2002 Bôcher Prize

The 2002 Maxime Bôcher Memorial Prize was awarded at the 108th Annual Meeting of the AMS in San Diego in January 2002.

The Bôcher Prize is awarded every three years for a notable research memoir in analysis that has appeared during the previous five years in a recognized North American journal (until 2001, the prize was usually awarded every five years). Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–10. Bôcher was also one of the founding editors of *Transactions of the AMS*. The prize carries a cash award of $5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2002 prize, the members of the selection committee were: Luis Caffarelli, Sergiu Klainerman (chair), and Linda Preiss Rothschild.

Previous recipients of the Bôcher Prize are: G. D. Birkhoff (1923), E. T. Bell (1924), Solomon Lefschetz (1924), J. W. Alexander (1928), Marston Morse (1933), Norbert Wiener (1933), John von Neumann (1938), Jesse Douglas (1943), A. C. Schaeffer and D. C. Spencer (1948), Norman Levinson (1953), Louis Nirenberg (1959), Paul J. Cohen (1964), I. M. Singer (1969), Donald S. Ornstein (1974), Alberto P. Calderón (1979), Luis A. Caffarelli (1984), Richard B. Melrose (1984), Richard M. Schoen (1989), Leon Simon (1994), Demetrios Christodoulou (1999), Sergiu Klainerman (1999), and Thomas Wolff (1999).

The 2002 Bôcher Prize was awarded to Daniel Tataru, Terence Tao, and Fanghua Lin. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Daniel Tataru

Citation

The Bôcher Memorial Prize in 2002 is awarded to Daniel Tataru for his fundamental paper "On global existence and scattering for the wave maps equations", *Amer. Jour. of Math.* 123 (2001) no. 1, 37–77. The paper introduces a remarkable functional framework which has played an important role in the recent breakthrough of T. Tao on the critical regularity for wave maps in two and three dimensions. The work of Tataru and Tao opens up exciting new possibilities in the study of nonlinear wave equations.

The prize also recognizes Tataru's important work on Strichartz estimates for wave equations with rough coefficients and applications to quasilinear wave equations, as well as his many deep contributions to unique continuation problems.

Biographical Sketch

Daniel Tataru was born on May 6, 1967, in a small city in the northeast of Romania. He received his undergraduate degree in 1990 from the University of Iasi, Romania, and his Ph.D. in 1992 from the University of Virginia. He was assistant, associate, and then full professor at Northwestern University (1992–2001) with a two year interruption when he visited the Institute for Advanced Study and Princeton University (1995–97). Since 2001 he has been a professor of mathematics at the University of California at Berkeley.

Response

I feel very honored to receive the 2002 Bôcher Prize, for which I am grateful to the selection committee and the American Mathematical Society. I would like to take this opportunity to acknowledge several people who have significantly influenced my work. My undergraduate mentor, Viorel Barbu, provided a model and an inspiration for me on what it means to be a mathematician. Later, my thesis advisors, Irena Lasiecka and Roberto Triggiani, through their professional support as well as their warmth, helped me grow, move on confidently, and adjust successfully here in the U.S. From them I learned control theory, which subsequently served both as a motivation and as a source of good problems in unique continuation. Sergiu Klainerman is the one who introduced me to nonlinear hyperbolic equations. I thank him for his constant support and for a fruitful collaboration during my years in Princeton. I am also grateful for the help and the encouragement that I received earlier in my career from M. G. Crandall, J. L. Lions, and P. L. Lions, as well as for the support of my friends and former colleagues at Northwestern. In addition, I continue to learn from my collaborators Herbert Koch and Hart Smith.

The wave maps equation is a semilinear second order hyperbolic equation which models the evolution of "waves" which take values into a Riemannian manifold. The starting point of the work in the citation was an earlier article of Klainerman and Machedon. At the time it was clear that bridging the gap between their result and the main wave maps conjecture required two distinct improvements of their argument. One was the so-called "division problem", which is related to controlling the bilinear interaction of waves at a fixed size of the frequency; the second was to control the interaction of low and high frequency waves in order to prevent the migration of the energy toward high frequency (which could lead to blow-up). These two problems correspond to two separate logarithmic divergencies in the work of Klainerman and Machedon, but, more importantly, they also correspond to the difference between a local and a global (in time) result. In the article cited I solved the first of these two problems; the second one was later solved by Tao. This was not an easy problem. Part of the difficulty lies in the construction of an appropriate functional framework. However, one does not have a good starting point for this, since the main condition this framework has to satisfy is a self-consistency condition. The solution was to start with a "reasonable" framework, proceed with the proof, and then backtrack and readjust the initial set-up whenever the argument did not work. While fairly intuitive, my approach is quite technical and I hope it can be simplified in the future. After the recent work of Tao there are still some finishing touches to be put on the study of the wave-maps equation. However, the more interesting problems which are still open are the other unsolved critical problems for the Yang-Mills equation, the nonlinear Schrödinger equation, and others.

My work on second order nonlinear hyperbolic equations was initially a byproduct of my attempt to use a phase space localization technique called the FBI transform for the analysis of partial differential operators with rough coefficients. Originally the FBI transform had been employed by Sjöstrand for the study of partial differential operators with analytic coefficients; as I learned later, it has also been used by physicists under the name of the Bergman transform. This approach produced sharp Strichartz type (dispersive) estimates for linear second order hyperbolic equations with rough coefficients, which in turn led to considerable progress in the local theory for second order nonlinear hyperbolic equations. At the same time an alternate approach for the nonlinear equations was pursued by Bahouri and Chemin, with comparable success. Later on, using Klainerman's vector fields method, Klainerman and Rodnianskii were able to improve my results. Around that time it became clear that the FBI transform is not robust enough for the study of nonlinear hyperbolic equations. My recent joint work with Hart Smith is based on another way of constructing a parametrix for the wave equation, using wave packets (which are highly localized solutions that stay coherent on a given time scale). The idea of constructing approximate solutions as superpositions of wave packets goes back to Fefferman, but its first effective use for second order hyperbolic equations is due to Smith. The joint work of myself and Smith largely completes the local theory for general second order nonlinear hyperbolic equations. The main open problem that remains is to understand whether the results can be improved for equations which have a special structure such as the Einstein equations, nonlinear elasticity, and other related problems.

Daniel Tataru

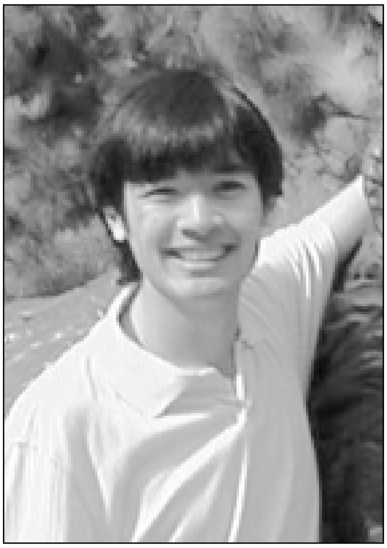
Unique continuation problems for PDEs have long been on my list of favorite topics. Originally my interest was motivated by problems in control theory, but later it took a life of its own. My view of the subject was influenced by the work of several mathematicians: L. Hörmander, G. Lebeau, L. Robbiano, C. Zuily, and others. Initially I worked on unique continuation problems which, up to that time, had received little or no attention: for boundary value problems, for operators with partially analytic coefficients, for anisotropic operators. Later on, in an ongoing joint project with Herbert Koch, I have returned to some of the more classical problems, but with a new twist: rough coefficients and/or unbounded potentials. The starting point for us was the seminal work of D. Jerison, C. Kenig, and T. Wolff.

Terence Tao

Citation

The Bôcher Memorial Prize in 2002 is awarded to Terence Tao for his recent fundamental breakthrough on the problem of critical regularity in Sobolev spaces of the wave maps equations, "Global regularity of wave maps I. Small critical Sobolev norm in high dimensions", *Int. Math. Res. Notices* (2001), no. 6, 299–328 and "Global regularity of wave maps II. Small energy in two dimensions", to appear in *Comm. Math. Phys.* (2001 or early 2002).

The committee also recognizes his remarkable series of papers, written in collaboration with J. Colliander, M. Keel, G. Staffilani, and H. Takaoka, on global regularity in optimal Sobolev spaces for KdV and other equations, as well as his many deep



Terence Tao

contributions to Strichartz and bilinear estimates.

Biographical Sketch

Terence Tao was born in Adelaide, Australia, in 1975. He received his Ph.D. in mathematics from Princeton University in 1996 under the advisorship of Elias Stein. He has been at the University of California, Los Angeles, as a Hedrick assistant professor (1996–1998), assistant professor (1999– 2000), and professor (2000–). He has also held visiting positions at the Mathematical Sciences Research Institute (1997), the University of New South Wales (1999–2000), and Australian National University (2001). He is currently on leave from UCLA as a Clay Prize Fellow.

Tao has been supported by grants from the National Science Foundation and fellowships from the Sloan Foundation, Packard Foundation, and the Clay Mathematics Institute. He received the Salem Prize in 2000.

Tao's research is divided into three areas: real-variable harmonic analysis (especially estimates for rough operators and connections with geometric combinatorics); nonlinear evolution equations (especially the global behavior of rough solutions); and algebraic combinatorics (specifically the understanding of the Littlewood-Richardson rule and its generalizations, and its applications to linear algebra, algebraic geometry, and representation theory).

Response

I am deeply flattered and honored to be nominated for the Bôcher Prize, and I am grateful to the prize committee for their recognition of this research. I have been extremely fortunate to have been supported, encouraged, and taught by many wonderful people and collaborated with many more. For the papers cited above I was particularly influenced by many invaluable conversations with Elias Stein, Tom Wolff, Jean Bourgain, Sergiu Klainerman, Chris Sogge, Daniel Tataru, Michael Christ, and my collaborators Mark Keel, Jim Colliander, Gigliola Staffilani, Hideo Takaoka, Ana Vargas, and Luis Vega. I am particularly grateful to Mark Keel and Sergiu Klainerman for giving me a thorough and expert introduction to the field of nonlinear wave and dispersive equations.

In the analysis of nonlinear dispersive equations, the tools used can be roughly divided into "analytical" tools and "algebraic" ones. By analytic tools I mean the use of function spaces such as Sobolev or Lebesgue spaces, coupled with linear, bilinear, multilinear, or nonlinear estimates in these spaces (which are often proven by harmonic analysis techniques). These estimates can allow one to apply perturbation theory and approximate a nonlinear equation by the linear analogue, at least for short times. By algebraic tools I refer to the use of conservation laws, symmetries, monotonicity formulae, special transformations, integrability, and explicit solutions. These algebraic identities give some partial control on the global development of solutions to the nonlinear PDE. To obtain satisfactory global control of solutions, one often combines the partial global control coming from the algebraic identities, with the more detailed but local control coming from the analytic techniques. For instance, perturbation theory might show that smooth solutions exist as long as the energy remains finite, while algebraic identities (i.e., integration by parts) might show that the energy remains constant. Combining the two one would then be able to show that smooth solutions exist globally in time, so that no singularities can ever form if the initial energy is finite.

Both the algebraic and analytic tools have been under development for many decades, the groundwork being laid by many excellent mathematicians. In the last ten years there has been immense progress, particularly in the analytic side of things, thanks to the efforts of Bourgain, Klainerman-Machedon, Kenig-Ponce-Vega, and many, many other authors. Indeed, our understanding of the local theory of nonlinear wave and dispersive equations has become quite satisfactory. Unfortunately, even when this local theory is completely understood, it does not always match up with the algebraic tools needed to create good global results; for instance, the local theory may need control of the solution in the Sobolev space H^2, but the conserved quantities might only control the solution in H^1.

One interesting development in recent years is that hybrid techniques, combining both analytical and algebraic ideas, have started to bridge some of the above gaps. In particular, the use of cutoff functions (in space, or in frequency, or in both), together with the latest linear and multilinear estimates, have been used to obtain "localized" conservation laws, "localized" evolution equations, "localized" gauge transforms, etc., which are more flexible than their global algebraic counterparts, and have had some recent successes. Notable applications of this type of philosophy include Bourgain's series of papers on nonlinear Schrödinger equations; the work by Martel and Merle on the stability of solitons for the generalized Korteweg de Vries equations; the many papers on global solutions below the energy norm (starting with work of Bourgain, and also including the papers by Colliander, Keel, Staffilani, Takaoka, and myself); the recent breakthroughs on quasilinear wave equations by Bahouri-Chemin, Tataru-Smith, and Klainerman-Rodnianski; and the recent series of papers on wave maps. For instance, one effective technique for constructing global solutions when the energy is infinite is to construct a smoothing operator, define the associated smoothed out energy, and show an approximate conservation law for the smoothed out energy. For wave maps, one new technique has been to localize the wave map to different frequency modes, and then gauge transform each frequency mode independently. The work on quasilinear wave equations is also interesting in that it seems to bring geometric optics back into the cutting edge of the theory. (Intriguingly, this has also occurred in the theory of oscillatory integrals, thanks to the work of Bourgain, Wolff, and others.)

In the future I believe we will see a more systematic synthesis of the analytic and algebraic techniques, perhaps ultimately leading to a unified theory for treating the development of nonlinear PDE; at present there are only tantalizing hints of such a theory. The end result should be a more powerful and flexible theory, allowing for much more detailed control on the global behavior of nonlinear PDE. (In particular, I hope to see finer control on the possible cascade of energy between frequencies, and on tracking particle-like behavior of solutions.)

Fanghua Lin

Citation

The Bôcher Memorial Prize in 2002 is awarded to Fanghua Lin for his fundamental contributions to our understanding of the Ginzburg-Landau (GL) equations with a small parameter. In a remarkable series of papers, among which we single out his pioneering work "Some dynamical properties of the GL vortices", *Comm. Pure Appl. Math.* (1996), 323–59, he has established, both in the stationary and evolutionary cases of GL equations, that the limiting phenomenon is governed by a finite dimensional system associated to the BBH renormalized energy.

The prize also recognizes his many deep contributions to harmonic maps and liquid crystals. Of particular note is his paper "Gradient estimates and blow-up analysis for stationary harmonic maps", *Annals of Math.* (2), 149 (1999), no. 3, 785–829.

Biographical Sketch

Fanghua Lin was born in China in 1959. He received a Ph.D. from the University of Minnesota (1985). He has held faculty positions at the Courant Institute (1985–88) and the University of Chicago (1988–89; 1996–97). Since 1989 he has been a professor of mathematics at New York University. Over the years, he has held numerous visiting positions, including those at MSRI, IAS, University of Paris-Sud, the Max Planck Institute, Academia Sinica, Hong Kong University of Science and Technology, and the University of Minnesota.


Fanghua Lin

Lin's awards and honors include an Alfred P. Sloan Fellowship (1989–91), a Presidential Young Investigator Award (1989–94), and the Chang Jiang Professorship (1999). He has served on the editorial committees of twelve mathematics journals, published over 110 research articles, and given numerous invited addresses.

Response

It is a great honor to be awarded the Bôcher Prize, and I am grateful to the members of the Bôcher Prize selection committee and the American Mathematical Society for their recognition of my work and this kind citation.

The Ginzburg-Landau (GL) equations with a small parameter were used to model superconductivity. They are also among the standard equations used to describe various phase transition phenomena. Many people have contributed to the study of these equations, and I interpret my receipt of this prize as a tribute to all of them.

My first paper on this subject established the connection between the critical points of the Bethuel, Brezis, Hélein (BBH) renormalized energy and the static solutions of the GL equations, one of many open problems posed in the seminal work of BBH. Soon after that, I learned the importance of various dynamical issues, particularly those concerned with vortex dynamics, and the scientific views of several of my colleagues at the Courant Institute (Weinan E and Andy Majda) and also of my collaborator Jack Xin have greatly influenced me. I had a lot of fun learning and solving some of these problems related to the vortex dynamics in 2-D.

The study of the GL equations in high dimensions is more subtle and involved. The analytical method that I developed jointly with T. Rivière can also be applied to study similar problems for the Seiberg-Witten and Yang-Mills equations. Nevertheless, many difficulties remain, especially those concerned with dynamical issues.

Much of my work relies on ideas from geometric measure theory, and I take this opportunity to thank my Ph.D. advisor, my long-time friend and collaborator, R. Hardt, for introducing me to this fascinating subject. I have been extremely lucky to have been at the Courant Institute, and I thank my colleagues there for their advice, support, and friendship during these past years. I also want to thank friends at the University of Chicago who have been very kind and offered a great deal of help during my career.