Two Reactions to *The Mathematical Education of Teachers*

In August 2001 the Conference Board of the Mathematical Sciences (CBMS) issued the report *The Mathematical Education of Teachers* (MET). The aim of the report is to set forth recommendations for bringing about significant improvement in the mathematical education of future teachers. The project to produce the report was supported by a grant from the U.S. Department of Education.

The members of the Steering Committee for the report were: James Lewis (chair), Richelle Blair, Gail Burrill, Joan Ferrini-Mundy (advisor), Roger Howe, Mary Lindquist, Carolyn Mahoney, Dale Oliver, Ronald Rosier (ex-officio), and Richard Scheaffer. The members of the Writing Team were: Alan Tucker (lead writer), James Fey, Deborah Schifter, and Judith Sowder. The members of the Editing Team were: Cathy Kessel (lead editor), Judith Epstein, and Michael Keynes.

Other *Notices* articles on teacher education include "Spotlight on Teachers" by James Lewis, April 2001, pages 396–403; and a review of Liping Ma's book, *Knowing and Teaching Elementary Mathematics*, reviewed by Roger Howe, September 1999, pages 881–7.

The *Notices* invited two individuals to give their reactions to the MET report. Their commentary follows.

—Allyn Jackson

Amy Cohen

We mathematicians at colleges and universities have a natural interest in mathematics education in elementary and secondary schools and therefore an interest in the education of school teachers. Admittedly, our efforts alone cannot guarantee that mathematics teachers will be effective. Nonetheless, we play a crucial role in educating not only teachers but also the faculty who educate teachers. It will not help for mathematics faculty simply to complain about the preparation of undergraduates; faculty should actually work to improve the education of prospective teachers.

The report *The Mathematical Education of Teachers* (MET) challenges mathematics faculty, their academic leadership, and particularly those engaged in designing programs and delivering courses to modify what they teach prospective teachers and how they teach it. In the first fifteen pages the report makes eleven numbered recommendations and provides a brief context for them. Further chapters lay out five strands of subject matter knowledge that are developed throughout grades 1–12, discuss the mathematical understanding essential to teach this material effectively, and suggest changes in content and delivery of the undergraduate education of prospective teachers to help them obtain that essential understanding.

This report, especially its first two chapters, deserves careful reading and consideration by all mathematicians, especially those who will take

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The Mathematical Education of Teachers

The AMS is publishing the full 145-page report in cooperation with the Mathematical Association of America (MAA). The MAA is also producing a shorter, 50-page form of the report. The CBMS is distributing about 3,000 free copies of the full report (to departments of mathematics and other organizations) and will distribute about 10,000 free copies of the short form in response to requests. The report is also available on the Web at `http://www.maa.org/cbms/`.

The full report is available for sale from the AMS (item code CBMATH/11) or the MAA (ISBN 0-8218-2899-1). Further information may be obtained by telephoning the AMS at 800-321-4267 or by visiting the AMS Bookstore at `http://www.ams.org/bookstore/`.

part in the discussion of educational issues. The later chapters are essential reading for those engaged in the education of teachers, either in policymaking or in classroom instruction, and will repay careful reading by those who want a deeper understanding of the mathematics involved in precollege grades. It may be more fun to try to derive one's account of reality from pure reason, but it is more useful in effecting change to work from fact.

The recommendations of this report can be the basis for productive change, provided that the necessary resources can be found to implement them. Money is not the only rare resource; so are time, energy, and wisdom to change teaching habits and requirements for graduation and certification. To improve the education of teachers, the various stakeholders must generate the political will to find and use these resources.

Most mathematicians will probably agree with many statements in the MET report, starting with the assertion in the preface that "teachers need a solid understanding of mathematics so that they can teach it as a coherent, reasoned activity and communicate its elegance and power." Repeatedly the report stresses that teachers should acquire "mathematical common sense", that the language of mathematics has meaning, and that students need both facts and understanding to make use of mathematical reasoning. The report argues that, in order to teach effectively and to develop professionally, teachers need fluent mastery of mathematics with intellectual substance and subtlety. Thus the report repeatedly suggests that at least some of the undergraduate courses taken by prospective teachers include explicit attention to connections between the material taught in schools and the material taught in college. Finally, since people teach as they were taught, the report argues that instructors of mathematics courses for prospective teachers should use a variety of effective teaching styles and should engage students in active modes of learning.

Many colleagues will, I hope, be a bit skeptical of the assertion that the "mathematical knowledge needed for teaching is *quite different* (emphasis added) from that required [for] other professions." Over the last year, members of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America (MAA) have met with mathematicians (and others) who hire and supervise B.A./B.S. mathematics majors in the private sector to inquire what knowledge and "habits of mind" they see in productive staff and therefore seek when hiring. Their responses describe qualities remarkably like those MET lists as important for effective teaching: seeing the mathematical content in mathematically unsophisticated questions, seeing underlying similarity of structure in apparently different problems, facility in drawing on different mathematical representations of a problem, communicating mathematics meaningfully to diverse audiences, facility in selecting and using appropriate modes of analysis ("mental", paper-and-pencil, or technological), and willingness to keep learning new material and techniques.

The MET report, at least as I read it, suggests implicitly that prospective teachers take courses that are different from those taken by other mathematics students. How many courses should be different is ambiguous; should some courses, or most courses, or all courses be different? Separating prospective teachers from other mathematics students might help develop an esprit de corps and a more professional attitude among future teachers, especially those aiming at K–8 certification. On the other hand, it seems ironic that the community of mathematics educators, which so often argues against tracking in middle and secondary grades, appears here to propose something like tracking for prospective teachers. It seems to me likely that for prospective teachers of elementary and middle grades most courses should be different. Prospective high school teachers might need only some courses that are different.

A larger related question is how to offer courses that lead to substantial learning by students not headed for doctoral programs while still meeting the needs of the very few who will follow in our footsteps as university professors. Perhaps once it was sufficient simply to present material, assign homework, give exams, and report which students learned well. But today it is not enough merely to inspire and identify those few students who will become our future research colleagues, regardless of how we treat them. The continued flow of resources to support basic research in mathematics depends in large part on the ability of mathematicians to prepare students who can use mathematical techniques and reasoning in many roles, including teaching in the schools.

Teaching is no longer a lifelong vocation. Teachers often move on to other employment. College graduates initially pursuing other employment sometimes return for postbaccalaureate certification programs. The education of teachers should not be limited to preparation for teaching. Finally, one might hope that well-conceived general education courses in mathematics could help recruit undecided majors for teaching careers at the elementary and middle grades. In some departments there is already evidence that mathematics majors with satisfying experiences as tutors or peer mentors sometimes decide rather late in their undergraduate years to enter teaching careers.

Each of the eleven numbered recommendations of the MET report should attract interest and discussion. Some will draw more immediate agreement than others will. Here are my reactions, formulated primarily with the intent of eliciting discussion.

Recommendations 1 and 5. These recommendations say that prospective teachers should develop a deep understanding of the mathematics they will teach and should also master mathematics that comes both before and after their own intended teaching level. The courses they take should connect college content to school content. These recommendations also say that mathematicians who teach these courses should "have a serious interest in teacher education" and should "cooperate with education faculty."

The later chapters of the MET report give arguments and examples to defend these recommendations. A university mathematician reading these chapters with a reasonably open mind is likely to gain both information about and insight into the challenges of teaching mathematics in American schools.

These recommendations pose cultural problems for departments in research universities and for departments staffed by faculty trained at research universities. Such faculty probably learned school mathematics easily. Ideas they have always found clear require time and effort to communicate to students who have never experienced mathematical clarity. It may be harder for a faculty member to explain an idea that has always seemed obvious than an idea that has not yet been fully understood. Will faculty invest the necessary time and effort? Will that investment command respect from peers and reward from chairs and deans?

Many university faculty have come through national educational systems where the job of a research department was purely to produce research results and research mathematicians. In such systems prospective teachers, like prospective economists or engineers, studied with entirely different faculties at the postsecondary level. Faculty with such backgrounds may find it hard to accept teaching nonmathematics majors as a respectable use of their talents. It may be hard for those with such backgrounds to enlarge their concept of the proper use of their talent in order to come to believe that it is respectable to work with nonmathematics majors. Even many mathematicians trained in the U.S. are influenced by a value system that rewards the "best" faculty with a reduced teaching load concentrated in courses for graduate or prospective graduate students.

How might U.S. universities convince faculty that it is a mark of respect to be asked to apply one's insights to the enthusiastic teaching of often unenthusiastic students, especially in courses for elementary or middle school teachers? Or, failing that, should we convince senior faculty and deans to appoint and promote some faculty on criteria counting teaching and research equally rather than to judge primarily on research? Should we consider reverting to a pattern of separate graduate and undergraduate departments of mathematics?

The committee producing the MET report was charged with offering recommendations, not with guiding implementation. Faculty sympathetic to the goals will have to work on implementation within the *Realpolitik* of their departments and universities. It is easier to say "the entire mathematics faculty [should] actively support teacher education efforts" than to figure out how to create such support. My experience with promoting educational change suggests that it is often better to enlist a respected cadre of supporters within a broad population of nonobjectors than to provoke overt opposition by insisting that everyone agree.

I would suggest further that innovators offer plausible benefits not only for students but also for faculty members and the department as a whole. One of my colleagues, who was initially dubious about a pedagogical initiative he was talked into trying, remarked midsemester that "It's more fun to teach when the students learn." This remark made it easier to recruit other colleagues to try the same innovation. If our state governments and university administrations are serious about increasing the number and quality of the teachers we educate, they must be willing to supply resources and rewards. Asking a department to steal resources from valued ongoing projects to fund new ones is a sure way to generate hostility. This last concern is especially important in departments that have typically used the same courses for the mathematical education of K–8 teachers and for the general education of students whose intended majors do not require calculus.

Recommendations 2, 3, and 4. These recommendations present specifics on the amount of mathematics instruction prospective teachers should receive: 9 semester-hours for teachers of grades 1–4, 21 semester-hours for teachers of grades 5–8, and at least the equivalent of a full mathematics major for high school mathematics teachers. The recommendations say that this coursework should convey both mathematical knowledge and habits of mathematical common sense and reasoning so that teachers can ask good questions, investigate problems, communicate understanding, and lead their students to do the same.

For many institutions, these three recommendations carry staggering implications for the allocation of resources and the construction of graduation requirements. It is not surprising that later recommendations discuss the need for cooperation among faculty members, various academic units, and public policy groups in implementing this increased coursework wisely. The MET report suggests in Chapter 9 a 6-credit-hour capstone course for future high school teachers to bring coherence to previous coursework in content and pedagogy. I am unsure whether this course is envisioned as an intense one-semester undertaking or a full-year sequence. I was intimidated by the sheer volume of topics suggested for inclusion in the capstone course. Nonetheless, this suggestion is worthy of careful attention and possible adaptation.

In my own institution we might pursue the goals of this suggestion in the preparation of high school teachers by adding 1-credit companion seminars to each of the two core courses in our major (advanced calculus, which is really an introduction to real analysis; and abstract algebra, which is really a fairly concrete introduction to rings before groups). These 4-credit courses already include a weekly workshop in which students work in groups on problems connecting various topics before writing up solutions individually. The goal is to develop communication skills. A seminar for prospective teachers, team-taught by faculty from mathematics and mathematics education, could discuss connections to high school content and pedagogy without totally segregating teachers from other majors.

Carrying out the report's recommendation that prospective teachers of grades 5–8 should have 21 credit hours of mathematical content and making sure that pedagogical issues are appropriately addressed would require substantial attention from mathematics faculty and substantial resources from university administrators, especially in those states that currently require only 3 credit hours each in mathematics, science, and technology. The traditional Ph.D. in mathematics does little to prepare college faculty for teaching such courses. The suggestion of optional minors within a Ph.D. program to address this issue is intriguing but not obviously compatible with calls to speed progress of graduate students to the doctoral degree. Would fellowships for postdoctoral training programs be a practical alternative? Or should some mathematicians pursue postdoctoral master's degrees in mathematics education as others now do in computer science or finance?

Recommendations 6 through 11. Recommendations 6, 7, and 8 call for cooperation between mathematics departments and mathematics education departments, between 2- and 4-year institutions, and between working teachers and instructors of prospective teachers. Recommendations 9 through 11 address policies supporting high-quality teaching in the schools. They call for the participation of college and university faculty in the accreditation and certification processes, for the participation of teachers throughout their careers in activities designed to enhance their understanding of mathematics and ways to teach it, and the use of mathematics specialists in the middle grades.

Prospective teachers often face disparate lists of requirements for general education, academic majors, and teacher certification—each set by a different authority inside or outside the university. Lengthening these lists won't help. Making them more coherent will help. To make room in students' programs for more mathematics will require tact lest other departments will complain of lack of respect for their contributions to the education of teachers and fight the potential loss of their resources to mathematics.

Efforts to coordinate the mathematics programs of 2- and 4-year institutions can build on the collegiality developed within the MAA and the American Mathematical Association of Two-Year Colleges. Articulation agreements (compacts negotiated between institutions to specify transfer-credit policy) can cause problems if faculty have insufficient input. The mission of a 2-year school often extends well beyond preparing students for transfer to 4-year schools. There may be pressures that make it harder for faculty at 2-year schools than for those at 4-year schools to develop in students the facility in reasoning and communication as well as in calculation needed for upper-level courses.

In summary, I recommend that my colleagues read this report looking for ideas to adopt (or at least adapt) rather than looking for excerpts to disparage. What I like best is the report's underlying assumption that we need an alliance of *content* and *process*, not a victory of one over the other. I hope we can agree that students and their teachers must understand as well as remember mathematics in order to use it well. It would follow that education for prospective teachers, like all college-level education, should aim for deep understanding not just coverage of material. I hope that the ideas and arguments of the MET report will both spur and assist the mathematical community to improve the mathematical education of teachers and thus to improve the status of mathematics in twenty-first-century America.

Steven G. Krantz

I had a dream last night. I dreamed that I was teaching a class on pseudodifferential operators. On the first day I asked the class what they thought a pseudodifferential operator should be. No good. I got nothing but blank stares. I then said, "OK. Who can tell us what a singular integral should be? *Hint:* Think Calderón and Zygmund." Still I got no response.

Undaunted, I smiled and said, "I'll make it easy for you. What is the concept of a distribution?" One young fellow finally raised his hand and described the idea of a probability distribution. "Good answer!" I cried. "In fact, in this course we instead use the distribution theory of Laurent Schwartz."

We spent the rest of the class time discussing how we felt about mathematical analysis, about the role of the mathematician in society at large, and about what kind of teacher David Hilbert was. It was a rewarding hour.

The reader who has stuck with me so far is probably thinking that old Steve Krantz has finally gone around the bend. But no, I am portraying a teaching process that is being purveyed by well-meaning individuals who have set themselves up as the arbiters of teaching standards for the next generation of school mathematics teachers. Students are supposed to cogitate and interact with each other and generate—hit or miss—the ideas for themselves. The volume under review is an instance of this new *Weltanschauung.*

Consider now a scene in the third grade: We want to teach the kids how to multiply two 2-digit numbers together. One way to do this would be to just *tell them how it is done.* But that would be crass indeed. In the words of the report under review (which I shall refer to hereinafter as MET),

> First, she [the teacher] must believe that mathematics is about ideas that make sense, rather than a collection of motiveless rules, and that her students have mathematical ideas that can be built upon; next, that there are many ways to solve a problem.

This is a commendable sentiment. Understanding cannot come without motivation. We must link up new ideas to what the student already knows. But the teacher must play a *dynamic* role in making those connections.

I like to think of the analogy of learning to play the piano. When the (young) student sits down on the first day with his or her teacher, it is unlikely that the teacher will say, "Now, how do you think you should hold your hands? What do these symbols on the sheet music likely stand for? And what song would you like to play first?" In fact, what the teacher does instead is *to show the student how to play the piano.* As the student learns finger exercises and scales and elementary tunes and ultimately Chopin études, he or she also develops an appreciation for why things are done the way they are, why the traditional hand positions are effective, and finally how music is structured and why it works.

Why can't we teach mathematics in the same way? It is, frankly, a waste of time to try to get students to "invent" the mathematics that it took us hundreds of years to develop. What can they accomplish in just one hour?

I certainly think that students (of any age) should be encouraged to contribute their ideas, and the teacher should be sufficiently well trained so that he or she can respond thoughtfully and meaningfully to any and all student questions. I thank God every day that I have the intelligence and training always to have useful and stimulating responses for most any comment or query that my students might formulate. Our K–12 teachers should, ideally, be able to do the same.

Certainly the teacher should at some point say to the students, "Why do we multiply in this fashion? Why do we carry in this fashion? What does the operation of 'carrying' actually mean? How would we do things if we were working in some number system other than base 10?" In this way the students can turn the methods over in their minds, can find hooks on which to hang the ideas, and (one hopes) can finally say, "Aha! Now I understand." This *must* be the goal of all teaching. The process that I have described in these last two paragraphs contains all the elements of the self-discovery/group-learning paradigm that MET and other new-age texts are promoting. But it builds *atop* a foundation of basic skills and concepts. It uses time more wisely, and it makes much more didactic sense. My "traditionalist" approach allows the teacher to play a more proactive role and to *guide* the students to the right ideas.

I once read an article in *UME Trends* in which an educator said, "I can no longer teach my students calculus. What I can do is teach them *about calculus.*" It took me a moment to understand what was being said; after that, I felt as though I had gaped into the pits of hell. The message is that bona fide calculus—taking limits and calculating derivatives and computing integrals and applying these ideas to physics—is just too recondite. What is better is just to chat about these things, more or less along the lines of *A Tour of the Calculus* by David Berlinski [BER]. Is this the state to which our concept of education has devolved?

If you read pages 33–35 of MET, then you might conclude that the answer is "yes". Instead of seventh- and eighth-grade teachers being told to show their students the rudiments of Euclidean

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geometry, it is suggested that they treat their students to the software The Geometer's Sketchpad.[1] Instead of these teachers being advised to introduce their students to symmetry and congruence and angle, they are being led to books on the occurrence of geometric shapes in primitive cave paintings and folk artifacts.

Chapter 5, page 41, considers the high school treatment of geometry. As the reader may know, this subject has been trivialized in the past twenty years. Gone are the days when students were presented with a few definitions (line, point, betweenness, etc.), presented with Euclid's five axioms, and then sent off to fight the forces of darkness armed with only the two-column proof. Many high school geometry texts in recent years present fifteen new axioms in each chapter. Why? So that they can avoid doing any proofs. In other words, Euclidean geometry has been turned into a phenomenological subject: point and click; look at the pretty triangle. The most modern texts go even a step further: they have no axioms and instead engage in an anecdotal description of various pictures, including much ado about fractals.

Unfortunately, MET does not inveigh against the downgrading of geometry that I have just described. It instead takes the position of "Let a thousand flowers bloom" and advocates the use of the software The Geometer's Sketchpad to facilitate the process.

To pile the ridiculous on top of the sublime, page 42 advocates that "future teachers should also be exposed to twentieth-century developments in geometry." Just what do these authors have in mind? Chern's version of the Gauss-Bonnet theorem? The proof of the Calabi conjecture? Perhaps Gelfand-Fuks cohomology? Everybody wants to be current and hip. But we would be better off if our high school teachers were well versed in mathematics *up to and including the time of Weierstrass*—nothing more.

Let me stress that MET, at its outset, enunciates a number of lofty and commendable goals—eleven of them in fact. Some of these are:

1. Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.
2. Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching.
3. The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty.
4. Teachers need the opportunity to develop their understanding of mathematics and its teaching throughout their careers, through both self-directed and collegial study, and through formal coursework.

These goals are splendid, and their actualization is sure to improve the quality of mathematics teaching at every level in the K–12 curriculum. But the methods recommended in this report for actually implementing many of these goals are too vague and too much oriented towards self-esteem and empowerment to actually have the intended effect. As an instance, page 35 contains the admonition

> Prospective teachers need experience with designing simple experiments, collecting, displaying, and analyzing data, and using software that helps them understand how to display and interpret data. *Fathom* can be used to enhance teachers' learning; *Datascope* and *ProbSim* are available for Macs for use in the middle grades.

Is this useful teacher-training information? Do we need a nationally recognized and touted report to bring us to this level of understanding?

We also must bear in mind that there are significant societal factors that impact on, and often hinder, our efforts to improve school education. In the mid-nineteenth century most school teachers were not paid at all, at least not in hard currency. They were given food to eat by the local farmers, clothes to wear by the local seamstresses, and a place to sleep thanks to the largesse of some pigs and cows with a spare manger. The way that we train and compensate school teachers in 2001 is a throwback to a slightly more recent time when many teachers were unmarried women. The feeling was that they needed little more than a subsistence living; after a few years they would marry and that would be the end of that. Starting salaries for school teachers today have remained dreadful, and even the salaries of the most senior and experienced teachers are mediocre.

Perhaps most important of all is that teachers get so little parental and societal support. They are actually beleaguered in their efforts to teach anything of substance. Teachers are among the most important and influential members of our society, and I think that they should be paid as well as a dentist or an engineer. We should aid them in every way that we can. And paying our teachers properly would give us some clout: we could expect more of them, demand that they be better trained, and also attract much higher-quality people to the profession.

[1] *The Geometer's Sketchpad is a popular software product that allows the user to render pictures of triangles and rectangles and parallelograms and other geometric figures and to move them around and fit them together.*

Hyman Bass likes to point out that you do not learn gourmet cooking by eating out in fancy restaurants, you do not learn how to sing opera by attending performances at the Met, and you do not learn how to play tennis by watching the U.S. Open on TV. You learn by doing. You get an education by beating your head against the ideas. You develop your intellect by forcing your brain to do isometrics. And so we come to the use of the computer in education (Chapter 6 of MET). My view is that the computer should be used to illustrate and to reinforce ideas. It can also be used to try things. *A computer cannot teach* any more effectively than an oscilloscope can bring about world peace.

MET drops the ball on the crucial topic of computers in mathematics education. It once again extols the virtues of The Geometer's Sketchpad and like software packages. It takes an attitude of "Different strokes for different folks". This is a topic on which MET could really have provided some guidance. Instead, it presents a mere two pages of fluff. This treatment typifies an inbred flaw of any work like MET. It is the work of a committee rather than of an individual. It has no voice, and it has no soul. It is afraid to say anything and therefore errs on the side of vapidity. Or else it says nothing.

Chapter 7 provides a detailed treatment of the training of elementary school teachers. The authors of MET identify a number of desiderata for such a teacher:

1. understanding how place value permits efficient representation of number;
2. seeing how the operations of addition, multiplication, and exponentiation are used in representing numbers;
3. recognizing the relative magnitude of numbers;
4. recognizing how the base-10 structure of number is used in multidigit computations.

There are many more. And they are all worthwhile. Much of the chapter is devoted to detailed and lengthy vignettes of classroom scenes in which young children consider mathematical ideas (in fact all parts of the report are laced with these devices). One such vignette depicts a group of kids measuring each other's heights and manipulating the data. Another has students putting shirts into drawers. A third involves counting mouse legs [sic].

I must confess that, after the first two vignettes, I had had enough. I was no longer entertained, and I was not learning anything. I also fail to see what these stories have to do with teacher education. They may illustrate ways in which teachers put ideas before the class. But if I were writing a book or report about teacher education, I would concentrate my efforts on specific units of mathematics that teachers ought to know, how we can train the teachers to know them, and *how we can test and confirm that they actually master these ideas and can use them flexibly in the classroom.* The device of the vignette is indicative of a rather dreary nonspecificity, a lame attempt to entertain, and an almost deliberate fogginess that marks the entire approach of MET to the task of teacher training.

One good feature of MET is that it stresses the (potential) importance of the schools of education. These entities *must* coordinate more closely with the math departments at our colleges and universities, and vice versa. Mathematics departments and their high-flown denizens must take an active interest in educational issues, and they must play a genuine role in seeking solutions. It seems to me that the most important function of a school of education is to show its students precisely how their mathematical knowledge plays a role in teaching, how the ideas from different courses fit together, and how everything they know can be applied in the classroom. Surely a symbiosis with the local department of mathematics can only help in this process.

The most important contribution of the teaching reform movement is that it has sensitized us all to teaching issues. The report MET is a contribution to this new educational dialogue, and I am happy that it was written. However, I hope that it will be but one contribution of many yet to come. The authors of this volume have a certain *Gestalt* that is well worth considering. But theirs is not the final word. Although MET espouses the importance of content and of ideas, it does not provide much hard information about how to imbue our school-teachers-in-training with these ideas. The MET report frequently makes facile reference to Liping Ma's remarkable book *Knowing and Teaching Elementary Mathematics* [LMA]. But the authors of MET seem to have none of Ma's insight or grace, and they are unable to follow her model.

I hope that MET is widely distributed and will generate useful discussions. The goal should be that the next-generation work on this subject will be more specific, more candid, more realistic, and—in the end—more useful.

References

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