Book Review

The Universal History of Numbers and *The Universal History of Computing*

Reviewed by Joseph Dauben

The main aim of this two-volume work is to provide in simple and accessible terms the full and complete answer to all and any questions that anyone might want to ask about the history of numbers and of counting, from prehistory to the age of computers.

—Georges Ifrah
The Universal History of Numbers (Foreword)

… historically unacceptable, a deception.

—Review of the French edition in the Bulletin ARMEP *(1995)*

Number systems, like hair styles, go in and out of fashion—it's what's underneath that counts.

—Abraham Robinson
Yale Scientific Magazine *(1973)*

This is the first installment of a two-part book review. The second part will appear in the next issue of the *Notices*.

The Universal History of Numbers. From Prehistory to the Invention of the Computer (Volume I)
Georges Ifrah
Translated from the French by David Bellos, E. F. Harding, Sophie Wood, and Ian Monk
John Wiley & Sons, New York, 1999
xxii + 633 pages

The Universal History of Computing. From the Abacus to the Quantum Computer (Volume II)
Georges Ifrah
Translated from the French and with notes by E. F. Harding, Sophie Wood, Ian Monk, Elizabeth Clegg, and Guido Waldman
John Wiley & Sons, New York, 2000
410 pages

The dust jacket blurbs for the first of these two books (referred to in the following as *Numbers*) could not be more promising: "Georges Ifrah is the man. This book, quite simply, rules," wrote a reviewer for *The Guardian*. *The International Herald Tribune* declared that "Ifrah's book amazes and fascinates by the scope of its scholarship. It is nothing less than the history of the human race told through figures." The popular press in France was just as enthusiastic about the original French version, which appeared in 1994. *Le Figaro* was impressed that "[Ifrah's] amazing undertaking, describing humankind's relationship with numbers from Paleolithic times to the computer age, spans the world from Mayan ruins to Indian museums, from Egyptian hieroglyphics to Greek philosophers to Chinese libraries." Similarly, *L'Express* dubbed Ifrah "the Indiana Jones of arithmetic…who decided in 1974 to begin the search for his Grail, the origin of numbers."

Ifrah himself shows little restraint in declaring what he has accomplished: "I think I have brought together practically everything of significance," he writes, adding that "this is also probably the only book ever written that gives a more or less universal and comprehensive history of numbers and numerical calculation," (*Numbers*, pp. xvii-xviii).

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This despite the contributions of Karl Menninger and Tobias Dantzig, whom he mentions, and others who have also written on the subject like Graham Flegg, whom he does not. If Ifrah's two volumes actually delivered on his promises, they would indeed be remarkable. But in fact the aim of these books is more restrictive than their titles suggest, for what Ifrah really offers is a history of the Hindu-Arabic place-valued number system, along with the evolution of their numerals and the complementary history of computational devices from the abacus to the modern computer. Although the cover of volume II mentions the "quantum computer," Ifrah never gets that far. Mathematicians expecting to find truly "universal" histories of numbers and computation will be greatly disappointed, for there is little or nothing here about the truly interesting numbers—the ones that have done most to make modern mathematics what it is—π and e, the irrational, transcendental, and transfinite numbers, quaternions and infinitesimals, for example. Similarly, modern aspects of computation related to software, parallel processing, fifth-generation supercomputers, or the quantum computer are left unexplored.

Ifrah describes himself in his introduction as an "intellectual tourist." He explains how these books were the result of his inability, as a school teacher, to respond to the simplest questions raised by his pupils, such as "Where do numbers come from?" and "Who invented zero?" Ifrah discovered that such questions "soon drew me into the most fascinating period of learning and the most enthralling adventure of my life." He gave up his teaching job, and he turned to archaeology, psychology, even ethnology to help supply answers. He began to travel the world in search of evidence, and as Ifrah puts it, he "was soon to conquer the whole world, from America to Egypt, from India to Mexico, from Peru to China, in my search for more and yet more numbers. But as I had no financial backer, I decided to be my own sponsor, doing odd jobs (delivery boy, chauffeur, waiter, night watchman) to keep body and soul together," (*Numbers*, p. xvii). One cannot help but admire such determination!

Ifrah's quest was aided by his own personal history. As he notes, "a Moroccan by birth, a Jew by cultural heritage, I have been afforded a more immediate access to the study of the work of Arab and Hebrew mathematicians than I might have obtained as a born European," (*Numbers*, p. xxii). (Neither, it might be added, plays a large a role in the history of number systems, at least as Ifrah tells the story.) Once back in France from his travels, he continued his research and "fired off thousands of questions to academic specialists in scores of different fields," (*Numbers*, p. xvii). But as will become apparent in a moment, he either wrote to the wrong experts, was indifferent to their responses, or was not prepared to settle for their inconclusive results and the tentative nature of their research.

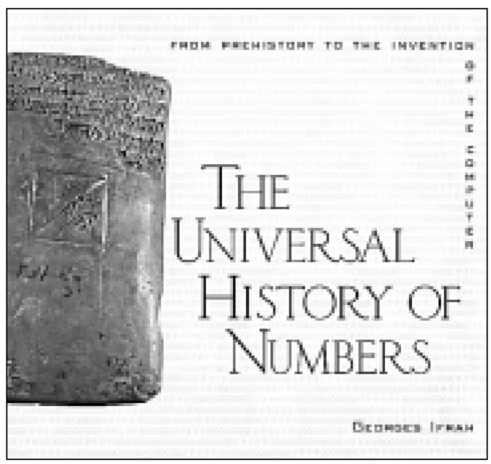
Ifrah is not a modest writer, and more than once he emphasizes the magnitude of what he has done, the importance of what he has accomplished, the new solutions he has to offer to old or neglected questions about numbers. He claims to be the first to have successfully deciphered the Elamite number system in use some 5,000 years ago in what is now modern Iran (the subject of study by a number of scholars whose work Ifrah should have known and acknowledged; more about this below). He also claims to have shown that Roman numerals derive from notching (but again, he is by no means the first to have suggested this; see for example [Menninger 1969, p. 241]). He writes, "There are also some new contributions on Mesopotamian numbering and arithmetic, as well as a quite new way of looking at the fascinating and sensitive topic of how 'our' numbers evolved from the unlikely conjunction of several great ideas. Similarly, the history of mechanical calculation culminating in the invention of the computer is *entirely* new" (emphasis added) (*Numbers*, p. xviii). This last remark is especially surprising because the story Ifrah tells of computing is basically chronological, conventional, and prosaic, adding nothing new to what is for the most part a well-established historical record.

The nonspecialist reader may well take these assertions at face value and believe that the two volumes under review here are truly universal in their coverage, that the information Ifrah delivers is genuine, and that his claims of breakthroughs are legitimate. However, those who specialize in the languages, texts, and documents with which Ifrah works have raised serious concerns about what he has written and about the possibly pernicious influence his books may have on students or those who would take his historical conclusions at face value.

Historians of mathematics in particular have voiced strong reservations about Ifrah's pronouncements on the history of number systems. *Histoire Universelle des Chiffres* was first published in 1981 (English translation, *From One to Zero*, 1986) and was considerably expanded in a revised version that appeared in 1994 with the same title (*Numbers* is a translation of the 1994 version). In 1995 a group of five experts in France agreed it was necessary to confront the popularity Ifrah's work was being accorded and to point out explicitly his numerous misreadings, misinterpretations, and pure fabrications. Earlier critics have also pointed up errors, some significant, that subsequently Ifrah apparently has chosen either to dismiss or ignore, for they almost all appear without change in the current English translation. Having had plenty of time to respond to questions and doubts raised about his facts and conclusions, Ifrah could have made the British and American editions more authoritative, more accurate, more useful to students and teachers alike. Given that these volumes have been produced on a lavish scale and translated into many languages, it is regrettable that he chose not to. Because he has ignored the criticisms of his French colleagues and earlier critics, it is all the more important to take notice of those criticisms here.[1]

The Reaction to Ifrah's Works in France

In 1995 the *Bulletin de l'Association des Professeurs de Mathématiques de l'Enseignement Publique* devoted two issues to a discussion, by recognized scholars, of the merits of Ifrah's work (references to the critiques cited in this review appear in the bibliography). Since the overarching theme of *The Universal History of Numbers* is the emergence of the decimal place-valued number system, which arose independently in four distinct cultures—in Mesopotamia, China, India, and Mayan meso-America—Ifrah's work in each of these areas was accorded special scrutiny. As Tony Lévy of the Centre National de la Recherche Scientifique asks in his opening remarks: Does Ifrah's work present the history of numbers fairly with respect to recent scholarship and current understanding of the sources? Can one regard as established the conclusions presented by Ifrah as "historical verities"? Lévy leaves no room for doubt: "La réponse est doublement négative"—in other words, "no" on both counts (Lévy, p. 532).



Lévy explains that he and his colleagues felt an obligation to "rectify [Ifrah's] deceptive, confused, even muddle-headed views." They felt compelled to do so he says because of Ifrah's relentless habit of presenting conclusions that are "often debatable, generally weak, and at times wholly imaginary," as if they were "historically valid theses" (Lévy, p. 532).

Lévy subjects to close scrutiny Ifrah's account of the evolution and transmission of a symbolic, written notation for the Hindu-Arabic number system. Whereas the specialists, Indianists and Arabicists alike, deplore the lacunary state of the sources related to this history, Ifrah offers nothing but certainties. He writes with no doubts or reservations that "[w]hen the Arabs learnt this number system, they quite simply copied it (Fig. 25.3). In the middle of the ninth century, the Eastern Arabs' 1, 2, 3, 4, 5, 6, and 9 could easily be confused with their Indian Nâgarî prototypes" (*Numbers*, p. 532). Where, Lévy asks, are the Arabic documents to support this thesis? It turns out that a single manuscript (Paris, BN, MS arabe 2547) cited in support of his claim (see Ifrah's Fig. 25.3, *Numbers*, p. 532) preserves the graphic forms of the ciphers 2, 3, and 6. What is its date? Ifrah says 969 AD, accepting a hypothesis of Franz Woepke (whose work on the subject, *Mémoire sur la Propagation des Chiffres Indiens* (1863), is now nearly a century-and-a-half old). But as Lévy points out, 969 is not really mid-ninth century, and in fact, this date is probably too early (and by nearly 300 years), since the manuscript in question includes mention of the date 1259 (which was perhaps introduced by a copyist but which nevertheless raises serious doubts about the exact date in question). Of the other manuscripts Ifrah uses, the three oldest are of the eleventh century, and depict only the numerals 1, 4, 5, and 9. Despite the uncertainties in the written record on which the historian must depend, Ifrah offers no reservations: "These are the forms that the Arabs used when they adopted Indian numeration" (*Numbers*, p. 380). But in the absence, Lévy underscores, of any satisfactory, detailed studies of either the Arab or Indian primary sources, what Ifrah presents with such confidence is "historically unacceptable, a deception." ("La description de M. Ifrah est historiquement irrecevable; elle est trompeuse" (Lévy, p. 534).)

If there is reasonable doubt about the general conclusions Ifrah draws about transmission of the Hindu-Arabic numerals and number system, what about the details he offers for each of the four civilizations that first advanced decimal place-valued systems?

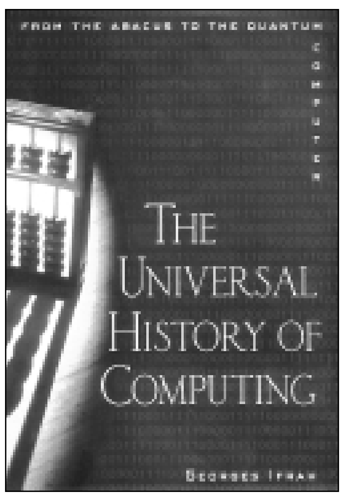
Mesopotamia

James Ritter (Université de Paris VIII) admits that his first reaction to Ifrah's "universal" history was one of perplexity, due to the errors that "appear

[1] *The author would like to thank William Aspray, André Cauty, Pierre Sylvain Filliozat, Joran Friberg, David Grose, Allyn Jackson, Tony Lévy, Jean-Claude Martzloff, Karen H. Parshall, James Ritter, and Christoph J. Scriba for responding to questions and reading earlier drafts of this review; while I have made every effort to reflect accurately the views of those cited here, and to do justice to Ifrah's own work as well, I alone am responsible ultimately for my views as expressed herein.*

on every page." Ritter, an Assyriologist, begins his critique with discussion of Ifrah's views on oral numbers in the Sumerian language, which played an important part, says Ifrah, in the origins of the base 60 system. On p. 82 Ifrah gives a list of Sumerian numbers, from which he concludes that the identification of the names for "one" and "sixty" reveals the existence of a base 60 in the oral Sumerian language of numeration prior to the invention of writing.

Ritter simply declares all of this to be false, due to an erroneous conflation of sources. First of all, he takes Ifrah's list to be a contrived amalgamation of names coming from all epochs. Over the more than 2,000 years of this history, the connections between names and numbers changed. If one is concerned with the origins of the base 60 system, it is the earliest texts that are relevant. Although rare, texts from the end of the third millennium, Ritter maintains, are perfectly clear— and in those texts, one (ash) and sixty (gesh) are not pronounced the same way.



As for the origins of the base 60 itself, Ifrah offers the hypothesis that it came from the combination of Sumerians using a base 5 system and another, supposedly indigenous, people using a base 12 system. Counting in distinct ways on their fingers, the two supposedly combined forces to create the base 60, the least common multiple of the two. But as Ritter objects, and as Ifrah himself admits, there is no hint, not a trace of anything like this in any written texts. This is further complicated by the fact (noted by Ifrah in passing) that there was no one system of written numbers in Sumerian by the beginning of the third millennium, but at least six—and more if one includes variants. What led to the emergence of the unique base 60 system, Ritter explains, was rationalization of all these different systems with the administrative centralization of the third millennium, a simplification that can be followed progressively in the Sumerian texts (Ritter, p. 683).

Ritter is even more adamant in rejecting Ifrah's account of the abacus in Mesopotamia. This time, Ifrah conflates provisional research with established fact. As Ritter explains: "After a wholly fabricated presentation of the 'calculists' and the 'abacists,' [Ifrah] takes, in an illegitimate way, several pages torn from their context of an article by the Assyriologist Stephen Lieberman" (Ritter, p. 683). Ifrah uses these pages to prove the existence of a word for "abacus" in the Mesopotamian texts, a suggestion made with many reservations and flagged with question marks by Lieberman. As Ritter cautions: "These readings are all controversial. Not all Assyriologists nor the great dictionaries accept them. All the terms under discussion are absent from all texts except for the tradition of lexical lists used in the schools" (Ritter, p. 683). For an instrument so universally used—as Ifrah maintains the abacus was—how can we account for the fact that it is passed over in complete silence in the thousands of mathematical texts and other documents left by Mesopotamian civilizations over three thousand years?

The final but greatest problem Ritter finds with Ifrah's interpretation of the Mesopotamian record is "ignorance pure and simple." Ifrah boasts of his having deciphered the proto-Elamite numerical systems, a means of writing developed in ancient Iran towards the end of the third millennium. But Ritter points out that this had already been accomplished by Assyriologists *before* any of Ifrah's publications, thanks primarily to the research of Joran Friberg, Peter Damerow, and Robert Englund. Ifrah has consistently ignored or downplayed most work done in this domain since the 1950s, and in particular he seems unaware of the significant advances of the last fifteen years, especially of the important contributions of the Berlin group of Damerow, Englund, Friberg, Nissen, and others (see, for example, the recent survey article [Friberg 1999]).

Chinese Mathematics

Jean-Claude Martzloff of the Institut des Hautes Études Chinoises in Paris, one of the world's leading authorities on the history of Chinese mathematics, finds similar fault with Ifrah's account of Chinese mathematics. Martzloff notes "errors and hazardous suppositions often repeated and sometimes amplified," which result in an "increasingly distorted image of the history of Chinese numeration" (Martzloff, p. 676). Martzloff takes Ifrah to task on three subjects: Chinese written calculation, the counting board, and the role of zero in Chinese mathematics.

As for Chinese written calculation, Martzloff warns of Ifrah's uncritical use of Karl Menninger's *Number Words and Number Symbols. A Cultural History of Numbers* (1957), where it is said that written calculations in China were explained in the *Ding Ju Suanfa* (*The Arithmetic of Ding Ju* (1355)), along with the method of checking a calculation by casting out nines. In fact, casting out nines does not appear in the *Ding Ju Suanfa*, and it was not until the seventeenth century that European missionaries introduced the method.

How could Menninger have made such an egregious error, one that Ifrah unfortunately repeats? It turns out that the text of the *Ding Ju Suanfa* was included with another, the *Tongwen Suanzhi* (*Treatise on European Arithmetic* (1613)), when the two were reprinted together in Shanghai in 1936. In using this edition, Menninger simply failed to distinguish one from the other! But the error is reproduced by Ifrah and made all the more damaging because it causes him to misdate the chronology of the appearance of written calculations in China by several centuries.

Another main source upon which Ifrah depends is volume three of Joseph Needham's important series, *Science and Civilisation in China*, the first part of which is devoted to mathematics. As Martzloff points out, although Needham does mention a counting board on occasion, he does so only in passing, and it is clear that it is nothing more than a hypothesis. But most who have written subsequently about Chinese mathematics based upon Needham's account have forgotten the hypothetical character of his remarks and have simply advanced from hypothesis to certainty. One of these is Geneviève Guitel, who in her *Histoire Comparée des Numérations Écrites* (1975) writes as if counting tables were not hypothetical but objects that really existed. Ifrah draws heavily on Guitel but goes even further, and, relying with great imagination on the rare pictorial images that exist relative to the practice of calculation, he produces from a late (sixteenth century) Chinese illustration a model "truer than nature" (Martzloff, p. 678) of a Chinese counting rod table.

There is nothing provisional about Ifrah's statement: "For arithmetical calculation, the Chinese used little rods made of ivory or bamboo called *chou* (calculating rods) which were placed on the squares of a tiled surface or a table ruled like a checkerboard" (p. 283). But this, says Martzloff, is pure fantasy, and the illustration (drawn by Ifrah himself) is not based on any real artifact or printed source describing such a table. What Ifrah starts from is an illustration from the *Suanfa Tongzong* (*General Source of Computational Methods*), a 1592 book about the abacus. But that work does not treat counting rods, and the illustration from it cannot plausibly be interpreted as a counting board.

The final criticism Martzloff levies against Ifrah involves a typographical error at a crucial point in Needham's study of Chinese mathematics, which misleads Ifrah into incorrectly dating by several centuries the first appearance of the zero in China. The error in question concerns a collection of Chinese manuscripts dating approximately to the tenth century, possibly earlier, and recovered from caves at Dunhuang at the beginning of the twentieth century. For more than twenty years, a French research team, of which Martzloff is a member, has catalogued and studied this collection in detail. Of the manuscripts dealing with mathematics, Martzloff reports that no zero in any form has been found among these materials! Assuming that the most ancient zero in China would have been contemporary with the more ancient Indian zero, Ifrah reconstructs the genesis of the Chinese zero based again on the counting table, Needham's faulty dating, and a "cascade of fantastic hypotheses which he takes to be established facts, but presented without the least justification" (Martzloff, p. 679). For example, Ifrah imagines the invention of a new system of Chinese numeration comprised in part of rods for calculating and Chinese characters for writing results (*Numbers*, p. 281). According to Martzloff "there is not the least shred of historical evidence" for such a system (Martzloff, p. 679).

The Mayans

André Cauty of the Université de Bordeaux specializes in the study of the Indians of ancient Mexico. He is critical of Ifrah's treatment of how the Mayas counted orally and the means by which they presumably carried out and recorded the results of arithmetic operations. Cauty also finds fault with Ifrah's reconstruction of the Mayan vigesimal system. Ifrah insists: "Even though no trace of it remains, we can reasonably assume that the Maya had a numeral system of this kind, and that intermediate numbers were figured by repeating the signs as many times as was needed. But that kind of numeral system, even if it works perfectly well as a recording device, is of no use at all for arithmetical operations. So we must assume that the Maya and other Central American civilisations had an instrument similar to the abacus for carrying out their calculations" (*Numbers*, p. 308).

But once more there is virtually no evidence upon which to base such assumptions. What Ifrah offers instead is the fact that the Incas used some sort of counting board to manipulate counters to facilitate their arithmetic. He refers to an illustration from the Peruvian Codex of Guaman Poma de Ayale of the sixteenth century (Ifrah's Fig. 22.20, *Numbers*, p. 308). However, this is a counting board, not an abacus, and there is no indication of how it would have been used and whether it followed a decimal, vigesimal, or some other arrangement. And in any case, this has no connection with the Maya! All Ifrah is able to provide is conjecture because, as Cauty puts it, he cannot conceive of a civilization without a highly developed sense of arithmetic and some means of both carrying out arithmetic operations and then recording the results. Cauty is rightly dubious, since there is not the least bit of archaeological or textual evidence for the existence of a Mayan abacus.

The Uncertainties of the History of Indian Numeration

Pierre S. Filliozat, a Sanskrit expert at the École Pratique des Hautes Études, is impressed by the attention Ifrah devotes to India, which he notes is usually given short shrift in most scholarly studies. Filliozat acknowledges the active interest, even the passion, in Ifrah's approach to the subject and says that "a leitmotiv of his book is praise for the works and genius of Indian mathematicians" (Filliozat, p. 542).

Nonetheless, Filliozat questions Ifrah's account of the Indian development of the place-valued number system and the appearance of the sign for zero, the *sunya*, meaning void. Ifrah (again!) maintains that this must have arisen in conjunction with use of an abacus, arranged according to powers of ten, the zero being necessary to write down the void places when no counters appeared on the abacus. Use of the zero, Ifrah maintains, would have freed scribes from having to use the abacus and permitted the direct notation of numbers. "This was the birth of the modern numerals, which signaled the death of the abacus and its columns" (*Numbers*, p. 437).

As Filliozat points out, the existence of the abacus at an early date is not documented in India; there is no archaeological evidence, and there are no literary descriptions or texts to bear out any of the speculations Ifrah presents. There is not even a word for "abacus" in Sanskrit, Filliozat notes (Filliozat, p. 547). And when an instrument did come into play in the fifth century, it was certainly not an abacus; it was nothing more than a common board upon which to write.

Possible Influences on Indian Mathematics

Ifrah briefly explores the possibility that the Indians may have been influenced by one of the other civilizations in which a decimal, place-valued system with zero arose: the Babylonians, the Chinese, or the Maya. He immediately eliminates the Maya for reasons of geography and rejects a Babylonian influence in part because the Babylonian base 60 is missing from the Indian system (*Numbers*, p. 408). This leaves the Chinese, but since the zero only appeared in Chinese mathematics around the eighth century, in all probability due to the influence of Indian Buddhist missionaries, Ifrah concludes that "it would seem highly probable under the circumstances that the discovery of zero and the place-value system were inventions unique to Indian civilisation" (*Numbers*, p. 409).

One Chinese source of which Ifrah is apparently unaware is the *Sun Zi Suanjing* (*The Mathematical Classic of Sun Zi*), written around 400 AD. This has been available in an English translation since 1992 in *Fleeting Footsteps*, an edition prepared with extensive commentary by Lam Lay Yong and Ang Tianse. This source not only gives a complete description of Chinese rod numerals but also describes in detail ancient procedures for multiplication and division. The most ambitious part of Lam and Ang's study, one not without controversy, argues that the Hindu-Arabic number system had its origins in the rod numeral system of the Chinese. The most persuasive evidence Lam and Ang offer is the fact that the complicated, step-by-step procedures for carrying out multiplication and division are identical to the earliest but later methods of performing multiplication and division in the West using Hindu-Arabic numerals, as described in the Arabic texts of al-Khwārizmī, al-Uqlīdisī, and Kūshyār ibn Labbān (for an extensive review of Lam and Ang's book, see Jean-Claude Martzloff, *Historia mathematica* **22** (1995), pp. 67–73).

It seems likely that the actual symbol for zero was introduced to China from India. Lam and Ang argue that the procedures for multiplication and division were in turn transmitted from China to the West via India, with Indian numerals taking the place of Chinese rod symbols for the purposes of writing down both the methods and results. This is certainly a hypothesis Ifrah should confront, especially given the many details that Lam and Ang provide and the relevance of their research for one of the main concerns of Ifrah's book.

Negative Numbers

There are numerous places where Ifrah has not asked what to a mathematician would have been the obvious or most pertinent question. One example will have to suffice here to give an idea of the technicalities that he either misses entirely, or has chosen to overlook. Ifrah cites a work by Brahmagupta of 628, the *Brahmasphutasiddhanta*, which defines zero as the result of the subtraction of a number from itself. This work also provides a table of results for operations involving negative numbers, in which the product of two negative numbers is given (according to Ifrah) as a negative number (*Numbers*, p. 439). But what Brahmagupta really writes about sign manipulation may be found in verses 30–35 of chapter 18 of the work in question, where the text clearly reads, "The product of a negative and a positive is negative, of two negatives positive." (I am grateful to Kim Plofker for having provided this translation from the actual text of the *Brahmasphutasiddhanta*, as well as the details that follow about terminology.) In any case, it would have been worth a few lines of commentary at this point to discuss the understanding necessary to see that the product of two negative numbers, or the division of two negative numbers, should yield a positive rather than a negative result, something that Brahmagupta clearly understood. Instead, what Ifrah says is that from the rules Brahmagupta gave for operations on "fortunes", "debts", and "nothing" (misunderstanding here that these are in fact technical terms in Sanskrit for positive, negative, and zero), "Modern algebra was born, and the mathematician had thus formulated the basic rules…We can see that at that time the Indian mathematicians knew the famous 'rule of signs' as well as all the fundamental rules of algebra" (*Numbers*, p. 439). This is another example of the great leaps Ifrah is willing to make even in the face of evidence that the true situation is not exactly as he represents it.

From One to Zero

Ifrah's *The Universal History of Numbers* is a reworked version of his earlier book *From One to Zero*, which first appeared in French in 1981. The time between the two books gave him ample opportunity to respond to critical reviews and to take advantage of expert judgments in order to correct errors or modify exaggerated interpretations—or simply to take into account readers' responses. When John Allen Paulos reviewed the English translation of *From One to Zero* in the *New York Times Book Review* in 1986, he acknowledged that the book had been "glowingly reviewed in France." But, calling the book "exhaustive and at times exhausting," Paulos wrote that "too much of this long book reads like a collection of appendices, and I often found myself saying 'enough already' as Mr. Ifrah piled up his historical documentation." Rather than heed Paulos's words in writing his new books, Ifrah did just the opposite. *The Universal History of Numbers* "translated afresh—is many times larger, and seeks not only to provide a historical narrative, but also, and most importantly, to serve as a comprehensive, thematic encyclopaedia of numbers and counting" (*Numbers*, p. v). As a result, readers are faced not with one volume but two, which expand Ifrah's earlier effort to nearly twice its original length. Consequently, Paulos's earlier objections are now doubly justified!

The second part of this review will appear in the next issue of the Notices.

References

[1] André Cauty, A propos de 'Comment comptaient les Mayas.' Histoire Universelle des chiffres (Ifrah 1994), *Bulletin de l'Association des Professeurs de Mathématiques de l'Enseignement Publique* **398** (April-May 1995), 534–542.

[2] ____ , Numération à deux 'zéros' chez les Mayas, *Repères. Revue des Instituts de Recherche sur l'Enseignement des Mathématiques* **41** (October 2000), 25–51.

[3] Pierre Sylvain Filliozat, Les incertitudes de l'histoire de la numération indienne, *Bulletin de l'Association des Professeurs de Mathématiques de l'Enseignement Publique* **398** (April-May 1995), 542–547.

[4] Joran Friberg, Counting and accounting in the proto-literate Middle East, *Journal of Cuneiform Studies* **51** (1999), 107–137.

[5] Lam Lay Yong and Ang Tianse, *Fleeting Footsteps*, Singapore: World Scientific, 1992.

[6] Tony Lévy, A propos de l'histoire des numérations et de l'ouvrage de G. Ifrah: Histoire Universelle des Chiffres, *Bulletin de l'Association des Professeurs de Mathématiques de l'Enseignement Publique* **398** (April-May 1995), 531–534.

[7] Jean-Claude Martzloff, L'histoire des numérations chinoises: Quelques erreurs et supputations hasardeuses souvent répétées et parfois amplifiées, *Bulletin de l'Association des Professeurs de Mathématiques de l'Enseignement Publique* **399** (June 1995), 675–680.

[8] Karl Menninger, *Zahlwort und Ziffer. Eine Kulturgeschichte der Zahl*, Göttingen: Vandenhoeck & Ruprecht, 1957; *Number Words and Number Symbols. A Cultural History of Numbers*, trans. P. Broneer, Cambridge, MA: MIT Press, 1969.

[9] James Ritter, La Mésopotamie et Monsieur Ifrah, *Bulletin de l'Association des Professeurs de Mathématiques de l'Enseignement Publique* **399** (June 1995), 681–685.

[10] ____ , Metrology, writing and mathematics in Mesopotamia, *Calculi* (J. Folta, ed.), special issue of *Acta Historiae Rerum Naturalium Necnon Technicarum* **3** (1999), 215–241.

About the Cover

This month's cover accompanies the review of George Ifrah's book on numbers. It is a recent photograph of the well known Babylonian tablet YBC 7289, from the Yale Babylonian Collection. There are three numbers on it, written in sexagesimal place notation: 30, the square root of 2, and $30\sqrt{2}$. Its exact function is not known, but it is surely one of the large number of "school tablets" from the Old Babylonian period, 1800–1600 B.C. The diagram forming a background to the numbers suggests that the Babylonians were aware of visual reasoning that leads to a proof of Pythagoras' Theorem for isosceles triangles.

More information can be found at `http://www.math.ubc.ca/people/faculty/cass/Euclid/ybc/ybc.html`. Our thanks to William W. Hallo and Ulla Kasten, Curator and Associate Curator of the Yale Babylonian Collection.

—*Bill Casselman* (`covers@ams.org`)