The Story of Mathematics

Reviewed by Karen Hunger Parshall

The Story of Mathematics
Richard Mankiewicz
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Richard Mankiewicz has done the unimaginable: he has written a "coffee-table book" on mathematics. From its richly colored dust jacket to its handsomely reproduced color plates and illustrations to its sumptuously thick paper, *The Story of Mathematics* is a beautiful example of the publisher's art that should adorn the coffee table of anyone who has ever wondered about the interrelations between mathematics and the broader culture.

Mankiewicz's book is not, strictly speaking, a book for mathematicians or even, one might argue, for undergraduate students of mathematics. Thumbing through its pages, one finds virtually no definitions of terms or mathematical notation—one exception occurs in the discussion of Sir William Rowan Hamilton's discovery in 1843 of the quaternions—for, as Mankiewicz explains in his preface, his intention was not to "tak[e] the reader through a sequence of 'great theorems'" but rather "to illustrate how the mathematical sciences were intimately linked to the interests and aspirations of the civilizations in which they flourished" [p. 8]. The story of mathematics that

Mankiewicz wants to tell, then, is not the history of mathematics viewed as a body of knowledge. It is the story of how mathematics developed in tandem with and in response to diverse social and cultural concerns. As Ian Stewart confesses in his foreword, this "is the sort of book I would have loved to have read when I was a teenager" [p. 7], but he sees the book's real target audience as something other than that small group of teenaged mathematics enthusiasts that searches the shelves of the high school library for something, anything, to read on mathematics. This book, in Stewart's view, is for that (large) segment of the general public that "still associates math[ematic]s with school, and *with nothing else*" [p. 6; his emphasis]. In particular, it is for those who remain unaware of the field's "unbroken history of involvement with the mainstream of human culture, a history that has been going on for at least five thousand years" [p. 7]. How does Mankiewicz attempt to reach and educate this readership?

First consider the book's physical presentation. Mankiewicz has produced an approachable book. Under two hundred pages in length, it could hardly

Karen Hunger Parshall is professor in the departments of history and mathematics at the University of Virginia, Charlottesville. Her e-mail address is `khp3k@virginia.edu`.

be considered intimidating by one who might want to read something—but not too much—about mathematics. Moreover, it is divided up into twenty-four chapters, the majority of which are five rather sparsely typeset pages long and include three or more illustrations, some full-page. Given this format, it is a book that can easily be read in thoroughly digestible, fifteen-minute chunks.

The volume is also visually seductive with over a hundred images, most in full color, illustrating what one might term the material culture of mathematics. These range widely. In the chapters from the prehistory of mathematics, what Mankiewicz calls "year zero," through the Middle Ages in the Latin West, we find, among many others, a photograph of a Mesopotamian clay tablet inscribed in cuneiform with a table of accounts that dates from the third millennium BCE [p. 9], pages from a medieval Islamic text [p. 25] and from a thirteenth-century Chinese manuscript [p. 35] dealing with the Pythagorean theorem, a page from a sixteenth-century chronicle of Moghul India [p. 41], and a photograph of an astrolabe from ninth-century Iraq [p. 45]. The book's middle third includes, for example, a reproduction of *The Flagellation of Christ* by the fifteenth-century Italian painter Piero della Francesca [p. 62], a sixteenth-century French illustration on how to use a cross staff [p. 70], a photograph of a calculating device based on the principle of Napier's rods [p. 76], and an early sixteenth-century map of North Africa [p. 113]. Images in the book's final third range from a drawing by Moritz Escher inspired by the Möbius band [p. 129], to a photograph of a positive electrical charge taken in 1892 [p. 146], to Marcel Duchamps's 1912 *Nude Descending a Staircase* [p. 169], to computer-generated snapshots of cellular automata [p. 187]. Even those who simply flip idly through this book cannot fail to be impressed at how deeply mathematics is and has been embedded in cultures around the globe.

Once readers have been drawn into the book through its presentation, what do they encounter in its text? To a large extent, they find much of what historians of mathematics would recognize as the now-standard textbook account, except that Mankiewicz *has* placed more emphasis than some textbook authors on the cultural aspects of mathematics.

Still, the now-standard account is not necessarily what one would find in popularizations from, say, fifty years ago. In his book, *Mathematics: Queen and Servant of Science* (1951), Eric Temple Bell took a thoroughly Eurocentric approach to the history of the subject. He saw modern, Western mathematics as springing from classical Greek antiquity and the works of men like Plato, Euclid, and Archimedes. In his view, mathematics lay fallow until the period of the Scientific Revolution brought geniuses like Galileo and Isaac Newton into the picture. Mathematics then developed explosively in the eighteenth—but especially the nineteenth—century to result in the twentieth-century pinnacle of mathematical achievement. Although Bell mentions in passing the Babylonians and their presumably empirical understanding of the Pythagorean Theorem, as well as the Chinese and their possible proof of Fermat's "little" theorem, neither medieval Islamic mathematics nor mathematics in the Latin West figure into Bell's historical account. These civilizations and these time periods were simply not deemed as having contributed to the mathematical progress that had resulted in the mathematics of Bell's own day.

In some sense, Bell must not be criticized for his Eurocentric approach, which was, after all, the norm, especially in 1931 and 1937 when he first wrote the two pieces that he later revised and combined to form the 1951 text. Even so, Florian Cajori, writing his *A History of Mathematics* in 1893 for an audience of teachers and students of mathematics, spent almost a dozen of his 400 pages on the Babylonians and Egyptians and fifty pages on the Middle Ages including some sixteen pages each on "The Hindoos" and "The Arabs" but omitting the Chinese. By the 1960s, these "other cultures" began to figure prominently both in popularizations like Lancelot Hogben's *Mathematics in the Making* (1960) and in textbooks on the history of mathematics such as Carl Boyer's *A History of Mathematics* (1968). Serious historical scholarship on mathematics in Mesopotamia, in (to a lesser extent) Egypt, in medieval Islam, and in South and East Asia has subsequently been incorporated into the standard histories, culminating in texts like Victor Katz's *A History of Mathematics* (1993, 2nd ed. 1998).

If Mankiewicz draws from this now-standard textbook presentation, he also gives it a more cultural, less technically mathematical spin. Consider the chapter "Mathematics for the Common Wealth" [pp. 68–76], in which he gives an overview of sixteenth- and early seventeenth-century Europe. This was a time when the possibilities of the printing press came to be appreciated fully. This was a time, following the Hundred Years War, of increasing economic prosperity. This was a time of myriad voyages of discovery and strange and wonderful stories of distant lands, peoples, creatures, and plants. All of these broader cultural phenomena had implications for and reverberations in mathematics.

The printing press allowed mathematical knowledge to spread more easily, and the economies of the new medium gradually forced the development of a kind of shorthand for what had traditionally been a purely rhetorical, word-for-word way of expressing mathematical notions. Not a true

Mesopotamian cuneiform · medieval Islamic · Escher

Piero della Francesca · printing press · Francis Bacon

notation, this so-called "syncopated" style of mathematical expression began to appear in printed works like Luca Pacioli's 1494 text *Summa*. The *Summa* also contained detailed explanations of how to use Hindu-Arabic numerals and discussions of, for example, the principles of accounting. Texts like Pacioli's—and so mathematics—thus contributed to Europe's economic development by permitting the training of numerate workers hired specifically to keep financial records.

The voyages of discovery were also intertwined with mathematics in significant ways. In particular, sailors needed to determine accurately their position on the globe. This problem, one that often involved calculations in terms of large numerical values, was made much simpler by the application of the logarithms that John Napier developed in the early seventeenth century. Napier's technique similarly made the calculational aspects of astronomers' work easier.

The complex interplay between mathematics and society illustrated in these examples was, moreover, not lost on contemporaries. In works like *The Advancement of Learning* (1605), *The Great Instauration* (1620), and the *Novum Organon* (1620), Francis Bacon argued that science, properly done, would be crucial to the prosperity of the commonwealth. In particular, Mankiewicz writes that "[t]he use of mathematics by merchants, navigators and scientists was seen as contributing to the creation of greater wealth for the nation. The promotion of mathematics was no longer the concern of a few scholars, but a full-blown call to arms" [p. 75]. Mathematics and science were fully embedded in the culture.

Another point of departure between Mankiewicz's book and the now-standard textbook accounts of the history of mathematics is its engagement with trendier contemporary topics like game theory, computing, fractals, and chaos. For example, chapter twenty-one, entitled "War Games", treats game theory, the mathematical analysis of "games" involving pure strategy. Mankiewicz opens with a brief description of the nineteenth-century Prussian game of *Kriegspiel*, a war simulation game that helped train the mighty Prussian army prior to World War I. Germany's defeat in that war may have marked the end of what Mankiewicz terms *Kriegspiel*'s "mythical status" [p. 160], but another lesson of the war was that the whole notion of military strategy needed rethinking. "The military thus needed mathematicians and scientists not only for developing military hardware, but also for strategic advice—hitherto the domain of generals steeped in military history" [p. 160]. Once again, the import of extrascientific and extramathematical factors on the development of mathematics becomes manifest in Mankiewicz's rendition of the story of mathematics.

A brief history of game theory follows. It starts with Émile Borel's series of papers in the 1920s, in which the mathematics of games is applied not only to situations like bluffing at cards but also to the realms of economics and politics. Next, it mentions the ground-breaking work of John von Neumann and Oskar Morgenstern on the *Theory of Games and Economic Behavior* (1944) and discusses their analysis of "the two-player two-strategy zero-sum game—a game in which two perfectly rational players are each intent on winning, and in which the total utility is zero, i.e. one player's gain is the other's loss" [p. 161]. The presentation here is not mathematical but rather analogical; the situation is likened to "a scenario replayed in many a household…the division of a cake between two children so that neither feels that the other has the larger piece" [p. 161]. Biographical information on von Neumann adds color and a human dimension to the discussion as it does in the glimpse that follows of the Nobel Prize-winning work of John Nash from the 1940s and early 1950s on nonzero-sum games and optimal strategies. Finally, after raising the question "[w]as there an optimal strategy for nuclear weapons?", the chapter concludes with a description of the RAND Corporation, a group founded in 1945 as a think tank to "think the unthinkable" and to devise national strategies in a nuclear world [p. 163]. Mankiewicz takes yet another opportunity to emphasize the intimate relationship between mathematics and society in his chapter's closing line: "[t]he whole global marketplace is a shifting scene between collaborations and competition—a world of game theory" [p. 164].

For all of its strengths as a popular work on mathematics in culture, *The Story of Mathematics* also has some weaknesses. First, despite its efforts to discuss in an integrated way the development of mathematics by many different peoples and cultures, the book fails to interweave the role of women in mathematics (with the exception of Hypatia) into its narrative. There are also some—but not too many—misleading statements and outright errors, typographical or otherwise. For example, the Babylonians are described as "highly proficient in algebra, although questions and methods of solution were stated rhetorically in words rather than symbols" [p. 11]. To label what the Babylonians did as "algebra"—even with the qualification about their lack of symbols—is to create the misperception that they approached a certain range of questions in ways reminiscent of high school algebra. A better way to finesse the linguistic problem of applying the word "algebra" ahistorically would have been to provide the briefest example of a Babylonian algorithmic problem solution and to have described it as "translatable" into what we would call algebraic terms. The allusion to "algebraic thinking" without further qualification or explanation again in the context of the medieval Islamic mathematicians perpetuates this misperception [p. 46].

One of the book's actual errors occurs in chapter sixteen on "New Geometries", where Euclid's fifth postulate is said to state that given a point not on a line "through this point there is one and only one line which is parallel to the first line" [p. 129]. While this is one of the many logically equivalent statements of the postulate in the context of Euclidean geometry, it is not Euclid's formulation of it; a correct statement appears earlier in the chapter [p. 126]. Finally, what must have been some sort of typographical error in interpreting 20^2 and 20^3 has resulted in this nonsensical rendering of the Mayan, partly vigesimal number system: "A true vigesimal system would have place values in the sequence 1, 20, 202, 203, and so on, but the Mayan system uses the sequence 1, 20, 18×20, 18×202, and so on" [p. 16]. Another unfortunate typographical error occurs in the title of chapter three on the so-called "Pythagorean theorum" [p. 21], and not enough attention was paid in the proofreading stages to the diacritical marks on words from foreign languages.

Admittedly, these final criticisms smack of the school marm, but one wishes that a book—especially one as beautifully produced as this—could be totally error-free. Mankiewicz's *The Story of Mathematics* is not a book for everyone, but, then, it is not meant to be. It is not a book for historians of mathematics. It is not a book for mathematicians (but perhaps I am being overly optimistic about the extent to which the modern mathematical community appreciates its cultural roots) or for those with technical expertise who want to read deeply into the history of mathematics. It is not a book that defines all of its terms, or covers every possible mathematical topic, or treats every conceivable geographical region or constituency. It *is*, however, an unintimidating point of departure into the world of mathematics. It is a book for all of those who never managed to see the point of mathematics. In short, it is a book for the vast majority of the English-reading public, who could and should read it with great benefit.