Leonard Carlitz (1907–1999)

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Leonard Carlitz died on September 17, 1999, at the age of ninety-one years. Born in Philadelphia on December 26, 1907, he received his doctorate in mathematics from the University of Pennsylvania in 1930, working under the direction of H. H. Mitchell. Carlitz leaves a remarkable mathematical legacy, including work in number theory, finite field theory, combinatorics, special functions, and the arithmetic of polynomials over a finite field.

Carlitz spent a postdoctoral year (1930–31) with E. T. Bell at the California Institute of Technology as a National Research Council Scholar. During 1931–32, he studied with G. H. Hardy at Cambridge University, supported by an International Research Fellowship. This was the era when Hardy and Littlewood led one of the great centers of research in number theory, and Carlitz found the mathematical atmosphere there exhilarating. His work in additive number theory derives from that period.

On his return from Cambridge in 1932, Carlitz accepted a position at Duke University, where he remained until his retirement in 1977. He was a member of the editorial board of the *Duke Mathematical Journal* from 1938 to 1973, often serving as managing editor. In 1964 he was named James B. Duke Professor of Mathematics, the first member of the Duke mathematics department to hold one of these distinguished professorships.

Carlitz directed forty-five doctoral students at Duke. They are listed below with their dissertation titles. The fact that the great majority of these dissertations involve finite fields reflects Carlitz's abiding interest in the area for which he is perhaps best known. His approach to his students was low-key and supportive.

Carlitz published 770 research papers during his active years, and a final paper, culled from his classroom notes, appeared in the April 1995 issue of *Finite Fields and Their Applications*. In 1953 he published a record 44 papers. His most active decade was 1960–69, when he averaged 27 papers per year. During the early 1960s, when I was one of his graduate students, Carlitz had a National Science Foundation grant that paid for a half-time secretary. On more than one day I observed him reading a journal paper raising a question that he found of interest, that evening writing up a paper of his own answering the question, and having it typed and sent off to a journal the following day.

The importance of some of Carlitz's most profound papers was not appreciated until many years after they appeared in print. This unfortunate circumstance is sometimes attributed to the large number of his research papers. However, his choice of nondescriptive titles for many of his papers is a more likely explanation. In 1938 he introduced [2] the first Drinfeld module, now called the Carlitz module, anticipating a portion of Drinfeld's 1974 paper. Carlitz's paper showed how one may explicitly construct the abelian extensions of the function field $\mathbb{F}_q(T)$, a new and elegant solution of Hilbert's Twelfth Problem over $\mathbb{F}_q(T)$. Nevertheless, Carlitz entitled this paper "A class of polynomials".

David Goss's book [7] contains a detailed exposition of the Carlitz module and, more generally,

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of Carlitz's deeply original papers in the arithmetic of polynomials over $\mathbb{F}_q$. Carlitz wrote many papers on permutation polynomials, which provide one approach to the design of random number generators. His interest in this area originated in a paper of Dickson [5], whom Carlitz regarded as a mentor. Carlitz conjectured [8] that there are only a finite number of permutation polynomials of any given even degree over the *totality* of finite fields of odd order. This deep conjecture was proved [6] in 1993 using the classification theorem for finite simple groups. See [1] for a more detailed account of Carlitz's contributions to finite field theory. Carlitz's work with Olson [4] provides a sharp bound [3], [9] for the minus part of the class number of $\mathbb{Q}(\mu(p))$, p prime.

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Morris, Stephen Brent, *Permutations by cutting and shuffling—a generalization to q-dimensions*, 1974.

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Vaughan, Theresa Phillips, *Linear transformations and polynomials over finite fields*, 1972.

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Kurtz, David Corey, *Concavity and asymptotic properties of arrays of numbers*, 1971.

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Howard, Fredric Timothy, *A sequence of numbers related to the exponential function*, 1966.

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