well, I was never much good at mathematics." I suppose this counts as education too, in a negative sense. A more important contact was with Isaiah Berlin, whose philosophical ideas Davis says were in many ways like his own. He quotes Berlin describing the ideas of the Italian philosopher Giambattista Vico, who believed "…the validity of all true knowledge, even that of mathematics and logic, can be shown to be such only by understanding how it comes about, i.e., its generic and historic development." This seemingly rather extreme position apparently resonates with Davis, who refers to his own beliefs as social constructivism (as opposed, in particular, to a Platonism which he has rejected earlier on).

For readers of this review the main interest of the book will no doubt inhere in the mathematical portions, but it is important to bear in mind that the intended audience will supposedly have minimal mathematical sophistication. In view of this, I will make a few more general remarks before turning to professional questions. As already noted, Davis's chatty style aims to entertain as well as enlighten. I found particularly amusing a chapter having nothing to do with mathematics in which the author takes up painting and goes through the somewhat traumatic transition from painting still life (in particular, broccoli!) to "live" life with nude models. At the same time, while the author's informal, highly personal style makes for pleasant reading, it has its down side. Sometimes Davis abruptly changes the subject and seems to be writing whatever pops into his head. In the middle of talking about rigor à la deltas and epsilons, for example, he quotes, "My kingdom for a horse", and in the section "What I Learned from Mary Cartwright" he breaks off to describe an episode in Aldous Huxley's *Point Counter Point*, whose relevance was not clear to me.

In its last hundred pages the book is concerned less with its author's personal experiences and more with his speculations and reflections on mathematics. Part IX, "An Interlude: A Bit of Mathematical History", ends with a response to the Jefferson-Adams letter on planetary orbits. Davis quotes from an article by Jürgen Moser on the subject: "In spite of modern advances in this field, this is still an *open problem!*" Davis goes on to write, "We have now seen that in his plough, his cryptography [this refers to other Jeffersonian interests mentioned earlier], and in his response to Adams about celestial mechanics, Thomas Jefferson put his finger on what have turned out to be very difficult, abiding, and significant problems of applied mathematics. Not bad for an amateur!" Agreed.

Part X, "Mathematics and the French Enlightenment", starts with a chapter, "Voltaire and Newton," which leads up to "Emilie du Châtelet: Divine Mistress", who besides being Voltaire's mistress was apparently an unusually talented mathematician. The next chapter, "Diderot, d'Alembert, and Rousseau", is curious in that Rousseau is given only one short paragraph and no connection is made between him and mathematics. The final section, "Updating the Ghost of Mr. Jefferson", is the first of the Dear Ghost letters. Davis uses it as a sounding board for some of his own views. He fantasizes a sort of dialogue with the ghost: "'How goes the United States of America; how goes the Enlightenment?' you asked me when we first made contact. And I answer that all of us are children…of the Enlightenment and without Enlightenment values we would find life unbearable. But I must add that the legacy of the Enlightenment…has proven to be a deeply troubling one.…" And later, referring to some other philosophers: "if the Enlightenment does not accommodate reflection on the destructive aspect of the progress it engenders, then it seals its own fate and ours." This is a major theme throughout the book. To oversimplify, Davis worries that we may be in danger of losing our souls by overdosing on technology and mathematization.

In Part XI, "Applied Mathematics of Jeffersonian Interest", the first section is a disquisition, "What's in a Can of Peaches? Or the Mathematization of Our Civilization", in which, to illustrate that mathematics is everywhere, Davis discusses various mathematical properties of the peach can. I found this section a bit of a letdown. The things discussed included the geometry of the can and the ratio of surface to volume. The fact that there are thirty-six separate numbers on the can's label suggests more mathematics. He notes on the label, "For maximum goodness, sell by May 15th," which leads him to talk about the calendar, "a complicated arithmetic-astronomic arrangement." Some telephone numbers on the label lead to a short paragraph on communication in which he mentions the Bell Telephone Laboratories and switching

systems. He taps the can and hears a musical note, leading to the mention of sounds and eigenvalues. The trouble is that all of these things, at least in their early stages, surely predate Jefferson and would hardly seem revolutionary to someone who was knowledgeable about LaPlacian celestial mechanics.

By contrast, I found one of the high points of the book to be the chapter "One, Two, Three, Count It Legally". At the end of his introductory letter to Jefferson, Davis writes, "Perhaps I should now tell you some new applications of mathematics in areas in which you had particular interest: law, government, diplomacy, war, and politics." The "One, Two, Three" chapter considers whether the methods of statistical sampling should be used in taking the ten-year national census rather than trying to count the entire population. The story is nicely told as contemporary history, and among other things we learn that some very eminent statisticians strongly disagreed on the issue and even testified for the Department of Commerce, presenting their opposing views. Excerpts from some of the testimony are presented. This is, of course, grist for the Davis mill, supporting one of his frequently made contentions that "mathematical certainty" is perhaps not as certain as many of us would like to think it is.

Part XII of the book, the second to last, is entitled "Echoes of Rousseau", and we are back to ethical and moral issues, in the form of not one but two sets of fantasy letters. In the first Davis's Danish colleague enters a Rousseau essay contest sponsored by the French Ministry of Culture. His entry is in the form of a letter to Rousseau, which ends up, "And so, Mr. Rousseau, my answer to the question of whether the restoration of the arts and sciences has had a purifying effect upon morals is the same as yours: it has not." As a counter to this, the last section of the Rousseau part is another Davis-Jefferson letter, more formal this time, in which Davis tries to moderate the pessimism of his Danish colleague.

The last part, "Still Carrying On", ends with the book's longest chapter, a "fleshed out" version of a talk, "The Prospects for Mathematics in Multimedia Civilization", which Davis gave in Berlin at the time of the 1998 International Congress of Mathematicians. In it he touches on many issues, some purely mathematical ("The Discrete vs. the Continuous", "The Deterministic vs. the Probabilistic"), others about how mathematics is affecting our society and culture ("Words or Mathematical Symbols vs. Icons", and especially apt, "Thinking vs. Clicking"). In "What Is Proved vs. What Is Observed" we find the provocative statement "I would hope that the notion of proof will be expanded so as to be acknowledged and presented as one part of the larger notion of mathematical evidence."

Finally in the book's concluding paragraph, under the heading "A Personal Illumination", Davis describes the following event. "Walking the streets of my home town, I got an illumination: the history of future mathematics will be seen as the increased tension… between the real and the virtual." I must admit I'm not sure what this means, and unfortunately Davis does not elaborate. Nevertheless, since it comes as a culminating episode in his "education", I will try to interpret and comment and hope that I have it right.

By "virtual" I assume Davis means numerical or, perhaps better, computer-generated. Let me step back for a moment. We are all familiar with C. P. Snow's treatise on the "two cultures", referring, of course, to the sciences and the humanities. I have felt for a long time that pure and applied mathematics can without exaggeration be described as separate cultures. The difference in this case is not between scientists and humanists but between scientists and engineers. Roughly speaking, the aim of scientists is to help people understand how nature works, while engineers, conversely, want to help nature to work for people. The distinction between pure and applied mathematics is the same. As it happens, I too spent time at Brown University, where there were and still are both a pure and an applied mathematics department, with the pure department in the liberal arts college and the applied department in the college of engineering. Both departments did mathematics, but in different ways. The members of the pure department were full-time theorem provers. The applied people were, and here you'll have to forgive me, "mathematizers" (to elaborate on the word "mathematization" that appears so often throughout the Davis book). Recent history provides many examples of the interaction between the two groups. Among the most notable are the remarkable experiments of Feigenbaum showing the unexpected things that can happen when one iterates the simplest quadratic polynomial. Of course, as soon as the virtualists discover something like this, the theorem provers jump in and try in the old-fashioned Greek way to show why the phenomenon must happen. There are, to be sure, individuals with a foot in both camps, like the redoubtable John H. Conway, who first invented the famous "Game of Life" (what could be more virtual than that?) and then proved theorems about it.

What then is this "tension" that Davis foresees? Is it between the disciplines themselves or their practitioners? And what can he possibly have in mind when he talks of an expanded notion of proof? Perhaps he should have written one more letter to the revered Founding Father to clear these things up. In any case, it is probably a sign of a stimulating book when it leaves the reader with some unanswered questions for the author to take up at some future date.