The Colossal Book of Mathematics

Reviewed by Ed Pegg Jr.

The Colossal Book of Mathematics
Martin Gardner
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> Flexagons are paper polygons, folded from straight or crooked strips of paper, which have the fascinating property of changing their faces when they are "flexed." Had it not been for the trivial circumstance that British and American notebook paper are not the same size, flexagons might still be undiscovered, and a number of top-flight mathematicians would have been denied the pleasure of analyzing their curious structures. (Martin Gardner, December 1956)

My own introduction to Martin Gardner occurred while I recuperated from being reckless as a boy. Perhaps hoping I could be a bit more intelligent, my father gave me a copy of one of Gardner's *Mathematical Games* books at the hospital. After recovering, I learned to use my school's microfiche for issues of *Scientific American*. Through Gardner's columns in that magazine, I learned of topology, hexaflexagons, sprouts, and graph theory. Above, Gardner starts an article recounting the history of the "Flexagon Committee". This group of four Princeton University students investigated flexagons and discovered a wide variety of them.

> After my piece appeared, people all over Manhattan were flexing flexagons. Gerry Piel, publisher of *Scientific American*, called me into his office to ask if there was enough similar material to make a regular column. I assured him there was and immediately made the rounds of Manhattan's used book stores to buy as many books on recreational math (there were not many) as I could find. Once the column got under way, I began to receive fresh ideas from mathematicians and writing the column became an easy and enjoyable task that lasted more than a quarter century.

Gardner's writings became the standard for popular mathematics. As I was writing this review, author Clifford Pickover emailed me about his next book: "I'm digging up the Gardner references." Written well and well researched, Gardner's columns discussed mathematicians, problems, or research areas that often attained greatness. Members of the Flexagon Committee provide a typical example: Bryant Tuckerman found the twenty-fourth Mersenne prime (1971), John Tukey developed fast Fourier transforms (1965), and Richard Feynman won the Nobel Prize (1965).

The Colossal Book of Mathematics is Gardner's own compilation of what he considers to be his best columns. And this is not just some "greatest hits" collection put out by the studio—Gardner rewrote

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and remastered almost every column in the book. He adds, updates, and appends quite a lot. As I read through the work, I marvelled at the *vast* amount I'd missed through the years. Somehow the same columns that entertained me as a boy had hidden material that appealed to me as a mathematician. His work has been so much admired among mathematicians that he was awarded the AMS Steele Prize for Mathematical Exposition in 1987.

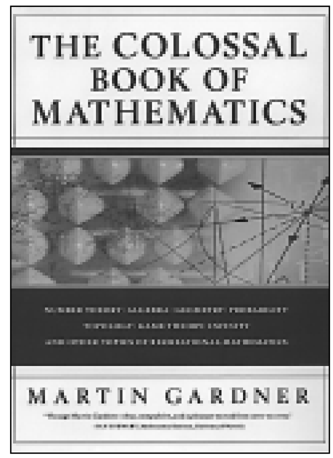
> Spheres of identical size can be piled and packed together in many different ways, some of which have fascinating recreational features. These features can be understood without models, but if the reader can obtain a supply of 30 or more spheres, he will find them an excellent aid to understanding. Ping-pong balls are perhaps best for this purpose. They can be coated with rubber cement, allowed to dry, then stuck together to make rigid models.

The chapter "Packing Spheres" opens with a friendly invitation. Many different facts follow about such interesting objects as figurate numbers, triangular numbers and squares with their algebraic interactions, tetrahedral pyramids, a sphere packing with density $\pi/\sqrt{18}$, random packings, densest packings, and loosest packings.

> In 1727 the English physiologist Stephen Hales wrote in his book *Vegetable Staticks* that he had poured some fresh peas into a pot, compressed them, and had obtained "pretty regular dodecahedrons." The experiment became known as the "peas of Buffon" (because the Comte de Buffon later wrote about a similar experiment), and most biologists accepted it without question until Edwin B. Matzke, a biologist at Columbia University, repeated the experiment. Because of the irregular sizes and shapes of peas, their nonuniform consistency and the random packing that results when peas are poured into a container, the shapes of peas after compression are too random to be identifiable.

I recently wound up doing some research on this very topic. Of course I started with Gardner's column. I later discovered that Kepler reported rhombic dodecahedral structures in a study of pomegranates. Kepler may have been wrong, but he was wrong first. Gardner ends his updated addendum with this aside: "Stanislaw Ulam told me in 1972 that he suspected that spheres, in their densest packing, allow more empty space than the densest packing of any identical convex solids." This is not true in two dimensions. Same-sized circles can cover the plane with density 0.90689, while an octagon covering has maximal density 0.90616. In 1934 Reinhardt constructed a smoothed octagon with maximal density 0.902414. (The worst packing convex shape in two dimensions is unknown.) At this time there is no known counterexample to Ulam's hypothesis, though Wlodek Kuperberg suspects that the rotation of a smoothed octagon along a diagonal axis is a good solid to check.

Gardner's "Mathematical Games" columns now make up fifteen volumes, and all of them are currently in print. A sixteenth book, *Gardner's Workout: Training the Mind and Entertaining the Spirit* (A K Peters, 2001), contains additional writings about mathematical recreations material from the period 1981 to 2001. Again, Gardner's writing and research are top notch and very light and enjoyable to read. A sampling of chapter topics includes magic squares, the minimal surface to make a cube opaque, the square root of 2, minimal Steiner trees, variations on the 12345679 trick, toroidal currency, three-point tilings, serial isogons, and new new math.



Every two years there is a Gathering for Gardner conference (`http://g4g4.com`). There, magicians and mathematicians take turns entertaining each other with tricks and mathematical fun. There are now two books of conference proceedings from these gatherings, both published by A K Peters: *The Mathemagician and Pied Puzzler: A Collection in Tribute to Martin Gardner* (1999) and *Puzzlers' Tribute: A Feast for the Mind* (2001). Among the many contributors to the latter volume are Elwyn Berlekamp, John Conway, Solomon Golomb, Scott Kim, Roger Penrose, Raymond Smullyan, and Martin Gardner.

Here are three updates Gardner might have made to *Colossal* had he started putting it together just a few months later:

1. In 2001 David Wilson found that the second player wins 3×5 Dots and Boxes. David: "If computers continue to double their capacity every 18 months, we should be able to analyze the entire 5×5 game in 2034." See `http://www.cae.wisc.edu/~dwilson/boxes/`.

2. Conway's Game of Life is still engendering discoveries. Between August 1989 and November 2000, spaceships with speeds of $c/3, c/12, 2c/5, c/5, 2c/7$, and $c/6$ were discovered. See `http://www.argentum.freeserve.co.uk/lex_s.htm` or `http://www.mirwoj.opus.chelm.pl/ca/index.html` for a program.

3. In 2001 Robertson, Sanders, Seymour, and Thomas proved the Snark Theorem: *Every Snark*

has a Petersen graph minor. Some history is in order. A four-coloring of a trivalent map is equivalent to a three-coloring of its edges (Tait's reduction). All bridgeless planar trivalent graphs can be three colored (by the four-color theorem), but rare nonplanar trivalent graphs cannot be. In his April 1976 column Gardner noted how difficult these graphs were to find and called them Snarks. The name stuck. In 1999 Robertson, Sanders, Seymour, and Thomas found a shorter proof of the four-color theorem, but this was just a warm-up exercise. In 2001 they proved what was known as Tutte's conjecture. It's now the Snark Theorem.

I spotted one flaw in typesetting: The book is arranged in twelve sections, each with three to six columns. The pictures heading the various sections, for example Topology and Probability, are swapped around. But this is a minor detail.

Anyone who has read Gardner's columns will enjoy seeing the many updates added to this book. Any math lovers who have not read Gardner's columns, should. The best way they can start is by reading this book.