Geek Chic

It's a thing that nonmathematicians don't realize. Mathematics is actually an aesthetic subject almost entirely.
—John H. Conway

We mathematicians know well the aesthetic pleasures of our subject. G. H. Hardy wrote in *A Mathematician's Apology* that a "mathematician's patterns, like the painter's or the poet's, must be *beautiful*; the ideas, like the colors or the words, must fit together in a harmonious way." How frustrating it is that these pleasures cannot be transmitted to the general public. How frustrating that our intelligent and otherwise highly literate friends have so little appreciation for our labors.

I have some *good news* to report. There is right now in the popular culture a *wave* of interest in mathematics. Even better, the image that has captured the public imagination is that of a mathematician working intensely and usually alone on the most difficult and abstract problems. What was once considered hopelessly geeky is suddenly au courant.

A recent manifestation of this phenomenon is the play *Proof*, in which three of the four characters are mathematicians. The plot centers on a notebook of uncertain authorship that may or may not contain a great proof. Ben Shenkman is excellent as Hal, a graduate student that many will recognize. Yes, he is geeky and callow (the repartee about his rock band is hilarious) but simultaneously very human and humane. The star, Mary Louise Parker playing Catherine, is amazing. I've often been disappointed by theatrical attempts to portray genius. Parker's Catherine has an authentic genius. She struggles with family and with mental illness. She has enormous intensity and vivid sexuality. The disparate pieces come together (this is magic to me) in an utterly believable and compelling portrayal. Reviews of the play, including one that appeared in the *Notices* (October 2000, pages 1082–1084), have been excellent. Buoyed by this success, *Proof* moved to Broadway in October 2000.

How can we mathematicians ride this wave? "Proof: A Symposium", held last October at New York University, provides an outstanding model. The symposium was a forum for discussion by mathematicians and nonmathematicians of some of the mathematical themes and ideas raised in *Proof*. The symposium's first panel discussion, on the nature of proof, was the most mathematical. The star was Thomas Nagel, a distinguished philosopher whose incisive views on objective truth were of interest to mathematician and nonmathematician alike. The second panel, "Women and Proof", had an all-star cast. The stories these women told were at turns funny and moving. Skillful organization kept the strong-minded panelists on common themes. The final panel, "Images of Proof", consisted of writers and actors. Hearing these panelists discuss my world was an eye-opener. They are always searching for the best image for their works. Andrew Wiles at lunch discussing French history is not a powerful image. Andrew Wiles in the attic for seven years is, for them, ideal. Most disconcerting to us are images that join mathematics and madness, a theme in almost all the popular works.

The key to the success of "Proof: A Symposium" and similar ventures is to bring together people from inside and outside the mathematical community. "Proof: A Symposium" was cosponsored by the Sloan Foundation, which is taking a leading role in the popularization of science and mathematics; the Manhattan Theatre Club, which produced *Proof*; and the Courant Institute. (Full disclosure: While Courant is my institutional home, I had no part in the organization of the symposium.) The Mathematical Sciences Research Institute in Berkeley has had a series of successful public events of this kind, beginning with the "Fermat Fest" in 1993. More recent events have included a conversation between mathematician Robert Osserman of Stanford University and playwright Tom Stoppard concerning Stoppard's work *Arcadia,* and a mix of conversations and theatre about the life of Galileo.

On the screen *Good Will Hunting* and π will soon be joined by a film based on *A Beautiful Mind*, Sylvia Nasar's biography of John Nash. *Proof* joins *Copenhagen* on Broadway. Bookstores are stocked with biographies of Paul Erdős; the latest book by the renowned mathematics expositor Keith Devlin; and novels with mathematical themes, such as *Uncle Petros and Goldbach's Conjecture* and *The Wild Numbers*. All this attention is most enjoyable. Let's take it as an opportunity to communicate to the public the beauty and centrality of our subject.

—Joel Spencer
Courant Institute, New York University

Proof: A Symposium
October 16, 2000
New York University

Panel I: *What's a Proof and What's It Worth?*: Peter Sarnak (moderator), Princeton University; Kit Fine, NYU; Arthur Jaffe, Harvard University and Clay Mathematics Institute; Dusa McDuff, SUNY Stony Brook; Thomas Nagel, NYU; Michael Rabin, Harvard University; Jack Schwartz, Courant Institute, NYU.

Panel II: *Women and Proof:* Margaret H. Wright (moderator), Lucent Technologies; Dusa McDuff, SUNY Stony Brook; Cathleen Morawetz, Courant Institute, NYU; Mary Pugh, University of Pennsylvania; Jean E. Taylor, Rutgers University; Karen Uhlenbeck, University of Texas at Austin.

Panel III: *Proof in Performance and Prose:* Michael Janeway (moderator), Columbia University; David Auburn, author of *Proof*; Rebecca Goldstein, novelist; Sylvia Nasar, Columbia University; Ben Shenkman, actor who plays Hal in *Proof*.

Letters to the Editor

Standards in School Mathematics

The *Notices* for September and October 2000 featured some discussion of the *Principles and Standards for School Mathematics* ("PSSM") of the National Council of Teachers of Mathematics (NCTM), this manifesto being for the most part a revision of NCTM's 1989 *Standards*. The earlier document, mainly unnoticed by the mathematical profession at the time, offered as its principal vision that school mathematics need not be difficult or dull and that the cure was to remove the mathematical content from it, leaving behind the mathematical concepts as a sort of Cheshire Cat grin. There is no place here for detail, for which see the "Mathematically Correct" Web page (`http://mathematicallycorrect.com/`) or find a copy in a library and see for yourself.

Needless to say, not everyone agrees with the above assessment of the import of the 1989 *Standards*, but by the end of the 1990s enough mathematicians—notable among them Richard Askey, the late Han Sah, and Hung-Hsi Wu—had developed a loathing for NCTM doctrine that managed to attract the attention of NCTM itself. Other opposition has also emerged, mainly from parents' groups enraged at the NCTM-blessed mathematics programs beginning to spread in their schools. ("Mathematically Correct", which speaks for some scientists and mathematicians as well, was a pioneer among these.) Clearly NCTM would have to take account of mathematicians in writing its scheduled new edition (i.e., PSSM), and it did.

As Joan Ferrini-Mundy, its principal editor, explained in her September *Notices* article, NCTM this time commissioned the commentary of many mathematicians, including committees of AMS, MAA, and SIAM, upon an earlier draft prepared for us. I myself served on the AMS committee and (by commission) as an individual too. NCTM solicited public advice at large, and I know several who also attempted to link the mathematical world with the new document, but the effort was to little avail; the message—the "vision" of PSSM—remains, in my vision, much the same as that of the original 1989 *Standards*.

PSSM continues to abhor direct instruction in, among other things, standard algorithms, Euclidean geometry, and the uses of memory. Though like its predecessor it has the word "standards" in its title, it is not a set of standards in the usual meaning of the term, for it refuses to say what exactly a child should learn in thirteen years of schooling. Long division? Quadratic formula? How to compute the quotient of two fractions? (See p. 218 of PSSM for an enlightening discussion.) Proof of a theorem on inscribed angles? Trigonometric identities? PSSM will neither affirm or deny, lest it seem to dictate content.

Joan Ferrini-Mundy has publicly averred that both PSSM and its predecessor have been misunderstood and that NCTM does indeed advocate learning the multiplication tables. This is almost true for the multiplication table, though only as a last resort (PSSM, p. 152). Other such concessions are harder to find. Almost anything in the way of content to be remembered can be omitted from a school mathematics program without running afoul of PSSM, providing the pedagogy is right and the process suitably "exploratory". "Explore", "develop", and "understand", and their variants, are much more prominent in the text than "know", "prove", and "remember".

Under the color of NCTM's vision of mathematics as expressed in the 1989 *Standards* have been written a number of school mathematics programs recently officially recognized as "exemplary" or at least "promising" by the U.S. Department of Education, but to a chorus of public protests, some of it from mathematicians. Because many of us with children—or grandchildren—in today's schools have now seen these programs in action, the public protests are still mounting. PSSM may prove a marginally better theoretical guide to further such projects than the 1989 version, but we deserve better than this. If the world of mathematics, sadly divorced from the world of school mathematics education, pays no more effective attention to the schools in the next ten years than it has in the past thirty, the country is in for a meager intellectual future.

True, it is not the primary business of mathematicians to study the problems of school mathematics programs, let alone engage in the political struggle needed to make a difference in the public schools themselves, but I appeal to all who read this letter to obtain a copy of PSSM (`http://www.nctm.org/`) and reflect on what such a document means to the future of the children in today's schools. I warn you that these "principles and standards" cannot be appreciated by reading only a few pages. In the small the document sometimes sounds good. But if PSSM in the large informs our vision, then self-esteem is better than knowledge, dictionaries can replace a ready (memorized?) vocabulary, and higher-order thinking skills will boil stones into soup.

—Ralph A. Raimi
University of Rochester

(Received October 20, 2000)

Hellmuth Kneser's Forgotten CR Extension Theorem

F. Treves' interesting and informative article in the November 2000 issue discusses several major themes in multidimensional complex analysis in the concrete context of the hyperquadric.

In particular, the author recalls the local extension theorem for Cauchy-Riemann functions (Theorem 1, page 1248), with reference to Hans Lewy's well-known 1956 paper. Treves, as well as virtually all other researchers in the field, had not been aware that this celebrated CR extension theorem was proved twenty years before H. Lewy's paper in a remarkable 1936 paper by Hellmuth Kneser, come back to light only recently (Die Randwerte einer analytischen Funktion zweier Veränderlichen, *Monatsh. Math. Phys.* **43** (1936), 364–380).

This important paper, together with earlier contributions by W. Wirtinger (1926) and F. Severi (1931), documents that CR functions have a history much older than commonly recognized.

Related to this matter is the widespread confusion regarding the global CR extension theorem, first proved by G. Fichera in 1957 and often attributed mistakenly to S. Bochner. For the record, there is no evidence whatsoever in Bochner's 1943 paper to suggest that Bochner had even remotely been thinking about CR functions and the corresponding version of Hartogs's famous extension theorem. More details may be found in my forthcoming historical article in *The Mathematical Intelligencer*.

—R. Michael Range
State University of New York at Albany

(Received October 31, 2000)

Correction to the History of Hilbert's Problems

In the August issue of the *Notices* appeared Grattan-Guinness's intriguing article "A Sideways Look at Hilbert's Twenty-three Problems of 1900". One claim made in that article calls for correction. Grattan-Guinness, discussing the comments after Hilbert's 1900 lecture at the International Congress of Mathematicians, stated that "Peano…remarked that [Hilbert's] Second Problem on [proving] the consistency of arithmetic was already essentially solved by colleagues working on his project of mathematical logic and that the forthcoming Congress lecture by Alessandro Padoa…was pertinent to it" (p. 756). Actually, Peano made a much more direct and unqualified assertion: "Monsieur Padoa's later communication will answer Hilbert's Second Problem" (p. 21 of the Congress proceedings). Despite Peano's claim, Padoa's article did not solve Hilbert's Second Problem by proving the consistency of arithmetic, but only stated that "to prove the consistency of a postulate system, one must find an interpretation of the undefined symbols which satisfies all the postulates simultaneously" (p. 249 of the Congress proceedings). What Hilbert wanted to do was, in fact, to find an absolute consistency proof, not a relative consistency proof using models. At that time only such relative consistency proofs were known, and this may account for Padoa's and Peano's confusion.

Next, Grattan-Guinness added: "Unfortunately Hilbert did not make amends in the *Archiv* version (presumably lack of Italian again), but in *L'Enseignement Mathématique* Padoa explicitly discussed this problem…". The fact that Hilbert did not modify his article when it was later reprinted in the *Archiv der Mathematik und Physik* to reflect Padoa's Congress article was not due to Hilbert's not knowing Italian, since Padoa's article was in French. Rather, it was because Padoa's Congress article contributed nothing to the solution of Hilbert's Second Problem, despite Padoa's claim in *L'Enseignement Mathématique* to have solved it.

Grattan-Guinness leaves the reader with the impression that Peano and Padoa were right in claiming that the Second Problem was solved and Hilbert wrong. But, in fact, the opposite is true. As is well known, the Second Problem was not solved until 1931 by Kurt Gödel in the profound result now known as his Second Incompleteness Theorem.

—Gregory H. Moore
McMaster University

(Received November 7, 2000)

Response to Moore's Letter

On consistency, it is clear that Hilbert required an absolute version for arithmetic, and I should have distinguished it from relative ones. It is a pity, though, that Padoa's foundational work has been overshadowed by that of others. His Paris lecture did not appear till 1901, after the *Archiv* version of Hilbert's paper anyway.

Two pieces of information from readers of my article are worth passing on.

The first has some kinship with the above point. Professor Rüdiger Thiele (Halle University) has found in a notebook in file 600 of Hilbert's mountainous *Nachlass* at Göttingen University Library an apparently undated passage in which Hilbert recalled including a 24th problem for his Paris address. It was concerned with finding criteria for finding simplest proofs of theorems in mathematics in general—once again, more a programme than a problem in the normal sense. He left it out of his final version, I suspect after realising that, ironically, simplicity is an extremely *complicated* notion to capture in a *general* way, so that nothing useful could be said here.

Concerning the complaints about the presentation of the lectures at the Paris Congress which I reported, Professor Wilfrid Hodges (University of London) tells me that recently he himself lectured in the same room that Hilbert had used. Apparently the acoustics there are terrible!

—Ivor Grattan-Guinness
Middlesex University

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