Mathematical Articles and Bottled Water

The system for publishing mathematical articles should be reformed, and the new system should resemble, on the economic side, the bottled water industry. My main theses are: (i) the results of mathematical research should be available to the public just like tap water, (ii) the role of the commercial (and noncommercial) publishers should be to sell an upgraded version of the product ("bottled water"), and (iii) only a coordinated action of big and powerful institutions (i.e., universities and the government) can bring about the change to the system.

The advent of electronic databases and the Internet changed the economics of mathematics publishing. In the past, a mathematical paper used to be sold only once, on paper. Recently commercial (and not only commercial) journal publishers started collecting journal articles in their private databases. Then they started selling access to their databases—an article can now be sold an unlimited number of times over an indefinite period. Electronic databases have a huge technical advantage over paper versions of journals—they offer search facilities unparalleled by anything one can possibly do with paper copies.

Distinct mathematical results do not compete with each other the way various models of automobiles do—a mathematician must have access to all known results in the field to be efficient and competitive. Hence, universities cannot choose between various publishers—they have to subscribe to all major journals and databases. This gives the publisher of a journal monopoly power, even if the company owns only a small proportion of the scientific literature in the field. As a result, some journal prices are outrageous. The emergence of commercial electronic databases only aggravated the situation.

Mathematical articles should be made available to the public in the same way water is. Public money is used to provide safe and cheap drinking water to most people. Companies selling bottled water exploit the fact that many people are willing to pay a premium price for bottled water for various reasons—taste, portability, etc. People have a choice—to drink tap water, provided free or almost free at many locations, or pay an extra fee for extra value.

The public and private universities and the government should create databases for mathematical papers in their least refined form. The Mathematics ArXiv is an example of such a database. The articles deposited there are not refereed. They are available in several electronic formats but the typesetting is only as good as the author chooses it to be. This is equivalent to tap water. The universities and the government should make it mandatory for mathematicians supported in any way by salaries or grants to deposit their articles in one of such databases.

Journal publishers should be in the business of selling "bottled water", i.e., the enhanced version of research articles. The enhancements would include elegant typesetting and linking to other articles, for example. If the new system is implemented, one might expect that journal prices would go down. The price of bottled water cannot be too high, as long as everyone has access to tap water.

The new proposed system would work only if the universities had a choice to opt out and cancel subscriptions to commercial publisher databases. But this will be a realistic possibility only when the free, university and government supported databases of articles in their raw form are complete. One cannot expect that voluntary submissions of some articles by some researchers would make a difference. An appeal to the universities to create free databases and an appeal to the mathematicians to deposit their manuscripts in them will have little effect if even a small but nonnegligible proportion of researchers ignore it. The universities and the government agencies such as the National Science Foundation have a legitimate claim to ownership of the mathematical results as they are the organizations who pay for the research. They should make it mandatory for mathematicians to submit their preprints to free public databases in the final (revised) form before transmitting them to the publishers for typesetting.

My proposal is far from a new camouflaged form of taxation in which the government takes away from mathematicians the fruits of their labor. It is much closer to the government imposed and enforced traffic laws—even extreme libertarians might approve traffic lights. I believe that all mathematicians would be happy to distribute their theorems for free to all other mathematicians and to the general public. Many mathematicians send their recent results to their colleagues as paper preprints or electronic files, post their articles on their personal web pages, or deposit them in public archives. The current system is chaotic with the result that for most articles, the access is free only to a limited number of people and only for a limited amount of time. Only government action could introduce order in the system and assure permanent free access to all of the mathematical literature for everyone.

—Krzysztof Burdzy
University of Washington

Letters to the Editor

AP Calculus Grading

I would like to report to the community on my experience grading the 2001 AP Calculus exams and the conclusions I have drawn from that.

The AP Calculus scoring system is one which is fundamentally unfair to the students who take the exam. The scoring rubrics in many ways fail to reflect the knowledge of calculus displayed by the students taking the exam, and in some instances egregiously so. Furthermore, the reading system is deliberately designed to restrict the exercise of the readers' judgment. Throughout the entire process, I was frustrated by being unable to apply my judgment in scoring books and infuriated by being forced to score books in ways that I felt were simply wrong. All in all, the AP Calculus reading was one of the worst professional experiences I have ever had. Unless and until there are major changes, I would never participate in it again, and I strongly advise colleagues not to do so either.

My desire here is that the necessary changes be made, and the reader may ask why I am writing a letter to the *Notices* rather than communicating about these issues directly with those responsible for the exam reading. The answer is that I have. I wrote a seven-page letter to the Chief Reader. His reply consisted of a one-paragraph brush-off. Thus I think that the only way to effect change is to bring the problem into the open.

I have already used some AP terminology, which I must define. We are not graders, but readers, and we do not grade exams, but rather read books. The standards for reading each of the problems in the books are called scoring rubrics, and readers are given these rubrics, which we must follow to the letter, at briefings. The Chief Reader is in charge of the whole process, with a hierarchy reaching down to the ordinary readers.

Let me propose the following two grading axioms:

Axiom 1. An answer which is mathematically correct, correctly expressed, and supported by correct reasoning, shall be given full credit.

Axiom 2. An answer, correct or incorrect, the reasoning for which is totally specious, shall be given no credit.

The readers of the *Notices* will undoubtedly question my sanity when I claim that these axioms are violated in the AP reading process, but they are.

I regard Axioms 1 and 2 as self-evident. I propose a third axiom, perhaps not self-evident, but still, I think, pretty clear.

Axiom 3. Readers who, using their best professional judgment, are convinced beyond a reasonable doubt of the correctness of a student's solution may overlook harmless omissions and inconsequential errors and assign full credit for the solution.

This axiom is also violated. Furthermore, there are many other instances where the scoring rubrics reflect extremely poor judgment, resulting in scoring books in ways that are unreasonable and unfair.

The readers of this letter will naturally want proof of these claims. Unfortunately, the space available for letters in the *Notices* precludes giving it here. Instead, I have posted a slightly modified version of my letter to the Chief Reader on my website, at `http://www.lehigh.edu/~shw2/ap2001.html`, where extensive detailed evidence may be found.

In summary, the AP calculus scoring system is badly broken. It urgently needs to be fixed—not tinkered with, but fixed. I regard adoption of my axioms as necessary elements of any repair, but in any event what is necessary is a complete reexamination of the scoring system from the ground up. Nothing less will do.

—*Steven H. Weintraub*
Lehigh University

(Received December 4, 2001)

Response to Weintraub

We regret that Steven Weintraub felt frustrated by his experience at the 2001 AP Calculus Reading. For a different perspective on the positive experiences of participating in the Reading, we invite everyone to read the letter by Roger Howe of Yale University that appeared in the November 1995 *Notices* (`http://www.ams.org/notices/199511/letters.pdf`).

Readers are asked to exercise their judgment in evaluating student solutions (not just answers) within the context of the philosophy and guidelines established by the scoring rubrics and in agreement with the instructions given in each problem and for the exam as a whole. We recognize that not every Reader will agree with every scoring decision, but we firmly believe that the final AP grade accurately, consistently, and fairly reflects student achievement. Weintraub has raised some important concerns, and we do take them seriously, as we do all comments from Readers.

The statistical evidence does not support the contention that the scoring system is fundamentally unfair to students. Analyses have shown that the AP Calculus exam grades of 1 to 5 correlate well with other measures of student achievement, including college grades in subsequent mathematics courses. Student scores on the free-response problems, the ones graded at the Reading, have a high correlation with the scores on the multiple-choice section of the exam. AP Calculus traditionally has one of the highest reliability coefficients among all AP exams (reliability of the AP grade is a statistical measure that shows to what extent the grade would have been the same if the student had taken a different form of the exam and if the responses had been scored by different faculty consultants).

Over 600 college and high school faculty participate in the Reading, grading over one million calculus problems. It is impossible to involve every Reader in the development of the rubrics, but they are set by professional mathematicians using their professional judgment. The Chief Reader develops preliminary rubrics in consultation with the AP Calculus Development Committee as the problems are written. Before the Reading starts, the Chief Reader meets with the exam and question leaders to discuss, argue, agree, and sometimes disagree about how to allocate partial credit among the nine possible points on each of the six problems. A final consensus on the rubrics emerges after examinations of hundreds of actual student responses and lengthy discussions with the eighty table leaders, all of whom have many years of experience with the AP Program and with the difficulty of fairly grading thousands of exams (almost 185,000 in 2001). After the Reading is completed, the composite scores based upon 108 points from the multiple-choice and free-response sections are converted to an AP grade of 1 to 5 by the Chief Reader in consultation with statisticians from Educational Testing Service. Decisions on cut scores are based on such factors as statistical information based on common multiple-choice items in the current exam and one or more past exams, studies comparing college student and AP student scores on both multiple-choice items and free-response problems, AP grade distributions from past years, and observations from the Reading.

The issues raised in Weintraub's "axioms" have been considered repeatedly and thoroughly over the years by the mathematics faculty who have participated in many Readings. Situations that arise in grading individual solutions revive these discussions. The positions taken at the 2001 Reading reflected established practices that these repeated considerations have eventually supported.

One of the primary goals throughout the process is that a student should receive the same score for a problem regardless of which Reader grades that problem. Axiom 1 is a guiding principle in every scoring rubric, but still allows interpretation about the depth that is needed to be "correct." One problem with Axioms 2 and 3 is interpreting "totally specious" and deciding what are "harmless omissions and inconsequential errors." Even if everyone agreed with these axioms, there is subjective evaluation about levels of "correctness," especially when "completeness" is part of the correctness. If Axioms 2 and 3 were to be adopted in full, consistency and fairness would certainly suffer, and students would be much more at the mercy of individual Reader attitudes. What would not be fair is to have the student's score depend to a large degree on who graded the paper.

We invite college faculty to participate in the AP Reading as the program continues to strive to improve even more the grading of the AP Calculus exam.

—Larry Riddle
Chief Reader, 2000–2003
Agnes Scott College

—Bernard Madison
Chief Reader, 1996–1999
University of Arkansas

—Ray Cannon
Chief Reader, 1992–1995
Baylor University

—George Rosenstein
Chief Reader, 1988–1991
Franklin and Marshall College

(Received February 5, 2002)

Contribution of John Nash to General Topology and Real Analysis

With the release of the movie *A Beautiful Mind* and the publication of several books, it is now widely known that John F. Nash Jr. did research in the theory of games which revolutionized economics. In other areas of mathematics his work is rated even more highly by mathematicians.

One of his lesser-known mathematical contributions is a problem he posed in 1956 that stimulated extensive research in general topology and real analysis. In "Research Problem 1: Generalized Brouwer Theorem" [*Bull. Amer. Math. Soc.* **62** (1956), 76], Nash defined a "connectivity map" from a space A into a space B to be one such that the induced map $A \to A \times B$ preserves the connectedness of any connected set in A. He asked if every connectivity map of a cell into itself must have a fixed point.

Motivation for this problem stems from the fact that to prove the Brouwer fixed point theorem for dimension 1, we need the function to have merely a connected graph, which is a consequence of continuity. So it is natural to ask if the result is true for higher dimensional cells for a *connectivity function.*

O. H. Hamilton showed that the problem has an affirmative solution ["Fixed points for certain noncontinuous transformations", *Proc. Amer. Math. Soc.* **8** (1957), 750–756], but his proof contained a gap. This gap was filled by J. Stallings ["Fixed point theorems for connectivity maps", *Fund. Math.* **47** (1959), 249–263], and the importance of connectivity functions was established. Moreover, Hamilton introduced *peripherally continuous functions* and Stallings introduced *almost continuous functions*, both of which are closely related to connectivity functions. These together with the previously known *connected* or *Darboux* functions and their generalizations began to be studied widely.

The following beautiful characterization of connectivity functions was discovered by S. K. Hildebrand and D. E. Sanderson ["Connectivity functions and retracts", *Fund. Math.* **57** (1965), 237–245].

Theorem. *Let $f : (X, T) \to (Y, T')$ be a function and consider the topology τ generated by $T \cup f^{-1}(T')$ on X. Then f is a connectivity function if and only if T and τ induce the same family of connected sets in X.*

Further references may be found by consulting MathSciNet and issues of the journals *Real Analysis Exchange* and *Fundamenta Mathematica*.

—Som Naimpally
Lakehead University

(Received January 17, 2002)

Software Patents

When asked about software patents, Donald Knuth [*Notices*, March 2002] conjectured that it might be possible to patent a 300-digit integer. The readers might be interested to know that I have already patented a 309-digit integer as claim 37 of U.S. Patent 5,373,560, issued in 1994. It relates to cryptography, and it is not as interesting as what Knuth had in mind. At the time, some people thought that patenting a number was a new extreme in silly software patents, but now we have business method patents that are even sillier.

—Roger Schlafly
(Received February 13, 2002)