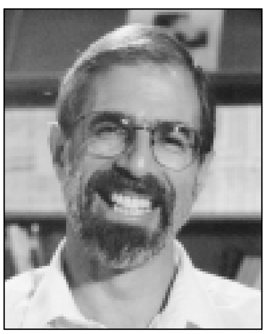
Nominations for President Elect

Nomination for David Eisenbud

Barry Mazur and Margaret H. Wright



It is a challenging exercise to convey David Eisenbud's deeply impressive combination of mathematical contributions, commitment to research in the mathematical sciences, creative and inspiring leadership, and ability to get things done—all of which make him an outstanding candidate for president of the American Mathematical Society.

This article has two parts: Barry Mazur describes David's accomplishments in mathematics, and Margaret Wright discusses his service to the mathematical sciences community.

Barry Mazur

It is a pleasure to have the opportunity to write some lines about David Eisenbud's mathematical work, in connection with his nomination for the presidency of the American Mathematical Society. David Eisenbud's research accomplishments extend broadly through algebra and its applications. His publications (over a hundred of them!) have made significant contributions to fundamental issues in the subject. David also has a marvelous gift for mathematical collaboration. The sweep of his interests and the intensity of his mathematical interactions have brought him into fruitful co-authorship with many mathematicians of different backgrounds and different viewpoints.

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Shortly after his graduate days, David began a joint project with Buchsbaum (cf. [1]–[4]). Among other things, they established an elegant geometric criterion for exactness of a finite free complex that has many applications in the homological study of commutative rings. For example, let $S = \mathbf{C}[x_1, \ldots, x_n]$ be the ring of polynomial functions on $\mathbf{C}^n$, and let

$$0 \to S^{r_t} \to \ldots \to S^{r_0}$$

be a complex of free S-modules S^{r_i} and maps $\phi_i(x) : S^{r_i} \to S^{r_{i-1}}$ that are matrices of polynomial functions on $\mathbf{C}^n$. If the ϕ_i were constant matrices, then this complex would be exact if and only if rank ϕ_i + rank $\phi_{i-1} = r_i$ for all i. Buchsbaum and Eisenbud showed that, in general, the complex is exact if and only if, for all i, the set of points $x \in \mathbf{C}^n$ where rank $\phi_i(x)$ + rank $\phi_{i-1}(x) \neq r_i$ is either empty or of codimension at least i in $\mathbf{C}^n$.

Eisenbud and Buchsbaum [4] also established a structure theorem for Gorenstein rings of codimension 3, which parallels a structure theorem for Cohen-Macaulay rings of codimension 2 found by Hilbert in the 1880s and generalized by Burch and others. Here is what they do, formulated in a geometric setting. If $V \subset \mathbf{C}^d$ is a subvariety of codimension e which can be cut out, locally, by precisely e equations, then its local rings satisfy certain dualities; let us refer to these dualities as the *Gorenstein* condition. Sometimes one encounters varieties of codimension e whose local rings satisfy this Gorenstein condition but require strictly more than e equations. Eisenbud and Buchsbaum show that, in codimension 3, such a variety V is defined locally by a set of polynomials $\mathcal{E}$ obtained from an $n \times n$ alternating matrix M of rank $n - 1$ with entries in $\mathbf{C}[x_1, \ldots, x_d]$, for some n, as follows. The elements of $\mathcal{E}$ are the Pfaffians of the $(n-1) \times (n-1)$ submatrices obtained by deleting a row and the corresponding column of M. (A *Pfaffian* of an alternating matrix is, in effect, the square root of its determinant.) This result has continued to play an important role in the local theory because of the access that it gives to interesting examples. It recently received attention also in the global case and has been extended by work on vector bundles and projective varieties in which Eisenbud continues to play a significant role [12].

In the middle 1970s David worked with Harold Levine on the topology of *finite* C^∞ map germs $f : (\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0)$ (cf. [5]). The requirement of "finiteness" is an algebraic property of the map germ which, among other things, guarantees that the restriction of f to a small sphere around the origin maps that sphere to a set which misses the origin. Such a finite map germ f therefore induces a map between $n-1$-spheres. The degree of this map of spheres is called the *topological degree* of f; it may also be regarded as the Poincaré index of the associated vector field. One can associate a finite dimensional vector space to such a map germ; its dimension is the multiplicity of the complexified germ. Eisenbud and Levine describe a quadratic form on this vector space and show that the signature of this quadratic form is equal to the topological degree [5].

V. I. Arnol′d once referred to this celebrated formula of Eisenbud-Levine, which links calculus, algebra and geometry, as a "paradigm" more than a theorem that provides a local manifestation of interesting global invariants and that "would please Poincaré and Hilbert (also Euler, Cauchy and Kronecker, to name just those classical mathematicians, whose works went in the same direction)."

Given this early work, it was natural for David's attention to turn to the study of singularities and their topology. In this period, David wrote a book with the topologist Walter Neumann [6] on the topology of the complements of the sort of knots that appear in the theory of plane-curve singularities.

David next became interested in algebraic geometry, beginning a long and important collaboration with Joe Harris (cf. [7], [8]). Together, they developed the theory of Limit Linear Series and used it to solve a number of classical problems about the moduli spaces of complex algebraic curves. This theory was published in a series of eight papers in *Inventiones*. One of the well-known applications of their theory is that it gives a nice proof of the following fact. For most algebraic curves C of genus $g \geq 23$, if we write down polynomial equations which define C and let the coefficients of the polynomials vary as rational functions of a complex parameter in such a way that for each value of the parameter the system of equations continues to define a curve, then all the curves obtained are isomorphic. In particular, this shows that the moduli space of curves of genus $g \geq 23$ is not rational, or even unirational, as had been conjectured by Severi. This result of Eisenbud and Harris sharpens what was priorly known and is part of a long classical development of the subject in which many open problems remain today. For example, it is still unknown if the same assertion is true for genus 22. The behavior described is in sharp contrast with what happens for low genus (all curves of genus 1, of course, fit into a single rationally parametrized family; for any genus $g \leq 13$ most curves of genus g are members of a single family of curves cut out by polynomial equations where the coefficients are rational functions of the appropriate number of complex parameters).

In the late '80s and '90s David published papers on many aspects of algebraic geometry and commutative algebra and became interested in combinatorics (collaborating with Dave Bayer on "graph curves" [8a]) and statistics (collaborating with Persi Diaconis and Bernd Sturmfels on random walks on lattices (cf. [9])).

David's most recent research represents projects with a number of mathematicians, mixing commutative algebra, algebraic geometry, and topology. Among other things this work includes significant applications of the theory of free resolutions over exterior algebras to:

Hyperplane arrangements,
Bernstein-Gel′fand-Gel′fand correspondence and Beilinson Monads,
Chow forms and elimination theory (including, among other things, new formulas for the resultant of three homogeneous forms in three variables),
Linearity of free resolutions and the existence of linear Cohen-Macaulay modules.

David has had twenty-one successful Ph.D. students, has organized many conferences here and abroad, and has written two textbooks (cf. [10], with Joe Harris; and [11]) which are among the best-selling texts in Springer's series of Graduate Texts in Mathematics. He is currently engaged with Harris in a new book project, a book for a second course in algebraic geometry.

Margaret Wright

David and I met in 1997 when we traveled to England with Don Lewis (then director of the Division of Mathematical Sciences at NSF) and Jim Crowley of SIAM to visit the Newton Institute and Hewlett-Packard Labs. David was just starting as director of the Mathematical Sciences Research Institute (MSRI) in Berkeley, and I was a member of the MSRI Scientific Advisory Committee. We've worked together closely during the past four years on a wide range of activities related to MSRI, as well as on broader efforts to increase support for mathematics research. Since my 1995–96 term as president of SIAM, I've continued an involvement in science policy—an interest that David and I share.

David has served the mathematical community as chair of the mathematics department at Brandeis, on advisory and evaluation committees for the National Science Foundation, as a member of the Board on Mathematical Sciences, and as vice president of the AMS. But his service that is most visible nationally and internationally has been as director of MSRI, where he moved in 1997 after twenty-seven years at Brandeis.

A fundamental strength of mathematicians is their ability to generalize, and I believe that David's performance as AMS president can be predicted with high accuracy by generalizing from his success at MSRI. In fact, his leadership at MSRI exemplifies the qualities needed by the AMS president.

With David as its director, MSRI has continued its tradition of superlative programs in fundamental mathematics while simultaneously expanding into a broader and more diverse selection of fields. David has furthered a deliberate policy of outreach into new areas, and MSRI's influence and reputation increasingly extend beyond core mathematics into areas on the boundaries between mathematics and science as well as into applications ranging from imaging to cryptography to finance. In addition to strengthening the intellectual heart of MSRI's mission, David has encouraged MSRI to present events that bring the public closer to the richness of mathematics. He has accomplished this through a multitude of thoughtful innovations—for example, the Journalist-in-Residence program that he began soon after he arrived at MSRI. David clearly understands that MSRI is not a one-person operation; he actively welcomes the ideas of other people, and he has worked to create an environment that encourages staff and volunteers to develop new projects.

Mathematics is, of course, necessarily linked to people—to individual mathematicians and to the mathematical sciences community. David recognizes and likes the human side of mathematics and has an intense interest in engaging and supporting young mathematicians. Within the past few years, MSRI has doubled its programs for graduate students and greatly increased the participation of women and minorities in these programs. The institute has regularly hosted workshops for women and minority mathematicians, as well as workshops conceived and organized by early-career mathematicians.

Some leaders are only "idea people": strong on concepts, weak on execution, who leave all the hard work to someone else. Others focus on details, their perspective limited by existing practice, feeling threatened by anything different. David is far removed from these extremes; he has many new ideas, but he also takes personal responsibility for transforming the best of them into reality. David follows through and does not shrink from the hard slog when it is necessary to get the job done. For example, to prepare MSRI for its recompetition two years ago, he spent countless hours consulting with others and writing (and rewriting) the NSF proposal. He has recently expended enormous energy raising funds for a building expansion that will greatly improve MSRI's operations.

Skill in communication—an essential quality for any scientific leader today—is one of David's greatest strengths. He is an articulate, original expositor in both writing and speech, able to describe eloquently the nature of mathematics, the links among branches of mathematics, the ties of mathematics with science and engineering, and the role of mathematics in applications. He also has the much rarer ability to communicate the excitement and content of mathematics to a general audience.

David is principled but not unbending; he listens to and respects the views of others, but does not shrink from taking a stand when necessary. He is passionately convinced of the importance of mathematics; his intellectual and personal dedication to mathematics is contagious and energizing to those around him. Even in difficult circumstances, he retains a sense of perspective and (sometimes just as important) a sense of humor. David is a persuasive, hard-working, and effective advocate for mathematics at all levels—precisely what the AMS needs.

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Nomination for David A. Vogan Jr.

Anthony W. Knapp

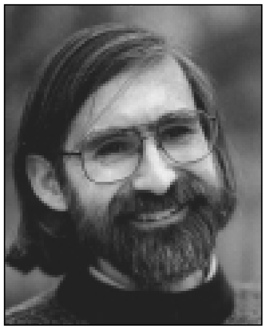

Photo courtesy Donna Coveney/MIT.

In recent years the tradition has been that the AMS president is someone of high stature in mathematics research whose presidential duties include representing American mathematics to the nonmathematical world and guiding the AMS committee system in its formulation and carrying out of policies. For this position, as one says in the sports world, David Vogan is the complete package. He has done stunning research over a long period of time, he is the Head of one of the very best mathematics departments in the world, he has been a quiet but forceful advocate for women in the profession, he is known for his extraordinary mentoring of graduate students, he is the author of four research-level books, and he has served the AMS long and well in several capacities.

David's field is the representation theory of Lie groups. A group representation is a group action by linear transformations, typically on a complex Hilbert space. Behind a representation is often an action of the group on a manifold, transitive or not, and one studies the manifold in part by studying the complex-valued functions on it. Group representations of nonabelian infinite groups were studied first by I. Schur and H. Weyl in the 1920s, and it was not long before this approach made the subject blend with quantum mechanics, as one examined the effect of symmetry and the breaking of symmetry on systems of differential equations. The decomposition of representations into sums or integrals of other representations and the identification of the ultimate irreducible pieces have remained as fundamental problems in the theory since the 1920s.

David has concentrated his research on reductive Lie groups, which one may view as closed subgroups of real or complex matrices that are stable under conjugate transpose. These are the groups whose normal subgroups offer few clues to their structure. Some early names associated with the representation theory of these groups are Bargmann, I. M. Gelfand, Naimark, Godement, and Mackey. But from the early 1950s until 1976, the year of David's thesis, the direction of the field was set by Harish-Chandra and R. P. Langlands.

Harish-Chandra's approach for such a G was ultimately analytic, using differential equations and asymptotic properties of the functions $g \mapsto (R(g)u, v)$ associated to a representation R to get a handle on R. The fundamental irreducible representations for Harish-Chandra were those in the "discrete series"—the ones that occur as subrepresentations of $L^2(G)$. Other representations of interest could be constructed by "parabolic induction" from the discrete series. Harish-Chandra classified the discrete series and then, in part using ideas that Langlands had developed for studying $L^2(G/\Gamma)$ for arithmetic subgroups Γ, completed the analysis of $L^2(G)$. Langlands, for his own part, went on to use asymptotic expansions to classify the irreducible representations. He used his classification as substantive evidence for a body of conjectures and questions that have come to be known as the Langlands program; these relate the solutions of Diophantine problems to infinite-dimensional representation theory, and later progress by Langlands on these conjectures was indispensable to the proof of Fermat's Last Theorem.

That much history brings us to David's thesis in 1976, which was written under the direction of B. Kostant and revolutionized the field. David introduced a completely algebraic theory for studying irreducible representations of reductive groups. The fundamental representations were not discrete series but representations behaving quite differently, and the tools were not differential equations and asymptotic expansions but cohomology theories. The final theorem of the thesis was a classification completely different from the one by Langlands. Building on ideas that G. J. Zuckerman introduced in 1978, David developed a construction now called "cohomological induction" that made his classification easier to formulate and to work with. His completed classification was published in 1981 in the first of his four research books. The Vogan-Zuckerman classification, as it is called, does not replace the Langlands classification; it gives a completely new way of looking at the field, and the passage back and forth between the two approaches is a powerful tool.

Left unaddressed by all this work was the question of which irreducible representations are unitary. Parabolic induction carries unitary representations to unitary representations, but cohomological induction does not necessarily. In a 1984 paper David proved, by a remarkably intricate algebraic construction, that cohomological induction does preserve unitarity when a certain positivity condition holds for the parameters. With this theorem he was able to classify the irreducible unitary representations for the general linear groups over the reals, the complexes, and the quaternions.

David's Hermann Weyl Lectures at the Institute for Advanced Study in 1986, published as an Annals of Mathematics Study in 1987, showed David's thinking about the classification of irreducible unitary representations for general G. The book gives great insight into the mind of a first-rate mathematician at work.

This classification problem for irreducible unitary representations remains unsolved in general, but it is now known that cohomological induction is an indispensable tool for the problem. A 1998 *Annals of Mathematics* paper by David with S. Salamanca-Riba reports on some recent progress.

In the 1980s J. Arthur made some conjectures related to the Langlands program. Like the program in general, Arthur's conjectures are first of all about automorphic forms, but they have consequences and analogs in

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representation theory for real and p-adic reductive groups. A 1992 book by David with two coauthors proves most of Arthur's conjectures for real groups. The results in the book provide evidence for the full Arthur conjectures about automorphic forms, as well as tools to approach those conjectures.

David received his Ph.D. from the Massachusetts Institute of Technology at age twenty-one, spent another year as an instructor at MIT, visited the Institute for Advanced Study for two years, and then returned to the MIT faculty. He rose through the ranks and is now Professor and Head of Mathematics. Over the years he served multiple terms as undergraduate director and graduate director.

He became Head of Mathematics in 1999. The MIT mathematics department has been especially successful at having pure and applied mathematics thrive together in a single department. The department has two subdepartments, one for pure mathematics and one for applied mathematics, and each has a select committee to deal with hiring and some other matters. Before becoming Head, David served on the select committee in pure mathematics. Now, as Head, he is responsible for representing the combined views of the two subdepartments to the dean and others. His appointment as Head indicates a level of trust in his ability to carry out this responsibility.

David takes seriously the status of women in the profession. He is a member of the AWM. As department Head, he has extended to mathematics instructors a good MIT faculty-leave policy for those who assume responsibility to care for a newborn child or a child newly placed for adoption or foster care. This extension of the policy is a serious step, as instructors are more likely to benefit from such a policy than are senior faculty.

David has supervised twenty-one Ph.D. theses. In addition, he has organized a weekly Lie Groups Seminar for twenty years whose speakers have kept the greater Boston mathematical research community abreast of current developments in many areas related to Lie groups.

David is admired as a teacher. At the time of his appointment as Head of Mathematics, the MIT News Office said, "Among these students, he is known for his loyalty and generosity with his time and his ideas."

David is married to his childhood sweetheart, and they have two children. He and his wife are pillars of one of the downtown Boston churches. Also, David is a director of The Giving Back Fund, a public charity that provides expertise to athletes, entertainers, and others to help them get the greatest possible impact from their philanthropy.

For the AMS David has been a member of the Council, has served on the Science Policy Committee, has coorganized three special sections at meetings, has been a member of the editorial staff of the *Bulletin* since 1987, and has served as founding editor of the electronic journal *Representation Theory*.

He has jointly organized three non-AMS conferences: a one-week conference at Oberwolfach, a special year at MSRI in representation theory, and the graduate component of one summer's Park City Mathematics Institute.