Working Together to Improve Mathematics Education

How we prepare children to understand and use mathematics is constantly evolving. The National Council of Teachers of Mathematics (NCTM) document *Principles and Standards for School Mathematics* urges all of us to start building the foundations of mathematics success even before students enter kindergarten. A key element of preparing for student success is teacher preparation in mathematics. NCTM's vision of a high-quality mathematics education for every child demands that we find approaches for improving the mathematics content knowledge and pedagogy of all teachers, especially elementary school teachers.

In most states, teachers in grades K–6 are not mathematics specialists, nor have states asked them to be. As we reach for higher standards, we are faced with a challenge. How should we prepare elementary school teachers in mathematics and mathematics pedagogy? According to recent National Science Foundation data, only 7 percent of elementary school teachers and 18 percent of middle grades mathematics teachers majored or minored in mathematics or mathematics education. Furthermore, 40 percent of elementary school and middle grades teachers of mathematics report that they do not feel qualified to teach the content that they teach. In some school districts, large percentages of middle grades and high school mathematics teachers lack the certification to teach mathematics. For example, in one major U.S. city, 57 percent of the middle grades mathematics teachers lack mathematics certification, and 20 percent have not satisfied their board of education college mathematics requirements.

The level of preparation in mathematics and mathematics teaching needed to implement solid, *Standards*-based mathematics programs may be largely inconsistent with the preservice preparation of the majority of elementary school teachers in the United States. Crucial questions we must address include: Do teachers lack the strong grasp of mathematics that would allow them to teach mathematical concepts with understanding? Can elementary teachers make appropriate connections (a) between arithmetic and real-life situations, and (b) among arithmetic concepts? Are elementary teachers being prepared to implement comprehensive and coherent mathematics programs that allow students to pursue high-quality mathematics in high school and college?

Amazingly, there is currently no comprehensive system in place in the United States to help elementary teachers of mathematics grow and develop professionally in their mathematics knowledge and understanding. We must change this.

AMS and NCTM should work closely to determine viable approaches for providing appropriate mathematics content to preservice and in-service elementary teachers. By "appropriate" I mean mathematics content that will help elementary teachers gain a greater understanding of the mathematics we expect them to teach. Such courses must help teachers become more skilled and fluent users of mathematics. Such courses must also help teachers better understand the underlying ideas and relationships that are the foundation of the concepts, skills, and algorithms that students are taught.

AMS and NCTM can work on many activities to improve mathematics education. For example, AMS and NCTM can identify content and pedagogical approaches that best serve both preservice and in-service teachers. In partnership, AMS and NCTM can make recommendations about the number of hours and the nature of appropriate mathematics courses that preservice teachers should take. AMS and NCTM could work jointly to create a mathematics specialist program for elementary teachers. AMS and NCTM can devise a realistic philosophy about awarding graduate credit for appropriate mathematics courses taken by in-service elementary teachers who seek to develop their understanding of and skills in mathematics while obtaining a graduate degree.

We cannot expect elementary school teachers to master and maintain new knowledge and skills and enhance their current teaching practices without support from their local or regional colleges and universities. In addition to providing structured, ongoing professional development beyond school hours, school systems and professional mathematicians must find ways to give teachers more time to learn more mathematics during the school week. Teachers need time to collaborate with colleagues and supportive mathematicians, time to examine reform curricula based on *Principles and Standards*, and time to incorporate new mathematics content and teaching strategies into existing or new curricula. Supportive mathematicians need opportunities to become better acquainted with the content and goals of school mathematics. Both mathematicians and teachers deserve opportunities to develop, analyze, master, and reflect on the content of school mathematics and new teaching approaches that increase the likelihood that every child will succeed.

NCTM has seen great success with its Academy for Professional Development, which consists of either two-day or five-day institutes focused on helping teachers put *Principles and Standards* into practice. There is certainly a role for organizations like the AMS to play in expanding the professional development opportunities of mathematics teachers in the United States and Canada via these institutes.

It is time to develop viable partnerships. Excellence in mathematics teaching and learning cannot wait. We must prepare knowledgeable students of mathematics. We must develop flexible and resourceful problem solvers for the future. We must eliminate classrooms in which only a few students gain access to the best mathematics teaching and learning we have to offer. We must staff our classrooms at every grade level with well-prepared, knowledgeable mathematics teachers. We can achieve all of this and more if we join our experiences and talents to make the vision of a high-quality mathematics education for all children a reality.

—Lee V. Stiff, Immediate Past President
National Council of Teachers of Mathematics

Letters to the Editor

Ramanujan's Partition Congruences

I feel that in the article "Addition and counting: The arithmetic of partitions" [October 2001], the authors have not done justice to the *elementary* work done in connection with Ramanujan's partition congruences.

(1) The authors have:

If $\delta = 5^a 7^b 11^c$ and $24\lambda \equiv 1 \pmod{\delta}$, then $p(\delta n + \lambda) \equiv 0 \pmod{5^a 7^{b'} 11^c}$ with $b' = \left[\frac{b}{2}\right] + 1$. This is incorrect for $b = 0$, when the right value of b' is 0, not 1.

(2) In 1968 Winquist was the first to find an elementary proof of the mod 11 congruence.

(3) Ramanujan (essentially) proved the identity, described by Hardy as Ramanujan's most beautiful,

$$\sum_{n\geq 0} p(5n+4)q^n = 5\prod_{n\geq 1}\frac{(1-q^{5n})^5}{(1-q^n)^6},$$

from which he deduced the congruence modulo 25 and stated without proof the identity

$$\sum_{n\geq 0} p(7n+5)q^n = 7\prod_{n\geq 1}\frac{(1-q^{7n})^3}{(1-q^n)^4} + 49q\prod_{n\geq 1}\frac{(1-q^{7n})^7}{(1-q^n)^8},$$

from which he deduced the congruence modulo 49.

Each of these is the first of an infinite sequence of such identities.

(4) In connection with Dyson's posited crank, the Andrews–Garvan crank works not only for 11 but remarkably also for 5 and 7.

(5) Finally, Rhiannon Weaver has recently given a large number (76065) of new congruences for $p(n)$.

—Michael D. Hirschhorn
University of New South Wales

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Response to Hirschhorn

The authors are grateful to Michael Hirschhorn for pointing out the typographical error which appears in their article. Regarding the remaining comments, they agree that the elementary results which he describes are important in partition theory. However, the purpose of this article was to describe to a general mathematical audience some of the important advances in partition theory over the past century and to highlight some of its connections to other areas of mathematics. It was of course impossible for us to list all of the relevant works in a subject which is hundreds of years old.

—Scott Ahlgren
University of Illinois, Urbana-Champaign

Ken Ono
University of Wisconsin, Madison

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Hypergeometric Identities

In Victor Moll's article "The evaluation of integrals: A personal story" [March 2002], one doesn't need the force of the Wilf–Zeilberger method! Identity (4) and the unnumbered identity for binomial coefficients on p. 313 are both straightforward applications of the (old!) Chu–Vandermonde theorem for a terminating ${}_2F_1$ with base 1. (See Andrews/Askey/Roy, *Special Functions.*)

The moral is that if one always uses the standard notation for hypergeometric sums, then one is likely to recognise known results.

As for the integral Moll considers, it is easy to prove **by induction** on m that

$$\int_0^\infty \frac{1}{(x^4+2ax^2+1)^{m+1}}\,dx = \frac{\pi}{2^{3m+\frac{3}{2}}}\binom{2m}{m}\frac{1}{(a+1)^{m+\frac{1}{2}}} \times {}_2F_1\left(\begin{matrix}-m, & m+1\\ -m+\frac{1}{2} & \end{matrix}; \frac{a+1}{2}\right).$$

—Michael D. Hirschhorn
University of New South Wales

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Public Repositories of Mathematical Articles

I would like to enthusiastically add my voice to the "Opinion" expressed by Krzysztof Burdzy in the May 2002 *Notices*. Free or minimal-cost repositories of mathematical papers would be of great benefit to mathematicians, educators (especially public school teachers), underprivileged persons with an interest in or curiosity about mathematics, and the public in general. The availability of such a repository or organized system of repositories could do much to bolster the general public perception of mathematicians and our work.

I am currently incarcerated, which makes access to mathematics journals next to impossible. While I am likely the only incarcerated mathematician in this country, I believe I represent a larger class: mathematicians or people who use mathematics who do not have ready access to a university library or other rich sources of material. To us, public article repositories would be, to use Burdzy's metaphor, a very welcome and refreshing drink of tap water.

—Calvin A. Curtindolph
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