George Keith Batchelor (1920–2000) and David George Crighton (1942–2000) Applied Mathematicians

G. I. Barenblatt

This is a memorial article about two great applied mathematicians, and I would like to start with a few words about the nature of applied mathematics. At the opening of the Congress on Industrial and Applied Mathematics at Hamburg, 3 July 1995, V. I. Arnold, unquestionably one of the most authoritative mathematicians of our time, said (see [1]),

> A common (though commonly suppressed) opinion both of pure mathematicians and theoretical physicists concerning "industrial and applied" mathematics is that it consists of a mafia of weak thinkers, unable to produce any important scientific results, but simply exploiting the achievements of pure mathematicians of past generations, and that the members of this mafia are more interested in cash than in science and are hopelessly corrupted by this.
>
> "They are so modest," a pure mathematician once said, "that they do not hope to achieve anything in a direct honest way; they distance themselves from mathematicians simply to avoid honest competition."
>
> I do not think that this characterization of applied mathematics was *completely* deserved. The achievements of Galileo devoted to business evoke no less admiration than the results of the pure philosopher Pascal.
>
> The difference between pure and applied mathematics is not scientific but only social. A pure mathematician is paid for uncovering new mathematical facts. An applied mathematician is paid for the solution of quite specific problems.

In fact, the characterization of "industrial and applied" mathematics by unnamed pure mathematicians and theoretical physicists mentioned by V. I. Arnold is *completely* untrue. Also, what was said by Arnold himself about the purely social difference between pure and applied mathematicians is to say the least doubtful. Indeed, according to this characterization, Andrew Wiles, who proved the Fermat Theorem, would be an applied mathematician, whereas Pierre Fermat himself would not be a mathematician at all, because his paid profession was as a judge. Also, Pascal's law, fundamental in hydrostatics, allows us to consider Blaise Pascal as an explorer of nature, not just as a pure philosopher.

In fact, the key to a correct understanding of the subject of applied mathematics and of the role and responsibilities of applied mathematicians lies in the famous saying of J. W. Gibbs:

> *Mathematics is also a language.*

G. I. Barenblatt is professor of mathematics at UC Berkeley and mathematician, Lawrence Berkeley National Laboratory. His e-mail address is `gibar@math.berkeley.edu`.

All people use language. However, we may distinguish among users of a language a particularly important group. They are the authors: poets, novelists, playwrights, essayists, etc., who create fictitious images and paradigms—idealized models of people and social phenomena. The greatest of these paradigms, like Phaedra, Francesca da Rimini, Romeo and Juliet, Dr. Faust, Natasha Rostova, Anna Karenina, and the events that surrounded them, continue to live for centuries. They transform human culture and, in particular, language itself.

To a certain extent, a similar role is played by applied mathematicians. Using the language of mathematics, developing and transforming it when necessary, applied mathematicians create their paradigms: models of phenomena, both in nature and in technology. These models give idealized but rather complete images of phenomena as a whole, enabling their mathematical analysis. The purpose of these models is to predict the behavior in unexplored ranges of the systems under study. When this goal is achieved, it leads to practical applications.

It is appropriate to cite here the opinion of John von Neumann [20]:

> The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretation, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work—that is, correctly to describe phenomena from a reasonably wide area.

Applied mathematicians create such models. It is difficult and exciting work to make models. This work is done through trial and error, starting and finishing by analysis of observations and experiments, both physical and numerical. It gives no less excitement and satisfaction than proving theorems in pure mathematics. We may say that to be a dedicated applied mathematician is an achievement, honour, and privilege. Great British applied mathematicians J. C. Maxwell, Lord Kelvin, Sir George Stokes, Sir Geoffrey Taylor, and, more recently, Sir James Lighthill created the style and atmosphere where G. K. Batchelor and D. G. Crighton lived and worked.

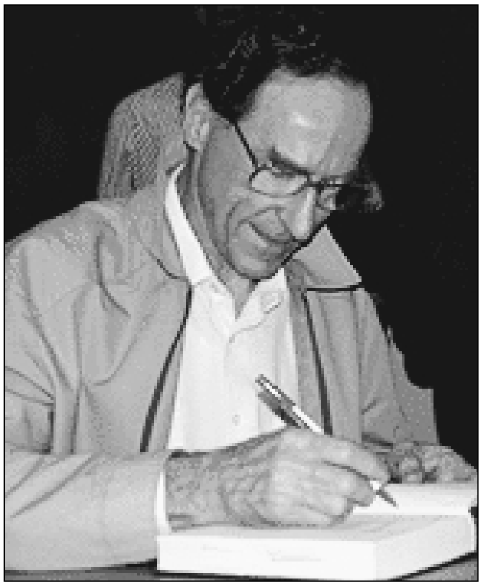
Turbulence

Turbulence is the state of vortex fluid motion where velocity, pressure, and other properties of the flow field vary in time and space sharply and irregularly and, it can be assumed, randomly. It remains to this day the greatest unsolved problem of classical physics. The phenomenon of turbulence was recognized and even named by Leonardo da Vinci. Osborne Reynolds performed the first fundamental studies of turbulence in the late 1800s. Since that time an army of researchers has participated in these studies, including, without any exaggeration, the best minds of mankind: A. N. Kolmogorov, W. Heisenberg, L. Onsager, L. Prandtl, G. I. Taylor, and Th. von Kármán. Nevertheless, up to now almost nothing has been obtained in the theory of turbulence from first principles, i.e., from the Navier-Stokes equations, which definitely should be applicable to describe the phenomenon for a wide class of fluids and external conditions. The phenomenon of turbulence is of basic importance; this is recognized even by laymen. At the same time, our fundamental and practical knowledge of turbulence is insufficient to say the least. This is a rather strange situation: we know more about the structure of remote stars than about the flow of water in the pipes in our homes!

The year 1941 marks a fundamental event in turbulence studies: A. N. Kolmogorov and his student of the time, A. M. Obukhov, were able to understand the local structure of "developed" turbulent flows, i.e., turbulent flows at large Reynolds number.[1] Their basic model of the phenomenon was as follows. Developed turbulent flow is, so to speak, stuffed by vortices. Due to the interaction of vortices in the flow (mutual cutting and self-cutting of vortices and their subsequent reconnections), an equilibrium cascade of vortices is formed in turbulent flow. (This idea was anticipated by the outstanding British physicist L. F. Richardson.) The cascade covers all scales of the vortices, from the largest ones determined by global flow configuration, to the length scale where the viscous dissipation of energy into heat becomes essential (now called the Kolmogorov length scale), to even lower scales. For flows having very large Reynolds numbers, this cascade should embrace a very large range of scales. The first basic idea was that the lower part of this cascade is stochastically universal for all developed turbulent flows. From universality follows statistical steadiness, local isotropy, and homogeneity of the relative motions of fluid particles. At the same time, according to the second basic assumption, the influence of viscosity in the upper part of this universal branch of the cascade is insignificant, because the vortices are still much larger than the length scale where the viscous dissipation of energy into heat becomes substantial. So energy moves through the cascade from large eddies to small ones, untouched by dissipation.

[1] *Reynolds number is the basic dimensionless parameter governing viscous fluid motion: the product of characteristic velocity with characteristic length of the flow divided by a property of the fluid, its kinematic viscosity.*

Photograph courtesy of Department of Applied Mathematics and Theoretical Physics, Cambridge University.



George K. Batchelor

This paradigm together with dimensional considerations, in fact the invariance with respect to a certain renormalization group, led to remarkably simple universal laws for the mean specific energy K of relative motions in turbulent flows at very large Reynolds numbers:

$$K = \frac{1}{2}\overline{(\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x}))^2} = C(\varepsilon r)^{2/3}$$

(Kolmogorov's law of two thirds), where $\mathbf{u}$ is the fluid velocity, $\mathbf{r}$ is the vector connecting two points, the bar denotes ensemble averaging, and ε is the energy dissipation rate per unit mass. In Fourier representation the following law was obtained for spectral energy density:

$$E(k) = C_1\, \varepsilon^{2/3}\, k^{-5/3}$$

(Kolmogorov-Obukhov law of five thirds). Here k is the wave number, while C and C_1 should be universal constants by the very logic of model construction.

Turbulent Times

It is impossible now to trace how the papers by Kolmogorov and Obukhov came to arrive at the Library of Cambridge University. They were published at the worst time of Hitler's invasion of the Soviet Union. Later, in September 1941, convoys delivering weapons and war materiel for the Soviet Union cruised from Britain to Murmansk and Archangel, ports in the north of Russia. On their way back the ships needed some ballast. Very often as part of the ballast books were used, in particular scientific publications of the USSR Academy of Sciences. It was apparently in this way that the volumes of *Doklady* with papers by Kolmogorov and Obukhov reached the Library at Cambridge University. From the other side of the world, in early 1945, before the end of the war, young George Batchelor traveled in a similar convoy of eighty ships from his native Australia to Cambridge in order to work with G. I. Taylor and to study turbulence.[2]

And so the great papers found a great reader! G. K. Batchelor studied Kolmogorov's short notes and published two papers [3], [4] explaining Kolmogorov's theory in detail. This required a titanic effort.[3] These papers, especially the first one,[4] made the Kolmogorov–Obukhov theory generally understandable and very popular. It is enough to mention that in the fifties and early sixties even students in the Soviet Union used Batchelor's paper as an introduction to the subject and that in 1991 the Royal Society of London published a special volume of Proceedings, edited by J. R. C. Hunt, now Lord Hunt of Chesterton, under the title *Fifty Years of Kolmogorov's Ideas in Turbulence.* G. K. Batchelor's role in disseminating and developing these ideas was of decisive value. The destiny of this theory would have been different without him!

For a long time (about twenty years) turbulence became his basic subject of research. At that time G. I. Taylor's interest moved from turbulence to other fields, so the work of G. K. Batchelor in turbulence was to a large extent independent.[5] Soon he understood that theoretical work in turbulence is impossible without permanent contact with experiment. He wrote to his friend and close colleague, A. A. Townsend, who remained in Australia: "You will come to Cambridge, study turbulence, and work with G. I. Taylor." The answer came immediately: "I agree, but I have two questions: what is turbulence and who is G. I. Taylor?" Townsend came and soon revealed himself as one of the most remarkable experimentalists working in turbulence. This shows what a strong personality G. K. Batchelor had; otherwise it would have been impossible to create the Department of Applied Mathematics and Theoretical Physics

[2] *Up to the end of his days, Batchelor remained an Australian citizen. This gave him some trouble in obtaining a French visa, in spite of his being a Foreign Associate of the French Academy of Sciences.*

[3] *A characteristic detail: G. K. Batchelor claimed in his paper that he was able to reproduce all the details of Kolmogorov's calculations except one: he was unable to derive the relation between longitudinal and normal components of structure functions without the simplifying assumption of complete isotropy of the flow; in Kolmogorov's paper there was the claim that this simplification is unnecessary. Up to now, as far as I know, no one has been able to perform the derivation free of this assumption, although no one doubts that such a derivation is possible.*

[4] *There was also a short preliminary publication in* Nature *in 1946. It is interesting that this publication immediately attracted the attention of J. von Neumann.*

[5] *GKB once told the author that he also considered A. N. Kolmogorov to be his teacher.*

(DAMTP), the *Journal of Fluid Mechanics*, and Euromech (see below).

New Directions

In the late forties and fifties G. K. Batchelor's activity in the study of turbulence was enormous, and it entered as a fundamental part of the subject in monographs and textbooks (see, e.g., [17]). Rather early he came to the conclusion that a book describing new ideas in turbulence was needed. His book [5], *The Theory of Homogeneous Turbulence*, was published by Cambridge University Press in 1953 when the author was thirty-three years old. In this book in particular the idea of two-dimensional turbulence appeared, which later attracted wide attention from fluid dynamicists, both theoreticians and experimentalists. The idea was very natural for the asymptotic description of atmospheric and oceanic turbulence: it seemed at that time that because the vertical length scale of atmospheric and oceanic motion is much less than the horizontal scale, the large scale turbulent motions can be considered to be two-dimensional.

G. K. Batchelor discovered that the events in the vortex cascade in two-dimensional turbulence should be completely different from the three-dimensional case: the energy flux goes from small to large eddies (inverse cascade). Therefore, the Kolmogorov-Obukhov "5/3" spectrum should be observed at small, not large, wave numbers. Later this work was continued by R. H. Kraichnan and other researchers and was confirmed by numerical and physical experiments. Apparently it was the first explicit demonstration of the fact, which became gradually evident, that two-dimensional fluid dynamics, and in particular two-dimensional turbulence, cannot represent real three-dimensional phenomena: the basic mechanism of the real phenomenon, the interaction of vortices, is missing in its two-dimensional counterpart. This interaction is the root of tremendous difficulties in mathematical investigations of the solutions to the three-dimensional Navier-Stokes equations.

It is impossible to present in this paper a complete picture of Batchelor's achievements of this period, but I can refer the reader to a forthcoming paper of H. K. Moffatt [16] where these results are presented in detail. The only thing I want to emphasize here is his interest and direct participation in experiments, in particular in the experiments (together with A. A. Townsend) concerning the turbulence decay behind the grid in a wind tunnel, which, after G. I. Taylor, was considered as a model of isotropic homogeneous turbulence. The variety of scaling laws of this decay continues to attract attention now, fifty years after the experiments were performed.

In the beginning of the sixties Batchelor stopped his work in turbulence rather abruptly and started a new project: for several years he wrote a treatise [6], *An Introduction to Fluid Mechanics.* This book reflected his experience in the new approach to the subject. It was published by Cambridge University Press in 1967 and since that time has been republished many times; now it is the most widely used textbook in fluid mechanics. At the end of his life Batchelor wanted to recast the book, but a severe illness did not give him much opportunity.

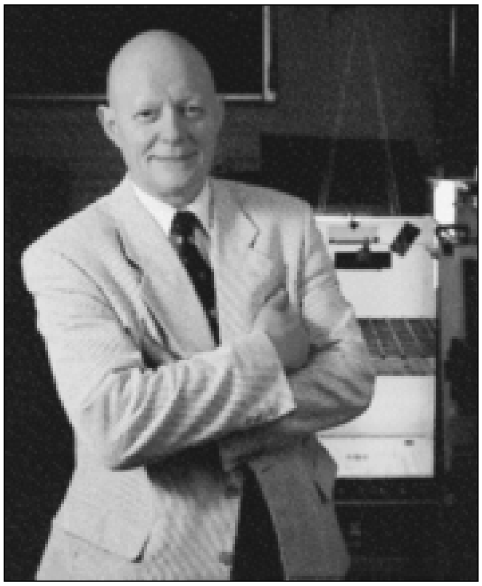
After the textbook Batchelor's research moved to a new direction, and so it happened that he became one of the founding fathers of *micromechanics* [7], an entirely new approach in continuum mechanics. According to the micromechanical approach, properties of the material microstructure, directly or indirectly observable, do not remain invariable; therefore they are explicitly introduced into consideration. The equations of macroscopic motions and those of the kinetics of microstructural transformations are considered simultaneously. Batchelor advanced this idea for fluid motions and applied it to shape the hydrodynamics of suspensions. In a parallel but unrelated series of works, American applied mathematician B. Budiansky advanced an analogous approach in application to solids. Nowadays micromechanics seems to be the most perceptive direction in all branches of continuum mechanics. It is apparent that this approach will give adequate mathematical models of many unexpected and sometimes counterintuitive phenomena that occur in the motions of suspensions, polymeric solutions and melts, biological fluids, etc.

New Institutions

Batchelor's organizing activity occupied much of his time, and here also he was successful. In 1959 he was able to create at (the very conservative, in the best sense of the word) Cambridge University a new Department of Applied Mathematics and Theoretical Physics, the now famous DAMTP. I want to emphasize that in this department there existed practically from the very beginning an experimental laboratory, something absolutely unexpected at Cambridge before, where the general idea was that mathematicians should work in college rooms or at home using only pen and paper. Such a combination of theoretical work and experiment happened to be very fruitful, and for a long time DAMTP was a Mecca for fluid mechanists, both from Britain and from overseas.

Batchelor's second major creation was the *Journal of Fluid Mechanics*, launched in 1956, nowadays the central journal in the field. It was also a very complicated task: leading journals like the *Proceedings of the Royal Society* feared strong competition, and with good reason.

Batchelor also created and chaired for twenty years the European Mechanics Committee, now the European Mechanics Society (Euromech), and



Photograph courtesy of Department of Applied Mathematics and Theoretical Physics, Cambridge University.

David G. Crighton

stimulated a wide program of colloquia covering new aspects of fluid and solid mechanics. His role in establishing scientific relations with scientists behind the Iron Curtain should be emphasized too.

Of substantial importance was the propaganda for G. I. Taylor's work and research style. Batchelor published G. I. Taylor's collected papers at Cambridge University Press in four volumes [19] and wrote a remarkable book [8] surveying Taylor's life and legacy. I definitely think that without Batchelor's activity the memory of Taylor's unique research style and his scientific achievements would decay in today's modern turbulent scientific ocean.[6] Nowadays they live on and continue to influence new generations.

New Leadership

In 1983, rather unexpectedly, Batchelor retired from his position as professor of applied mathematics at Cambridge University and head of DAMTP. His natural successor was his student and close collaborator for many years, H. K. Moffatt. I had a chance to present a survey of Moffatt's achievements at the last IUTAM Congress in September 2000, but since it is inappropriate to speak of Moffatt here, I mention only that it was he who proposed *helicity*, a new invariant in fluid mechanics, and shaped its new branch, topological fluid dynamics. He inherited the best features of Batchelor, and under his leadership the traditions established by Batchelor were carefully preserved and developed at DAMTP and outside of it. Now Moffatt is the director of the Isaac Newton Institute for Mathematical Sciences at Cambridge.

In 1991 Moffatt stepped down, and David George Crighton became the third head of DAMTP. Soon it became clear that there was no better choice throughout the world. Since 1986 Crighton had been professor of applied mathematics at Cambridge, following Batchelor. Before Cambridge he worked for twelve years at Leeds and created there a strong group of applied mathematicians.

Asymptotics

David Crighton was an applied mathematician in genuine British style. He was a student of John E. Ffowcs Williams, who was at that time at Imperial College in London, later at the engineering department at Cambridge. When David came to his future mentor for the first time, he was asked a natural question, "What do you want to study?" The answer was "Turbulence." "Walk away" was the immediate response. Of course, this was a clear exaggeration; Crighton was accepted, and soon they became close friends. John's speech at Crighton's memorial service at Great St. Mary University Church in Cambridge was unforgettable. Two other eminent persons were considered by Crighton as his mentors: Joseph B. Keller and Sir James Lighthill. Indeed, Crighton's first two papers [13], [14] (with J. E. Ffowcs Williams) were related to turbulence. However, turbulence did not enter his basic scientific life, although he was always interested in the events in turbulence studies. His basic subject became fluid-mechanical acoustics, which he shaped (see especially his essay [10]), although a strong diversity was characteristic of his creative work.

Crighton used diverse mathematical methods in his research, but his basic tools in applied mathematics were asymptotic methods. In his hands asymptotic methods were like a piano in the hands of Vladimir Horowitz.[7] I strongly advise the reader to read Crighton's paper [11] devoted to asymptotic methods in applied mathematics. This paper was published in the proceedings of a conference on Mathematics in Industry and is not as well known as it deserves: I think that this paper was to a certain extent his scientific "credo".

[6] *Compare G. I. Taylor with Sir Harold Jeffreys, also a great applied mathematician. For instance, it was Jeffreys who invented the WKB method fifteen years before Wentzel, Kramers, and Brillouin, but his priority remains unnoticed. Such a situation is inconceivable in the case of G. I. Taylor.*

[7] *This comparison is by no means an accident: Crighton's second passion besides mathematics was music. His colleagues knew that every year he disappeared for two weeks to the Bayreuth Wagner Festival. His fiftieth birthday was celebrated in Cambridge by a concert by Russian pianist Tatyana Nikolaeva, which he sponsored. Three weeks before his tragic death he conducted the orchestra of Jesus College (where he was Master), performing the overture to Tannhäuser; listeners were deeply impressed.*

Several examples: Using asymptotic methods, Crighton was able to construct a model of noise radiation by a propeller—a practical problem of immense importance. He proposed the "large-blade-number asymptotics" (later it appeared that in fact these asymptotics work very well already for the number of blades equal to four: such situations happen in applications of asymptotic methods). Crighton and his students were able (see [12], [18]) to find which section of the propeller blade produces major acoustic radiation. They were able to propose to designers, in particular those of the renowned Rolls Royce Company, detailed recommendations on how to reduce noise both in subsonic and supersonic propellers and in so-called propfans (systems of counterrotating blade rows). I want to emphasize that the asymptotic analysis performed was free of additional assumptions, and technically very complicated, but the final results appeared in a transparent form. Important results were obtained by Crighton and his students in hydro-aeroacoustics, also a field of high practical importance: sound generation due to interaction of fluid with deformable bodies (structures). Here I would like to mention a characteristic paper [2] where a model of vibration of submerged elastic bodies was proposed. It was shown (again by elegant application of asymptotic methods) that the vibrational modes generally speaking depend upon properties of the entire flow-body system; they cannot be separated into "solid modes" and "fluid modes". It was also demonstrated that the system of vibrational modes is incomplete, a practically important mathematical property.

The diversity of Crighton's scientific activity can be demonstrated by two papers [15], [9]. In the first paper an asymptotic model of void formation in fluidized beds was proposed. "Fluidized bed" (a layer of catalytic particles suspended by rising flow) is an important technological process in many branches of the chemical industry. If voids are formed, the efficiency of the process drops drastically. Here again an elegant asymptotic method was used and the final result was obtained in transparent form. In [9] the asymptotic method, based on an expansion in a small parameter ($\gamma - 1$) (γ being an adiabatic index), was used in modeling ignition of a combustible mixture by a shock wave.

Apropos: A Comment on Cash

The textbook Batchelor wrote was very successful financially. The royalties it has generated were bequeathed for the purpose of organizing a fluid dynamics laboratory at DAMTP, which will be named after him. He also left large sums to help research students at Cambridge and in his native Melbourne and to organize annual Batchelor lectures in the Department, where distinguished fluid dynamicists will lecture every year about their new achievements.

Both Batchelor and Crighton had strong fund-raising abilities. They collected large amounts of money to create new chairs at DAMTP and to construct new buildings for the department. Leading industrial giants like Rolls Royce and Schlumberger generously participated, because they had seen how useful applied mathematics could be for their business.

Now Cambridge University has established the G. K. Batchelor and David Crighton Memorial Funds. A steady flow of contributions goes to these funds, from modest donations by students to generous contributions by industrial companies and Cambridge Colleges. All are welcome.

The Legacy at DAMTP

The situation at DAMTP nowadays is to a certain extent similar to the situation at another distinguished Cambridge body, Cavendish Laboratory, after the unexpected death of Lord Rutherford in 1937. Cambridge University was lucky to choose Sir Lawrence Bragg. The new Cavendish professor had, according to Freeman Dyson, a definite strategy: he did not try to restore the previous fame related to "smashing the atom", and he did not start investigations in fashionable directions only because they were fashionable. Also, he did not pay attention to the mockeries of more traditional physicists, especially theorists. What he really did was to attract talented people (he was able to make a precise selection), supporting them and giving them freedom to work in their own directions. Batchelor was one of these people, and together with Batchelor a constellation of bright young people of different specialties was collected at Cavendish. At first such a transformation of the laboratory repelled devoted physicists from it, but later a remarkably large number of Nobel Prizes and other exceptional recognitions for Cavendish researchers persuaded the scientific community that Sir Lawrence had serious reasons for doing as he did.

Now Cambridge University has made its choice: Professor Timothy Pedley, whose field is biological fluid mechanics. He shaped this new branch of fluid mechanics together with Sir James Lighthill. Pedley, a Cambridge man, replaced David Crighton at Leeds, and during his time the applied mathematics there achieved further development. He returned to Cambridge to become G. I. Taylor Professor of Fluid Mechanics in 1996.

The problems before him are enormous: DAMTP is now a huge body, around four hundred people, including graduate students. A move to new buildings will come soon. However, remarkable people are around: Stephen Hawking for one, whose name and achievements are widely known; Professors, Readers, and Lecturers of different generations; and also bright young talents. The mathematical community can expect that like Sir Lawrence Bragg in the past, Timothy Pedley will be successful in preserving and developing the traditions of his predecessors.

George Batchelor and David Crighton were separated in age by one human generation and two scientific generations, yet they passed away nearly simultaneously. George Batchelor lived to enjoy a happy, harmonious retirement. David Crighton succumbed to cancer at the moment of liftoff into a new orbit of his distinguished career: at the time of his death he was, in addition to many other duties, president-elect of the London Mathematical Society. Both of these applied mathematicians were outstanding personalities. It is very difficult to become outstanding at Cambridge: everyone entering the great city becomes surrounded by eternal walls and eternal shadows and subject to exacting comparisons.

The lives and achievements of George Batchelor and David Crighton will inspire many generations to come. Although we grieve at their absence, we take comfort in celebrating their lives, and we rejoice in the time we were lucky enough to spend in their orbits.

References

[1] V. I. Arnold, Topological problems of the theory of wave propagation, *Russian Math. Surveys* **51** (1996), 1–47.

[2] P. E. Barbone and D. G. Crighton, Vibrational modes of submerged elastic bodies, *Appl. Acoustics* **43** (1994), 295–317.

[3] G. K. Batchelor, Kolmogorov's theory of locally isotropic turbulence *Proc. Cambridge Philos. Soc.* **41**, no. 4 (1947), 533–559.

[4] ____ , Recent developments in turbulence research, *Proc. VII Internat. Congress App. Mech.* (H. Levy, ed.) (London, September 1948).

[5] ____ , *The Theory of Homogeneous Turbulence*, Cambridge University Press, Cambridge, 1953.

[6] ____ , *An Introduction to Fluid Dynamics*, Cambridge University Press, Cambridge, 1967.

[7] ____ , Development in microhydromechanics, *Theoretical and Applied Mechanics* (W. T. Koiter, ed.), North-Holland, 1976, pp. 33–55.

[8] ____ , *The Life and Legacy of G. I. Taylor*, Cambridge University Press, Cambridge, 1996.

[9] P. A. Blythe and D. G. Crighton, Shock generated ignition: The induction zone, *Proc. Royal Soc. London* **A426** (1989), 189–209.

[10] D. G. Crighton, Acoustics as a branch of fluid mechanics, *J. Fluid Mech.* **106** (1981), 261–298.

[11] ____ , Asymptotics — an indispensible complement to thought, computation and experiment in applied mathematical modelling, *Proc. VII European Conf. Math. Industry* (A. Fasano and M. Primicerio, eds.), B. G. Teubner, Stuttgart, 1994, pp. 3–19.

[12] D. G. Crighton and A. B. Parry, Asymptotic theory of propeller noise, *AIAA J.* **29** (1991), 2031–2037.

[13] D. G. Crighton and J. E. Ffowcs Williams, Real space-time Green's functions applied to plate vibration induced by turbulent flow, *J. Fluid Mech.* **38** (1969), 105–131.

[14] ____ , Sound generation by turbulent two-phase flow, *J. Fluid Mech.* **36** (1969), 585–613.

[15] S. E. Harris and D. G. Crighton, Solitons, solitary waves, and voidage disturbances in gas-fluidized beds, *J. Fluid Mech.* **266** (1994), 243–276.

[16] H. K. Moffatt, G. K. Batchelor and the homogenization of turbulence, *Ann. Rev. Fluid Mech.* **34** (2002), to appear.

[17] A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics: Mechanics of Turbulence,* vol. 2 (J. L. Lumley, ed.), MIT Press, Cambridge, MA, and London, 1975.

[18] N. Peake and D. G. Crighton, An asymptotic theory of near-field propeller acoustics, *J. Fluid Mech.* **232** (1991), 285-301.

[19] G. I. Taylor, *Scientific Papers* (G. K. Batchelor, ed.), Cambridge University Press, Cambridge: vol. I (1958), vol. II (1960), vol. III (1963), vol. IV (1971).

[20] J. von Neumann, Method in physical sciences, *The Unity of Knowledge* (L. Leary, ed.), Doubleday, New York, 1955, pp. 157–164.