New Publications Offered by the AMS

Algebra and Algebraic Geometry

The Regulators of Beilinson and Borel
José I. Burgos Gil, Universidad de Barcelona, Spain

This book contains a complete proof of the fact that Borel’s regulator map is twice Beilinson’s regulator map. The strategy of the proof follows the argument sketched in Beilinson’s original paper and relies on very similar descriptions of the Chern-Weil morphisms and the van Est isomorphism.

The book has two different parts. The first one reviews the material from algebraic topology and Lie group theory needed for the comparison theorem. Topics such as simplicial objects, Hopf algebras, characteristic classes, the Weil algebra, Bott’s Periodicity theorem, Lie algebra cohomology, continuous group cohomology and the van Est Theorem are discussed.

The second part contains the comparison theorem and the specific material needed in its proof, such as explicit descriptions of the Chern-Weil morphism and the van Est isomorphisms, a discussion about small cosimplicial algebras, and a comparison of different definitions of Borel’s regulator.

Contents: Introduction; Simplicial and cosimplicial objects; $H$-spaces and Hopf algebras; The cohomology of the general linear group; Lie algebra cohomology and the Weil algebra; Group cohomology and the van Est isomorphism; Small cosimplicial algebras; Higher diagonals and differential forms; Borel’s regulator; Beilinson’s regulator; Bibliography; Index.

CRM Monograph Series, Volume 15

Boundary Cohomology of Shimura Varieties, III: Coherent Cohomology on Higher-Rank Boundary Strata and Applications to Hodge Theory
Michael Harris, Université Paris, France and Steven Zucker, Johns Hopkins University, Baltimore, Maryland

A publication of the Société Mathématique de France.

In this book, the authors complete the verification of the following fact: The nerve spectral sequence for the cohomology of the Borel-Serre boundary of a Shimura variety $Sh$ is a spectral sequence of mixed Hodge-de Rham structures over the field of definition of its canonical model. To achieve that, they develop the machinery of automorphic vector bundles on mixed Shimura varieties, for the latter enter in the boundary of the toroidal compactifications of $Sh$; and study the nerve spectral sequence for the automorphic vector bundles and the toroidal boundary. They also extend the technique of averting issues of base-change by taking cohomology with growth conditions. They give and apply formulas for the Hodge graduation of the cohomology of both $Sh$ and its Borel-Serre boundary.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Automorphic vector bundles on mixed Shimura varieties; Mixed growth conditions and coherent cohomology; The nerve spectral sequence for coherent cohomology; Hodge theoretic applications; On the comparison of Hodge structures; Bibliography; Mémoires de la Société Mathématique de France, Number 85 July 2001, 116 pages, Softcover, ISBN 2-85629-107-4, 2000 Mathematics Subject Classification: 14G35, 11G18, 14C30, 11F75, Individual member $30, List $33, Order code SMFMEM/85N
New Publications Offered by the AMS

**Theorie d'Iwasawa des Représentations p-Adiques Semi-Stables**

Bernadette Perrin-Riou, Université Paris-Sud, Orsay, France

A publication of the Société Mathématique de France.

Let $F$ be a finite unramified extension of $\mathbb{Q}_p$ and $V$ a $p$-adic galois semi-stable representation on $F$ of dimension $d$. The author develops Iwasawa theory for $V$ and the $Z_p$-cyclotomic extension: she constructs a logarithm (regulator map) from the Iwasawa module associated to the Galois cohomology of $V$ in a very explicit module on an algebra generated by analytic functions on the annulus $\{p^{-1/2} < |x| < 1\}$ and $\log x$.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Anneaux de fonctions; Modules d'Iwasawa associés à un $(\wp, N)$-module; Construction d'éléments de $\mathcal{D}_{m,s}(D)$; Théoremes de structure des $\mathcal{D}_{m,s}(D)$; Exponentielle; Normes universelles; A. DIGRESSION: Le polylogarithme; B. Étude d'éléments de $\mathcal{D}_{m,s}(D)$; Exponentielle; C. ÉTUDE: Les logarithmes modérés; D. ÉTUDE: Les logarithmes modérés; E. Exemple: Le cas des courbes elliptiques.

Mémoires de la Société Mathématique de France, Number 84

**Proceedings on Moonshine and Related Topics**

John McKay, Concordia University, Montreal, PQ, Canada, and Abdellah Sebbar, University of Ottawa, ON, Canada, Editors

This volume contains the proceedings of the Moonshine workshop held at the Centre de Recherches Mathématiques (CRM) in Montréal. A glance at the contents will reveal that the connection of some papers to Moonshine is not immediate; however, Moonshine has proved to be a very fertile area, and it does not stretch the imagination to believe that many more threads will be drawn together before we understand what is really going on.

In this volume, all the classical Moonshine themes are presented, namely the Monster simple group and other finite groups, automorphic functions and forms and related congruence groups, and vertex algebras and their representations. These topics appear in either a pure form or in a blend of algebraic geometry dealing with algebraic surfaces, Picard-Fuchs equations, and hypergeometric functions.

Contents: A. Baker and H. Tamanoi, Invariants for finite dimensional groups in vertex operator algebras associated to basic representations of affine algebras; C. Dong and G. Mason, Transformation laws for theta functions; C. F. Doran, Algebro-geometric isomonodromic deformations linking Hauptmoduls: Variation of the mirror map; G. Glauberman and S. P. Norton, On McKay's connection between the affine $E_8$ diagram and the monster; K. Harada and M. L. Lang, Sylow 2-subgroups of simple groups; W. L. Hoyt and C. F. Schwartz, Yoshida surfaces with Picard number $p \geq 17$; M. Kaneko and N. Todaka, Hypergeometric modular forms and supersingular elliptic curves; C. H. Lam, Fusion rules for ternary and $Z_2 \times Z_2$ code vertex operator algebras; H. Li, The regular representations and the $A_n$-algebras; J. McKay, Linear dependencies among completely replicable functions; J. McKay and A. Sebbar, Arithmetic semistable elliptic surfaces; M. Miyamoto, Modular invariance of trace functions on VOAs in many variables; N. Narumiya and H. Shiga, The mirror map for a family of $K3$ surfaces induced from the simplest 3-dimensional reflexive polytope; S. Norton, From moonshine to the monster; Y. Ohyama, Hypergeometric functions and non-associative algebras; K. Saito, Extended affine root systems. V. Elliptic eta-products and their Dirichlet series; C. S. Simons, Delling infinite Coxeter groups to finite groups; M. P. Tuite, Genus two meromorphic conformal field theory; H. Verrill, Picard-Fuchs equations of some families of elliptic curves.

CRM Proceedings & Lecture Notes, Volume 30

**Invariant Theory of Finite Groups**

Mara D. Neusel, University of Notre Dame, IN, and Larry Smith, Mathematishes Institut, Göttingen, Germany

The questions that have been at the center of invariant theory since the 19th century have revolved around the following themes: finiteness, computation, and special classes of invariants. This book begins with a survey of many concrete examples chosen from these themes in the algebraic, homological, and combinatorial context. In further chapters, the authors pick one or the other of these questions as a departure point and present the known answers, open problems, and methods and tools needed to obtain these answers. Chapter 2 deals with algebraic finiteness. Chapter 3 deals with combinatorial finiteness. Chapter 4 presents Noetherian finiteness. Chapter 5 addresses homological finiteness. Chapter 6 presents special classes of invariants, which deal with modular invariant theory and its particular problems and features. Chapter 7 collects results for special classes of invariants and coinvariants such as (pseudo) reflection groups and representations of low degree. If the ground field is finite, additional problems appear and are compensated for in part by the emergence of new tools. One of these is the Steenrod algebra, which the authors introduce in Chapter 8 to solve the inverse invariant theory problem, around which the authors have organized the last three chapters.

The book contains numerous examples to illustrate the theory, often of more than passing interest, and an appendix on...
commutative graded algebra, which provides some of the required basic background. There is an extensive reference list to provide the reader with orientation to the vast literature.

Contents: Invariants, their relatives, and problems; Algebraic finiteness; Combinatorial finiteness; Noetherian finiteness; Homological finiteness; Modular invariant theory; Special classes of invariants; The Steinrad algebra and invariant theory; Invariant ideals; Lannes’ T-fuctor and applications; Review of commutative algebra; References; Typography; Notation; Index.

Mathematical Surveys and Monographs, Volume 94

Analysis

Entire Functions in Modern Analysis
Boris Levin Memorial Conference

Yuri Lyubich, Technion-Israel Institute of Technology, Haifa, Israel, Vitali Milman, Tel Aviv University, Israel, Iossif Ostrovskii, Bilkent University, Ankara, Turkey, Mikhail Sodin, Tel Aviv University, Ramat-Atliv, Israel, Vadin Tkachenko, Ben Gurion University of the Negev, Beer-Sheva, Israel, and Lawrence Zalcman, Bar Ilan University, Ramat Gan, Israel, Editors

A publication of the Bar-Ilan University.

Distributed Worldwide by the American Mathematical Society.

This volume presents the proceedings from the conference, “Entire Functions in Modern Analysis” held at Tel-Aviv University (Ramat-Atliv, Israel) in memory of Professor Boris Levin, an outstanding mathematician and a brilliant teacher whose mathematical activity spanned over 60 years. Levin’s scientific interests lay principally in the theory of analytic functions and its applications to harmonic analysis, functional analysis, and operator theory. His ideas and results in this area, as expressed both through his personal influence and his books and papers, have influenced several generations of mathematicians.


Israel Mathematical Conference Proceedings, Volume 15
January 2002, 392 pages, Softcover, 2000 Mathematics Subject Classification: 30Dxx; 30Fxx, 30H05, 31Axx, 31C10, 39Bxx, 11A05, 42A75, Individual member $78, List $130, Institutional member $104, Order code IMCP/15N

Discrete Mathematics and Combinatorics

q-Series with Applications to Combinatorics, Number Theory, and Physics

Bruce C. Berndt, University of Illinois, Urbana, and Ken Ono, University of Wisconsin, Madison, Editors

The subject of $q$-series can be said to begin with Euler and his pentagonal number theorem. In fact, $q$-series are sometimes called Eulerian series. Contributions were made by Gauss, Jacobi, and Cauchy, but the first attempt at a systematic development, especially from the point of view of studying series with the products in the summands, was made by E. Heine in 1847. In the latter part of the nineteenth and in the early part of the twentieth centuries, two English mathematicians, L. J. Rogers and F. H. Jackson, made fundamental contributions. In 1940, G. H. Hardy described what we now call Ramanujan’s famous $\psi_1$ summation theorem as “a remarkable formula with many parameters.” This is now one of the fundamental theorems of the subject.
Despite humble beginnings, the subject of \( q \)-series has flourished in the past three decades, particularly with its applications to combinatorics, number theory, and physics. During the year 2000, the University of Illinois embraced The Millennial Year in Number Theory. One of the events that year was the conference \( q \)-Series with Applications to Combinatorics, Number Theory, and Physics. This event gathered mathematicians from the world over to lecture and discuss their research.

This volume presents nineteen of the papers presented at the conference. The excellent lectures that are included chart pathways into the future and survey the numerous applications of \( q \)-series to combinatorics, number theory, and physics.

**Contents:** B. C. Berndt and K. Ono, \( q \)-series Piano recital; Levis faculty center; Congruences and conjectures for the partition function; MacMahon’s partition analysis VII: Constrained compositions; Crystal bases and \( q \)-identities; The Bailey-Rogers-Ramanujan group; Multiple polylogarithms; A brief survey; Swinnerton-Dyer type congruences for certain Eisenstein series; More generating functions for \( L \)-function values; On sums of an even number of squares, and an even number of triangular numbers: An elementary approach based on Ramanujan’s \( \phi \) \( \psi \) summation formula; Some remarks on multiple Sears transformations; Another way to count colored Frobenius partitions; Proof of a summation formula for an \( \mathbb{A}_n \) basic hypergeometric series conjectured by Warnaar; On the representation of integers as sums of squares; 3-regular partitions and a modular \( K_3 \) surface; A new look at Hecke’s indefinite theta series; A proof of a multivariable elliptic summation formula conjectured by Warnaar; Multilateral transformations of \( q \)-series with quotients of parameters that are nonnegative integral powers of \( q \); Completeness of basic trigonometric system in \( \ell^p \); The generalized Borwein conjecture. I. The Burge transform; Mock \( \vartheta \)-functions and real analytic modular forms.

**Contemporary Mathematics,** Volume 291


**General Interest**

Taniguchi Conference on Mathematics

Nara 1998

Masaki Maruyama, Kyoto University, Japan, and
Toshikazu Sunada, Tohoku University, Japan, Editors

*An publication of the Mathematical Society of Japan.*

Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

In 1929, Mr. Toyosaburo Taniguchi established the Taniguchi Foundation with the goal of promoting research in the basic sciences in Japan and to engender mutual understanding on an international level via the exchange of ideas and research. In 1956, he instituted a division for mathematics within the Foundation and sponsored the first summer seminar. Since that time, the seminar has been held each year on various mathematical topics.

In 1974, Mr. Taniguchi promoted and sponsored an International Symposium in various fields of science on a smaller scale. His aim was to raise the level of scientific thought and research, while providing a forum where promising young scholars the world over could gather informally to exchange thoughts and to contribute their knowledge. These gatherings were held until 1999.

This volume is a collection of the research manuscripts written by the invited speakers at the final conference set up by the Taniguchi Foundation, the “Taniguchi Conference on Mathematics ’98”, held in Nara, Japan. The conference was aimed at gathering all previous participants of Taniguchi Symposia. The subject areas were chosen to include all important and active fields of mathematics. Hence the topics in this volume are quite diverse. The contributors are world-class mathematicians who are generally reporting on subjects for which they are well known. For example, contributions include R. E. Borcherds on vertex algebras, M. Kontsevich on non-commutative algebraic manifolds, P.-L. Lions on fluid mechanics, M. Kashiwara on micro-localization, J. Kollar on the topology of algebraic varieties, S. Mori on rational curves in algebraic varieties, and others.


**Advanced Studies in Pure Mathematics,** Volume 31

Knots, Braids, and Mapping Class Groups—Papers Dedicated to Joan S. Birman

Jane Gilman, Rutgers University, Newark, NJ, William W. Menasco, State University of New York, Buffalo, and Xiao-Song Lin, University of California, Riverside, Editors

There are a number of specialties in low-dimensional topology that can find in their “family tree” a common ancestry in the theory of surface mappings. These include knot theory as studied through the use of braid representations and 3-manifolds as studied through the use of Heegaard splittings. The study of the surface mapping class group (the modular group) is of course a rich subject in its own right, with relations to many different fields of mathematics and theoretical physics. But its most direct and remarkable manifestation is probably in the vast area of low-dimensional topology. Although the scene of this area has been changed dramatically and experienced significant expansion since the original publication of Professor Joan Birman’s seminal work, Braids, Links, and Mapping Class Groups (Princeton University Press), she brought together mathematicians whose research span many specialties, all of common lineage.

The topics covered are quite diverse. Yet they reflect well the aim and spirit of the conference: to explore how these various specialties in low-dimensional topology have diverged in the past 20–25 years, as well as to explore common threads and potential future directions of development. This volume is pasted to explore common threads and the classification of the components of the Fatou set and Sullivan’s non-wandering theorem.

Contents: J. Cantarella, D. DeTurck, and H. Gluck, Upper bounds for the writhe of knots and the helicity of vector fields; O. T. Dasbach and B. S. Mangum, The automorphism group of a free group is not subgroup separable; R. Ghrist, Configuration spaces and braid groups on graphs in robotics; J. Gilman, Alternate discreteness tests; S. P. Humphries, Intersection-number operators for curves on discs and Chebyshev polynomials; O. Kharlampovich and A. Myasnikov, Implicit function theorem over free groups and genus problem; M. E. Kidwell and T. R. Stanford, On the z-degree of the Kauffman polynomial of a tangle decomposition; W. Li, Knot invariants from counting periodic points; X.-S. Lin and Z. Wang, Random walk on knot diagrams, colored Jones polynomial and Ihara-Selberg zeta function; F. Luo, Some applications of a multiplicative structure on simple loops in surfaces; W. W. Menasco, Closed braids and Heegaard splittings; J. H. Przytycki, Homotopy and q-homotopy skein modules of 3-manifolds: An example in algebra Situs; T. Stanford and R. Trapp, On knot invariants which are not of finite type.

AMS/IP Studies in Advanced Mathematics, Volume 24


Previously Announced Publications

Rudiments de Dynamique Holomorphe
Michèle Audin, Université Louis Pasteur et CNRS, Strasbourg, France
A publication of the Société Mathématique de France.

This book is an introduction to rational iteration theory. In the first four chapters, the authors deal with the classical theory. The basic properties of the Julia set and its complement, the Fatou set, are presented; the highest points of the treatment are the classification of the components of the Fatou set and Sullivan’s non-wandering theorem.

The second part of the book studies several topics in more detail. The authors begin by considering at length two classes of rational maps: the chaotic maps and the hyperbolic maps. In the closing chapters, they include respectively a study of holomorphic families of rational maps with a view to discussing Fatou’s famous problem concerning the density of hyperbolic maps and an exposition of the methods of potential theory, touching on questions of ergodicity, which may serve as a preparation for generalizations in higher dimensions.

A number of the developments treated here appear for the first time in book form. Several original proofs are presented.

Cours Spécialisés—Collection SMF, Number 7

Les Systèmes Hamiltoniens et Leur Intégrabilité
Michèle Audin, Université Louis Pasteur et CNRS, Strasbourg, France
A publication of the Société Mathématique de France.

This book presents some modern techniques in the theory of integrable systems viewed as variations on the theme of action-angle coordinates. These techniques include analytical methods coming from the Galois theory of differential equations, as well as more classical algebro-geometric methods related to Lax equations. Many examples are given.

Cours Spécialisés—Collection SMF, Number 8

Geometrization of 3-Orbifolds of Cyclic Type
Michel Boileau, CNRS, Université Paul Sabatier, Toulouse, France, Joan Porti, Universitat Autònoma de Barcelona, Bellaterra, Spain, and Michael Heusener, Université Blaise Pascal, Aubière, France
A publication of the Société Mathématique de France.

In this book, the authors prove the orbifold theorem in the cyclic case: If Ω is a compact oriented irreducible atoroidal 3-
Bäcklund and Darboux Transformations.
The Geometry of Solitons

Alan Coley, Dalhousie University, Halifax, NS, Canada
Decio Levi, University of Rome III, Italy, Robert Milson, Dalhousie University, Halifax, NS, Canada, Colin Rogers, University of New South Wales, Sydney, NSW, Australia, and Pavel Winternitz, Université de Montréal, QC, Canada, Editors

This book is devoted to a classical topic that has undergone rapid and fruitful development over the past 25 years, namely Bäcklund and Darboux transformations and their applications in the theory of integrable systems, also known as soliton theory.

The book consists of two parts. The first is a series of introductory pedagogical lectures presented by leading experts in the field. They are devoted respectively to Bäcklund transformations of Painlevé equations, to the dressing method and Bäcklund and Darboux transformations, and to the classical geometry of Bäcklund transformations and their applications to soliton theory. The second part contains original contributions that represent new developments in the theory and applications of these transformations.

Both the introductory lectures and the original talks were presented at an International Workshop that took place in Halifax, Nova Scotia (Canada). This volume covers virtually all recent developments in the theory and applications of Bäcklund- and Darboux transformations.


Recommended Text

Function Theory of One Complex Variable
Second Edition

Robert E. Greene, University of California, Los Angeles, and Steven G. Krantz, Washington University, St. Louis, MO

From a review of the First Edition:
The book is carefully and precisely written in a lively and soft style. It is extremely clear ... and very detailed. Moreover, it is stimulating and very suitable for self-study ... Certainly, the book reflects the authors' experience in teaching. The other features include the fruitful connection with real analysis ... The authors have produced a modern, quality work that could serve as an excellent model for writing and teaching graduate texts ... it will occupy a distinguished place in the extensive literature on the subject ... I read this book with great pleasure and I warmly recommend it for all those who are interested in complex analysis of one variable.

—Mathematical Reviews

Complex analysis is one of the most beautiful subjects that we learn as graduate students. Part of the joy comes from being able to arrive quickly at some "real theorems". The fundamental techniques of complex variables are also used to solve
real problems in neighboring subjects, such as number theory or PDEs.

This book is a text for a first-year graduate course in complex analysis. It is an engaging and modern introduction to the subject, reflecting the authors' expertise both as mathematicians and as expositors. All the material usually treated in such a course is covered here, but following somewhat different principles. To begin with, the authors emphasize how this subject is a natural outgrowth of multivariable real analysis. Complex function theory has long been a flourishing independent field. However, an efficient path into the subject is to observe how its rudiments arise directly from familiar ideas in calculus. The authors pursue this point of view by comparing and contrasting complex analysis with its real variable counterpart.

Explanations of certain topics in complex analysis can sometimes become complicated by the intermingling of the analysis and the topology. Here, the authors have collected the primary topological issues in a separate chapter, leaving the way open for a more direct and less ambiguous approach to the analytic material.

The book concludes with several chapters on special topics, including full treatments of special functions, the prime number theorem, and the Bergman kernel. The authors also treat $H^p$ spaces and Painlevé’s theorem on smoothness to the boundary for conformal maps.

A large number of exercises are included. Some are simply drills to hone the students’ skills, but many others are further developments of the ideas in the main text. The exercises are also used to explore the striking interconnectedness of the topics that constitute complex analysis.

Graduate Studies in Mathematics, Volume 40

December 2001, approximately 561 pages, Hardcover, ISBN 0-8218-2905-X, LC 2001046415, 2000 Mathematics Subject Classification: 30-01; 30-00, 30-02, All AMS members $55, List $69, Order code GSM/40RT201

Lectures on Hilbert Modular Varieties and Modular Forms

Eyal Z. Goren, McGill University, Montreal, PQ, Canada

This book is devoted to certain aspects of the theory of $p$-adic Hilbert modular forms and moduli spaces of abelian varieties with real multiplication. The theory of $p$-adic modular forms is presented first in the elliptic case, introducing the reader to key ideas of N. M. Katz and J.-P. Serre. It is re-interpreted from a geometric point of view, which is developed to present the rudiments of a similar theory for Hilbert modular forms.

The theory of moduli spaces of abelian varieties with real multiplication is presented first very explicitly over the complex numbers. Aspects of the general theory are then exposed, in particular, local deformation theory of abelian varieties in positive characteristic.

The arithmetic of $p$-adic Hilbert modular forms and the geometry of moduli spaces of abelian varieties are related. This relation is used to study $q$-expansions of Hilbert modular forms, on the one hand, and stratifications of moduli spaces on the other hand.

The book is addressed to graduate students and non-experts. It attempts to provide the necessary background to all concepts exposed in it. It may serve as a textbook for an advanced graduate course.

CRM Monograph Series, Volume 14


An Introduction to Morse Theory

Yukio Matsumoto, University of Tokyo, Japan

In a very broad sense, “spaces” are objects of study in geometry, and “functions” are objects of study in analysis. There are, however, deep relations between functions defined on a space and the shape of the space, and the study of these relations is the main theme of Morse theory. In particular, its feature is to look at the critical points of a function, and to derive information on the shape of the space from the information about the critical points.

Morse theory deals with both finite-dimensional and infinite-dimensional spaces. In particular, it is believed that Morse theory on infinite-dimensional spaces will become more and more important in the future as mathematics advances.

This book describes Morse theory for finite dimensions. Finite-dimensional Morse theory has an advantage in that it is easier to present fundamental ideas than in infinite-dimensional Morse theory, which is theoretically more involved. Therefore, finite-dimensional Morse theory is more suitable for beginners to study.

On the other hand, finite-dimensional Morse theory has its own significance, not just as a bridge to infinite dimensions. It is an indispensable tool in the topological study of manifolds. That is, one can decompose manifolds into fundamental blocks such as cells and handles by Morse theory, and thereby compute a variety of topological invariants and discuss the shapes of manifolds. These aspects of Morse theory will continue to be a treasure in geometry for years to come.

This textbook aims at introducing Morse theory to advanced undergraduates and graduate students. It is the English translation of a book originally published in Japanese.

Translations of Mathematical Monographs (Iwanami Series in Modern Mathematics), Volume 208


Advances in Wave Interaction and Turbulence

Paul A. Milewski, Leslie M. Smith, and Fabian Waleffe, University of Wisconsin, Madison, and Esteban G. Tabak, New York University-Courant Institute of Mathematical Sciences, NYC, Editors

We often think of our natural environment as being composed of very many interacting particles, undergoing individual chaotic motions, of which only very coarse averages are perceptible at scales natural to us. However, we could as well think of the world as being made out of individual waves. This is so not just because the distinction between waves and parti-
cles becomes rather blurred at the atomic level, but also because even phenomena at much larger scales are better described in terms of waves rather than of particles: It is rare in both fluids and solids to observe energy being carried from one region of space to another by a given set of material particles; much more often, this transfer occurs through chains of particles, neither of them moving much, but each communicating with the next, and hence creating these immaterial objects we call waves.

Waves occur at many spatial and temporal scales. Many of these waves have small enough amplitude that they can be approximately described by linear theory. However, the joint effect of large sets of waves is governed by nonlinear interactions which are responsible for huge cascades of energy among very disparate scales. Understanding these energy transfers is crucial in order to determine the response of large systems, such as the atmosphere and the ocean, to external forcings and dissipation mechanisms which act on scales decades apart.

The field of wave turbulence attempts to understand the average behavior of large ensembles of waves, subjected to forcing and dissipation at opposite ends of their spectrum. It does so by studying individual mechanisms for energy transfer, such as resonant triads and quartets, and attempting to draw from them effects that should not survive averaging.

This book presents the proceedings of the AMS-IMS-SIAM Joint Summer Research Conference on Dispersive Wave Turbulence held at Mt. Holyoke College (MA). It drew together a group of researchers from many corners of the world, in the context of a perceived renaissance of the field, driven by heated debate about the fundamental mechanism of energy transfer among large sets of waves, as well as by novel applications—and old ones revisited—to the understanding of the natural world. These proceedings reflect the spirit that permeated the conference, that of friendly scientific disagreement and genuine wonder at the rich phenomenology of waves.

This item will also be of interest to those working in differential equations.


Contemporary Mathematics, Volume 283

Supplementary Reading

Variational Problems in Geometry

Seiki Nishikawa, Mathematical Institute, Tohoku University, Sendai, Japan

A minimal length curve joining two points in a surface is called a geodesic. One may trace the origin of the problem of finding geodesics back to the birth of calculus.

Many contemporary mathematical problems, as in the case of geodesics, may be formulated as variational problems in surfaces or in a more generalized form on manifolds. One may characterize geometric variational problems as a field of mathematics that studies global aspects of variational problems relevant in the geometry and topology of manifolds. For example, the problem of finding a surface of minimal area spanning a given frame of wire originally appeared as a mathematical model for soap films. It has also been actively investigated as a geometric variational problem. With recent developments in computer graphics, totally new aspects of the study on the subject have begun to emerge.

This book is intended to be an introduction to some of the fundamental questions and results in geometric variational problems, studying variational problems on the length of curves and the energy of maps.

The first two chapters treat variational problems of the length and energy of curves in Riemannian manifolds, with an in-depth discussion of the existence and properties of geodesics viewed as solutions to variational problems. In addition, a special emphasis is placed on the facts that concepts of connection and covariant differentiation are naturally induced from the formula for the first variation in this problem, and that the notion of curvature is obtained from the formula for the second variation.

The last two chapters treat the variational problem on the energy of maps between two Riemannian manifolds and its solution, harmonic maps. The concept of a harmonic map includes geodesics and minimal submanifolds as examples. Its existence and properties have successfully been applied to various problems in geometry and topology. The author discusses in detail the existence theorem of Eells-Sampson, which is considered to be the most fundamental among existence theorems for harmonic maps. The proof uses the inverse function theorem for Banach spaces. It is presented to be as self-contained as possible for easy reading.

Each chapter may be read independently, with minimal preparation for covariant differentiation and curvature on manifolds. The first two chapters provide readers with basic knowledge of Riemannian manifolds. Prerequisites for reading this book include elementary facts in the theory of manifolds and functional analysis, which are included in the form of appendices. Exercises are given at the end of each chapter.

This is the English translation of a book originally published in Japanese. It is an outgrowth of lectures delivered at Tohoku University and at the Summer Graduate Program held at the Institute for Mathematics and its Applications at the University of Minnesota. It would make a suitable textbook for advanced undergraduates and graduate students. This item will also be of interest to those working in analysis.

This item will also be of interest to those working in analysis.

Translations of Mathematical Monographs (Iwanami Series in Modern Mathematics), Volume 205
Nilpotent Orbits, Associated Cycles and Whittaker Models for Highest Weight Representations

Kyo Nishiyama, Kyoto University, Japan, Hiroyuki Ochiai, Tokyo Institute of Technology, Japan, Kenji Taniguchi, Aoyama Gakuin University, Tokyo, Japan, Hiroshi Yamashita, Hokkaido University, Sapporo, Japan, and Shohei Kato, Nakakasai, Edogawa-ku, Tokyo, Japan

A publication of the Société Mathématique de France.

Let \( G \) be a reductive Lie group of Hermitian type. The authors investigate irreducible (unitary) highest weight representations of \( G \) which are not necessarily in the holomorphic discrete series. The results of three articles of this volume include the determination of the associated cycles, the Bernstein degrees, and the generalized Whittaker models for such representations. They give a convenient description of \( K \)-types via branching rules for representations of classical groups. An integral formula for the degrees of small nilpotent orbits is established for arbitrary Hermitian Lie algebras. The generalized Whittaker models for each unitary highest weight module are specified by means of the principal symbol of a gradient-type differential operator, and also in relation to the multiplicity in the associated cycle. They also present expository introductions of the key notions treated in this volume, such as associated cycles, Howe correspondence for dual pairs where one member of the pair is compact, and the realization of highest weight representations on the kernels of the differential operators of gradient type.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.


Astérisque, Number 273

Discrete Groups

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This book deals with geometric and topological aspects of discrete groups. The main topics are hyperbolic groups due to Gromov, automatic group theory, invented and developed by Epstein, whose subjects are groups that can be manipulated by computers, and Kleinian group theory, which enjoys the longest tradition and the richest contents within the theory of discrete subgroups of Lie groups.

What is common among these three classes of groups is that when seen as geometric objects, they have the properties of a negatively curved space rather than a positively curved space. As Kleinian groups are groups acting on a hyperbolic space of constant negative curvature, the technique employed to study them is that of hyperbolic manifolds, typical examples of negatively curved manifolds. Although hyperbolic groups in the sense of Gromov are much more general objects than Kleinian groups, one can apply for them arguments and techniques that are quite similar to those used for Kleinian groups. Automatic groups are further general objects, including groups having properties of spaces of curvature 0. Still, relationships between automatic groups and hyperbolic groups are examined here using ideas inspired by the study of hyperbolic manifolds. In all of these three topics, there is a “soul” of negative curvature upholding the theory. The volume would make a fine textbook for a graduate-level course in discrete groups.

Translations of Mathematical Monographs (Iwanami Series in Modern Mathematics), Volume 207


Advances in Moduli Theory

Yuji Shimizu and Kenji Ueno, Kyoto University, Japan

The word “moduli” in the sense of this book first appeared in the epoch-making paper of B. Riemann, Theorie der Abel'schen Funktionen, published in 1857. Riemann defined a Riemann surface of an algebraic function field as a branched covering of a one-dimensional complex projective space, and found out that Riemann surfaces have parameters. This work gave birth to the theory of moduli.

However, the viewpoint regarding a Riemann surface as an algebraic curve became the mainstream, and the moduli meant the parameters for the figures (graphs) defined by equations.

In 1913, H. Weyl defined a Riemann surface as a complex manifold of dimension one. Moreover, Teichmüller’s theory of quasiconformal mappings and Teichmüller spaces made a start for new development of the theory of moduli, making possible a complex analytic approach toward the theory of moduli of Riemann surfaces. This theory was then investigated and made complete by Ahlfors, Bers, Rauch, and others. However, the theory of Teichmüller spaces utilized the special nature of complex dimension one, and it was difficult to generalize it to an arbitrary dimension in a direct way.

It was Kodaira- Spencer’s deformation theory of complex manifolds that allowed one to study arbitrary dimensional complex manifolds. Initial motivation in Kodaira-Spencer’s discussion was the need to clarify what one should mean by number of moduli. Their results, together with further work by Kuranishi, provided this notion with intrinsic meaning.

This book begins by presenting the Kodaira-Spencer theory in its original naïve form in Chapter 1 and introduces readers to moduli theory from the viewpoint of complex analytic geometry. Chapter 2 briefly outlines the theory of period mapping and Jacobian variety for compact Riemann surfaces, with the Torelli theorem as a goal. The theory of period mappings for compact Riemann surfaces can be generalized to the theory of period mappings in terms of Hodge structures for compact Kähler manifolds. In Chapter 3, the authors state the theory of Hodge structures, focusing briefly on period mappings. Chapter 4 explains conformal field theory as an application of moduli theory.

This is the English translation of a book originally published in Japanese. Other books by Kenji Ueno published in this AMS series, Translations of Mathematical Monographs, include An Introduction to Algebraic Geometry, Volume 166, Algebraic Geometry 1: From Algebraic Varieties to Schemes, Volume 185, and Algebraic Geometry 2: Sheaves and Cohomology, Volume 197.

Translations of Mathematical Monographs (Iwanami Series in Modern Mathematics), Volume 206
Probability Theory

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This volume presents topics in probability theory covered during a first-year graduate course given at the Courant Institute of Mathematical Sciences. The necessary background material in measure theory is developed, including the standard topics, such as extension theorem, construction of measures, integration, product spaces, Radon-Nikodym theorem, and conditional expectation.

In the first part of the book, characteristic functions are introduced, followed by the study of weak convergence of probability distributions. Then both the weak and strong limit theorems for sums of independent random variables are proved, including the weak and strong laws of large numbers, central limit theorems, laws of the iterated logarithm, and the Kolmogorov three series theorem. The first part concludes with infinitely divisible distributions and limit theorems for sums of uniformly infinitesimal independent random variables.

The second part of the book mainly deals with dependent random variables, particularly martingales and Markov chains. Topics include standard results regarding discrete parameter martingales and Doob's inequalities. The standard topics in Markov chains are treated, i.e., transience, and null and positive recurrence. A varied collection of examples is given to demonstrate the connection between martingales and Markov chains.

Additional topics covered in the book include stationary Gaussian processes, ergodic theorems, dynamic programming, optimal stopping, and filtering. A large number of examples and exercises is included. The book is a suitable text for a first-year graduate course in probability.

Cohomological Analysis of Partial Differential Equations and Secondary Calculus

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This book is dedicated to fundamentals of a new theory, which is an analog of affine algebraic geometry for (nonlinear) partial differential equations. This theory grew up from the classical geometry of PDE's originated by S. Lie and his followers by incorporating some nonclassical ideas from the theory of integrable systems, the formal theory of PDE's in its modern cohomological form given by D. Spencer and H. Goldschmidt and differential calculus over commutative algebras (Primary Calculus). The main result of this synthesis is Secondary Calculus on diffeivities, new geometrical objects which are analogs of algebraic varieties in the context of (nonlinear) PDE's.

Secondary Calculus surprisingly reveals a deep cohomological nature of the general theory of PDE's and indicates new directions of its further progress. Recent developments in quantum field theory showed Secondary Calculus to be its natural language, promising a nonperturbative formulation of the theory.

In addition to PDE's themselves, the author describes existing and potential applications of Secondary Calculus ranging from algebraic geometry to field theory, classical and quantum, including areas such as characteristic classes, differential invariants, theory of geometric structures, variational calculus, control theory, etc. This book, focused mainly on theoretical aspects, forms a natural dipole with Symmetries and Conservation Laws for Differential Equations of Mathematical Physics, Volume 182 in this same series, Translations of Mathematical Monographs, and shows the theory "in action".

This item will also be of interest to those working in algebra and algebraic geometry.