Book Review

Triangle of Thoughts

Reviewed by Yuri I. Manin

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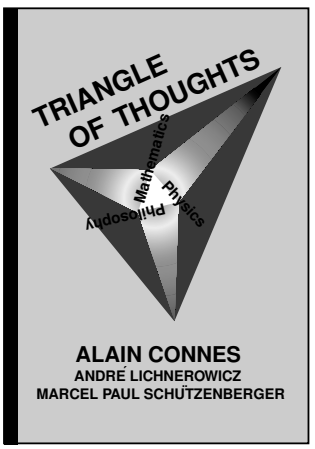
A. Connes, A. Lichnerowicz, M. P. Schützenberger
American Mathematical Society, 2001
ISBN 0-8218-2624-X
179 pages, $30.00

The literary form of philosophic dialogue inherited from Plato and revived during the Renaissance almost fell into oblivion in the last century, precisely when reflections on the implicit dialogical character of all human culture became the focus of the moral and cultural studies of Martin Buber and Mikhail Bakhtin. In fact, voices of most philosophers, before and after Plato, were authoritative and authoritarian, without a pretense of seeking truth in the clash of contrasting intellectual attitudes and varying viewpoints.

The central figure of a philosophic dialogue is a wise man, whereas modernity generally and systematically replaces wisdom by training. Wisdom seems to be an inborn faculty slowly ripened by life experience; as such it is rarely met and even more rarely put to any use. Training is a democratic surrogate for wisdom which, in spite of all of its (mainly aesthetic) drawbacks, is superior in one respect: it produces professionals.

This delightful book was conceived and written (told?) by wise professionals, mathematicians with a strong penchant for theoretical physics, the history of culture, and epistemology. It ought to be read slowly, perhaps no more than one dialogue at a time, and re-read in order to follow, say, a thread of argument that vanishes and reappears in a different context a dozen pages later. It is a difficult book, as its complete understanding requires a high level of professional training from the reader as well.



The participants discuss different images of the world, created by physicists. The core content of these images is expressed, and can only be expressed, in the language of mathematics, as we have known since the time of Galileo. But mathematics itself is not exclusively, or even predominantly, a language, and insofar as it is one, the semantics of this language does not reduce to any single physical interpretation, although it has its roots in the physical world.

As Alain Connes, professor at the Collège de France and a 1982 Fields Medalist, puts it in his opening statement, "without seeking to reduce every science to its object, it is simple for a physicist, chemist, geologist, or astronomer to define the object of his work: it consists of studying, on various levels, the structure and organization of matter. …Things are different in mathematics." And he proceeds: "To launch the debate, I would like to present right away two diametrically opposed points of view on mathematical activity:

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the viewpoint of the 'Platonists', who see themselves as the explorers of a 'mathematical world' about whose existence they have absolutely no doubt, and whose structure they uncover; and the 'formalists', who take refuge behind a sceptical attitude, considering mathematics as no more than a series of logical deductions in a formal system or, in a sense, as a sort of purified language."

Much of the first three dialogues ("Logic and reality", "The nature of mathematical objects", "Physics and mathematics: the double-edged sword") is an elaboration of this statement and the participants' positions towards it.

To summarize: Alain Connes believes in a "primordial reality" of mathematical objects and views the axiomatic method as a tool for studying this reality (cf. his other book of dialogues [ChCo]). André Lichnerowicz (who died in Paris in 1998) reveals a reserved stance towards formalist philosophy but uses this opportunity in order to learn more about arguments of formalists (not surprisingly, heavily relying upon Gödel's incompleteness theorem). Marcel Paul Schützenberger (who died in 1996) is more of a gadfly, haphazardly venturing outrageous statements to enliven the atmosphere, as in the following excerpt:

> M. P. S. — It is very presumptuous on my part to speak after you two. Some days, I support Alain's Leninist thesis. On other days, I would tend to support André's Stalinist thesis.
>
> A. L. — Why Stalinist?
>
> M. P. S. — Stalinism is opposed to Leninism by the massive injection of the free will lacking in Lenin, who had a mechanistic view of history. It does not take into account free will.

Structurally, these first three chapters serve not only to introduce some basic themes, but also the masks, the *personae* of the actors, even if they are real people and not invented persons. (The book ends with two short biographical notes: one about Lichnerowicz, written by Connes, and one about Schützenberger, written by Moshe Flato. An attentive reader will compare portraits of these two remarkable men with her or his own impressions.)

The remaining chapters are dominated by physics. What distinguishes them from many other books written for the general public is the implicit awareness of the distance between the physical world and the means we can use to grasp it, the distance that our technological advances can bridge but not do away with.

A revealing remark made by Lichnerowicz is pertinent here: "…if we compare what was called 'physics' or 'mathematics' in the nineteenth century with today's physics, what would surprise us would not be all the equations we write, but rather the pseudo-rational entities we make up to give them meaning. What has changed is the discourse, not the form of the equations."

Concerning equations, this is not literally true: with the advent of general relativity and quanta, the twentieth century added a lot of new equations to the classical arsenal. However, it is a fact that "new physics" brought with it new modes of discourse, in particular, by generating in the natural language numerous expressions referring directly to the mathematical descriptions of reality rather than to reality itself, understood in whatever sense we are prepared to concede to this much abused word.

As an example, consider the "probability amplitude" and the "superposition principle", two central notions in quantum mechanics. Richard Feynman in his beautiful lectures made a heroic attempt to explain their physical meaning to the general public, bypassing their mathematical content, because he could not count upon understanding of $\sqrt{-1}$, much less of Euler's formula for $e^{i\phi}$ and of the notion of complex linear space. In my opinion, he failed, but he could not have done better.

For examples from classical physics, see quotations from Maxwell on page 65 (about "proper names" for the p's and q's in analytical mechanics), and consider the mentions of this or that Lagrangian that continually pop up. (A whole history of theoretical physics could be written by focusing on the evolution of this remarkable abstraction.)

A further complication is that even a full mastery of the Euler formula, the Schrödinger equation, and, say, electron microscopy, does not help one to formulate a convincing epistemology, but only brings a troubled feeling that most interesting things cannot be expressed in words, or in words *alone*.

We have to accept this, with a sigh, as a professional risk for everybody trying to write about science, the author of this review included (cf. [Ma]). What is marvelous about this book is how many interesting things it manages to convey in words.

Here is a discussion on fire:

> M. P. S. — …I could take fire as an example of emergence. Fire is totally impossible to explain. The conjunction of specific factors in fire…
>
> A. L. — …I am convinced that fire, in the history of the human mind, is without parallel…
>
> M. P. S. — That is one way of putting it. It is a unique phenomenon in nature, and there will be others. But what I want

to emphasize is that there is no fire that is not on a human scale. You cannot make a fire that is one tenth of a millimeter.

A. L. — Conversely, the Sun is not a ball of fire.

M. P. S. — Conversely, as soon as you make a fire too big, it is no longer a fire, it is a firestorm. That is what the Allies produced in Hamburg, and what they repeated in Dresden. …The phenomenon is quite rare. It sometimes happens in forest fires. Instead of being 600 or 700°, the temperature goes up 1200 or 1300°. This is why the number of victims was so high in Hamburg and Dresden. The English high command deliberately wanted to set off a firestorm.

And here is a discussion about how well general relativity is confirmed by recent observations of binary pulsars and what exactly this confirmation means:

M. P. S. — If I understand correctly, the first Fourier coefficients, in fact the first seven, are sufficient to determine the physical parameters of the system. Once these parameters are known, the theory predicts the other Fourier coefficients and may therefore be refuted each time one of them is observed, making it possible to test the theory's validity.

A. C. — Exactly — since the 5 Keplerian parameters are measured directly, they can be forgotten. Then, as soon as we measure n post-Keplerian parameters (such as the precession of the periastron, the time dilatations, and the secular variation of the orbital period), we have n equations with 2 unknowns which are the two masses, whence we have $n - 2$ possible refutations of the relativistic theory of gravitation.

For example, for the binary pulsar 1913+16 we measure 3 post-Keplerian parameters, and so we have $3 - 2 = 1$ test of general relativity. For another binary pulsar, $1534 + 12$, we measure 5 post-Keplerian parameters, thus we have $5 - 2 = 3$ new tests of general relativity.

On language, music, multiculturalism, and quantum computing:

M. P. S. — …language begins with poetry rather than with grammar; euphony plays a big role here.

A. C. — Your point of view coincides with my own, since I sincerely believe that music is at its very beginnings, like language when it was at the stage of euphony. I think we might succeed in this way to educate the human mind to deal with polyphonic situations in which several voices coexist, in which several states coexist, whereas our ordinary logic allows room for only one.

Finally, we come back to the problem of adaptation which has to be resolved in order for us to understand quantum correlation and interrelation which we discussed earlier, and which are fundamentally schizoid in nature. It is clear that logic will evolve in parallel with the development of quantum computers, just as it evolved with computer science. This will no doubt enable us to cross new borders and to better integrate the mathematical formalism of the quantum world into our metaphysical system.

This is the concluding paragraph of the last chapter, "Reflections on Time", and the whole chapter is fascinating and frustrating.

This book can play an important role, if it helps the general intellectual public to avoid "the lure of unreason", invoked by John Weightman ([W]) in his fine and sensible review of Sokal's and Bricmont's critical contribution [SoBr] to the socio-philosophical controversies in which some leading minds of France and the USA got hopelessly entangled.

Basically, the authors celebrate the happy cohabitation of common sense with its most sophisticated refinements, developed in mathematics and physics, rather than that "strange blend of post-modernism with the ancient cult of the charismatic leader" ([W]).

This is the wisdom of professionals.

References

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[Ma] Yu. Manin, *Mathematics and Physics*, Birkhäuser, Boston, 1981.

[W] J. Weightman, The lure of unreason, *The Hudson Review*, vol. L, No. 3 (1998), 475–489.