VIGRE Turns Three

Rick Durrett

Three years ago the National Science Foundation (NSF) embarked on an ambitious plan to change how the United States trains its mathematicians and statisticians. VIGRE (short for Vertical Integration of Research and Education in the Mathematical Sciences, and pronounced "vigor") is a program of grants to mathematics departments, its goal being to increase the number of well-prepared U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences. In four years of competitions, thirty-five awards have been made ranging in size from $400,000 to $1 million per year. The VIGRE program thus represents a sizeable (and growing) part of the budget of the NSF's Division of Mathematical Sciences (DMS).

In order to "share information on the success and challenges of the VIGRE initiative and plans for its future", a workshop was held May 3–4, 2002, in Reston, Virginia. The program was organized by Richard Brualdi (University of Wisconsin), Robert Greene (University of California, Los Angeles), and Gary Mullen (Pennsylvania State University). This meeting was sponsored by four professional societies (AMS, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics) whose representatives took care of the meeting details. The meeting attracted representatives from sixty to seventy mathematics and statistics departments, many of whom received partial support of their local expenses from an NSF grant. A full report, being prepared by the workshop presenters and to be posted on the Web, will contain a complete account of the presentations and the discussions in the breakout sessions. Our brief account here will focus on the main issues raised at the meeting as seen through the eyes of the coordinator of the Cornell VIGRE grant.

The meeting began early on Friday afternoon with an overview of the goals and implementation of the VIGRE program given by Henry Warchall, a DMS program officer who is on the VIGRE management team. As Warchall explained, VIGRE was designed to fix a badly leaky educational pipeline. From 1993 to 1999, the most recent years for which there is data, the number of U.S. mathematics majors dropped by 23 percent and the number of U.S. students in Ph.D. programs dropped by 27 percent. In 1998, a blue-ribbon panel chaired by retired General William Odom concluded that the talent drain was threatening not only the nation's leading position in mathematics worldwide but also its ability to innovate in related disciplines.

The recommendations of the Odom report[1] led to the VIGRE program, whose goals are to make the study of mathematical sciences more attractive for undergraduate and graduate students and to render undergraduate, graduate, and postdoctoral training more effective and broadly applicable. VIGRE provides funds to Ph.D.-granting departments for undergraduate research stipends, graduate research traineeships, and enhanced postdoctoral research opportunities. Institutions receiving VIGRE funds are required to carry out educational activities that result in: (1) integration

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[1]*Report of the Senior Assessment Panel of the International Assessment of the U.S. Mathematical Sciences, NSF publication 98-95.*

of research with education at all levels, (2) enhanced interaction among students, postdoctoral fellows, and faculty members, (3) broad educational experiences that prepare students and postdocs for a wide range of career opportunities, (4) close mentoring of students and postdoctoral fellows, and (5) improvement of communication skills and teaching skills of students and postdoctoral fellows.

The second session in the meeting was a panel discussion on "Exemplary Practices". Richard Hain offered his thoughts on mentoring based on his experiences at Duke University. "Undergraduate research provides an ideal vehicle for mentoring, as well as an effective way of getting students interested in continuing their educational studies." Duke tries to recruit undergraduates into the Research Experiences for Undergraduates (REU) program at the beginning of the junior year. A mentor can then direct students towards courses that will help provide background before the actual research is undertaken in the summer between the junior and senior years. During the senior year, students typically continue their projects through reading courses and finish with a paper or senior thesis.

"In mentoring graduate students, it is important to keep the overall goals in mind—decreasing the time to degree, improving retention rates, broadening education, and improving teaching and communication skills," Hain said. Many mentoring roles have traditionally been filled by the thesis advisor. However, many places, including Duke, are finding it very useful to assign to incoming students mentors who can help guide the students through the oral or written exams that come early in the program and through the process of finding a thesis adviser.

The mentoring of postdocs in VIGRE programs is designed to prepare postdocs to take on and balance a wide spectrum of activities: performing research, finding new problems and directions, writing proposals, teaching introductory and advanced courses, and serving their departments and the mathematics community as a whole (e.g., reviewing papers and proposals). In the past many postdocs learned these skills through an association with a senior faculty member with related research interests. The VIGRE program has led in most cases to explicitly assigned mentors. At the University of California at Los Angeles (UCLA), for example, postdocs have one mentor for teaching and one for research. In addition to one-on-one mentoring, a number of programs have grant-writing workshops for postdocs and graduate students.

James Meiss from the University of Colorado, Boulder, discussed broadening the education of students in the mathematical sciences. The VIGRE program in the Applied Mathematics Department there is organized around "tetrahedra". Its Web page, which has a rotating labeled tetrahedron, states, "the vertices represent faculty, postdoctoral fellows, graduate students, and undergraduates, while the faces represent the modes of interaction among the members: teaching, researching, learning, communicating."

There are four tetrahedra that parallel the concentration areas of the department: nonlinear waves, dynamical systems, multigrid computation, and fast algorithms. Each group fosters collaborative research, runs a seminar series to encourage interaction and discussion at multiple levels, and disseminates research ideas into course projects. Testimonials on the Colorado Web page indicate that undergraduate research experiences have changed students' career plans to include mathematical research and that postdocs have benefited not only through faculty interaction with their research but also through the supervision of undergraduate projects.

The research program is only one of a number of ways that students in Boulder have experiences beyond those traditional in mathematics education. Internships with nearby national laboratories or high-technology companies are another broadening activity for students at all levels. Sophomore classes now require a series of projects that demand group interaction, often involve modeling or experimentation, and lead to written reports. Each undergraduate is required to have a minor in an area with significant mathematical applications. A graduate student must complete a yearlong sequence of courses at the graduate level in some area of application. In many cases this has led to graduate students doing research with an advisor from another department.

At the University of Arizona, as Michael Tabor described, a special feature of graduate training is a system of Research Tutorial Groups (RTGs) for first-year students. It is designed to facilitate the transition between consumer (of coursework) to producer (of research). In the fall semester, students are exposed to a variety of topics through blocks of seminars (essentially a hybrid between a "case study" and a minicourse). In the spring semester the students choose to pursue one of these topics with the faculty involved. The semester ends with all the students presenting their findings at a small research conference. The RTGs lead to vertical integration when research projects are co-advised by postdocs and advanced graduate students. Another benefit Tabor points out is that "better mentoring, and monitoring, of students come about through meaningful activities in which the students and their mentors are intellectually engaged, such as the RTGs."

A distinctive feature of the Arizona VIGRE program is competitive proposal writing by graduate students seeking to continue VIGRE support beyond

their first year. Students must submit a proposal laying out a clear plan of study, research, and vertical integration activities. A first draft is reviewed by a faculty committee, and an assigned faculty member discusses the proposal with the student, who then resubmits a revised proposal for final consideration.

The VIGRE activities at Duke, Colorado, and Arizona, while exemplary, do not begin to exhaust the innovative ideas that have been developed in response to the VIGRE programs. Many VIGRE programs have extensive Web pages to solicit applications for postdocs and graduate fellowships and to advertise their successes. These Web pages can be accessed from `http://www.vigre.org/`, which is maintained as a community service by the mathematics department at Penn State. A survey of those descriptions shows that, like ice skaters at the Olympics, universities implement the required elements of the VIGRE program (postdoctoral positions, graduate fellowships, and undergraduate research) in ways that showcase their particular strengths.

UCLA builds on its already close cooperation between pure and applied mathematics and the new national Institute for Pure and Applied Mathematics to achieve vertical integration and broadening through research groups in both traditional and emerging areas and through strengthening its ongoing internships with industry and government laboratories. UCLA also provides an example of VIGRE revitalizing existing parts of the curriculum. For a long time, UCLA has had "graduate participating seminars" that involve graduate students, postdocs, and permanent faculty and that can last one term or more or can continue indefinitely. Before VIGRE there were an average of twelve of these involving a total of thirteen students. In spring 2002 there were twenty-one involving sixty-one students.

At the University of Washington in Seattle, a VIGRE award shared by the Departments of Applied Mathematics, Mathematics, and Statistics seeks to extend the collaborations that have developed due to the recently implemented Bachelor of Science Program in Applied and Computational Mathematical Sciences.

At Penn State, the VIGRE grant supports the already successful MASS (Mathematics Advanced Study Semesters) program for undergraduates, which each year brings undergraduates from around the country to spend a semester at Penn State, where they are literally immersed into the study of mathematics. They take three core mathematics courses, do individual research projects ranging from theoretical mathematics to computer implementation, and participate each week in a two-hour interdisciplinary seminar and a one-hour colloquium.

A number of universities have implemented seminars and working groups in response to the VIGRE program. At the University of Georgia, five research groups in different areas of mathematics (prime number races, ropelength of knots, wavelets and applications, time scales, and flag varieties) are the main vehicle for vertical integration. At the University of Michigan, there is a VIGRE seminar aimed at advanced undergraduates and graduate students with talks related to diverse applications of mathematics and career possibilities in the mathematical sciences. Open problem seminars are designed to help postdocs and advanced graduate students to develop their own independent research agendas. Two "Pro-Seminars", one for first-year graduate students and one for postdocs, help incoming graduate students learn the ropes of graduate school and help young faculty to develop professional "street smarts" about submitting papers for publication and applying for grants.

At the University of Illinois at Urbana-Champaign, two types of peer groups have been formed to carry out mentoring and interpersonal, professional support. Across Level Peers (ALPs) brings together undergraduate students, graduate students, and faculty of all types in order to pursue wide-ranging agendas focused on topics in mathematics, both in teaching and research. Research Among Peers (RAPs), which includes graduate students, postdoctoral associates, and other faculty, focuses on specific research interests and is a source for forming research ideas, studying various topics, and presenting completed work.

Our final examples concern the optional component of VIGRE grants (outreach to grades K–12) and the mandatory one that comes with no funding attached: curriculum review. At Cornell, the benefits of our VIGRE grant are felt by local high-school students through the Math Explorer's Club, organized by Bob Strichartz, who has for a number of years run a successful REU program. The club has four six-week sessions during the school year, Saturdays from 10:30 a.m. to 1:00 p.m. In the first one-hour period students are presented with a mathematical idea, problem, or exploration as part of a six-week module. This is followed by a half-hour break at which refreshments are served. The final hour is devoted to problem-solving or computer laboratory activities.

At the University of Chicago, VIGRE funds help support four outreach programs. The Young Scholar's Program brings large numbers of students in grades 7–12 to the University of Chicago for a summer mathematics enrichment program. SESAME (Seminars for Elementary Specialists And Mathematics Educators) is a three-year, 270-hour program for elementary-school teachers in the Chicago Public Schools. Around twenty high-school teachers attend a seminar program in algebra and

Departments Awarded VIGRE Grants

1999
Carnegie Mellon University*
Columbia University
Harvard University
North Carolina State University
Pennsylvania State University
Princeton University
Rutgers University*
U. of Arizona
U. of California, Berkeley, Mathematics Dept.*
U. of Colorado, Boulder
U. of Michigan, Ann Arbor
U. of Washington, Seattle
U. of Wisconsin, Madison

* = terminated after three years

2000
Brown University
Cornell University
Duke University
New York University
Purdue University
Rensselaer Polytechnic Institute
SUNY Stony Brook
Texas A&M University
U. of California, Los Angeles
U. of Chicago
U. of Illinois, Chicago
U. of Illinois, Urbana-Champaign
Yale University

2001
Indiana University
Iowa State University
U. of Georgia
U. of Texas, Austin
U. of Utah

2002
Georgia Institute of Technology
Ohio State University
U. of California, Davis
U. of California, Berkeley, Statistics Dept.

geometry. The Summer Seminar in Calculus, which involves twenty high-school teachers and twenty high-school students, focuses on the calculus and its applications.

To quote the current solicitation for proposals (NSF 02-120), "a prerequisite to a VIGRE proposal is a thorough review of existing undergraduate and graduate curricula, with attention to how well the curricula prepare students for the diverse career opportunities now available to mathematical scientists." An example of the substantial changes that can occur is the program at Columbia University, where the core curriculum has been restructured to provide all mathematics graduate students—independently of their ultimate specializations—with a broad, yet tightly interwoven, introduction to the basic themes and techniques of modern mathematics. The material is covered in a series of six one-year courses: modern geometry, complex analysis, analysis and probability, groups and representations, algebraic topology, and commutative algebra/algebraic number theory and algebraic geometry. All first-year students are required to take three of the six courses and pass qualifying exams on them. Second-year students are strongly encouraged to attend the courses they did not take the first year.

As the many examples cited above should indicate, the VIGRE program has in its three years of operation inspired a tremendous amount of innovation in mathematics education and research at all levels. The more than 100 postdocs offered each year under VIGRE grants have turned the buyer's market seen in the mid-1990s into a seller's market where well-trained Ph.D.'s in mathematics who are U.S. citizens (or nationals or permanent residents) can expect to receive several job offers. The presentations of the three postdocs at the Reston meeting gave a more personal view of the benefits of the VIGRE postdoc program. It is unlikely that these three postdocs constitute a random sample because they come from the same institutions as the meeting organizers. However, their stories show that (i) VIGRE postdocs derive considerable benefits from the mentoring, reduced teaching loads, and travel money associated with their positions, and (ii) the citizenship requirements of the VIGRE program do not condemn us to hiring second-rate talent.

Jon Jacobsen is an example of a person whose career received a significant boost from his VIGRE postdoc. With the goal of being a high-school mathematics teacher, he entered the mathematics program at California Polytechnic State University at San Luis Obispo in 1988. During his early field experience in his junior year he found out that teaching was not for him, so he stayed and received a Master's degree, and later went to graduate school at the University of Utah. When he graduated in 1999, he had two options: a job at a small liberal arts school in Wisconsin and a VIGRE postdoc at Penn State. He took the VIGRE postdoc, and, after three years of interacting with researchers in the William G. Pritchard Fluid Mechanics Laboratory and having the experience of developing a new course to introduce the art of mathematical modeling through experiment, his second job search was a much different experience. Twenty-five applications (roughly the same number as in 1999) resulted in fifteen interviews at the AMS annual

meeting, several campus visits, and finally a tenure-track job at Harvey Mudd College.

Skip Garibaldi, a VIGRE postdoc at UCLA, knows first hand the difference between an ordinary postdoc and a VIGRE postdoc, since he had the former for one year and then the latter for two years. Two of his comments exemplify the difference: About the ordinary postdoc he said, "I found that simultaneously teaching two unrelated courses left me little time for research," and about the VIGRE postdoc he said, "the $2,500 for travel gave me the freedom to attend many more conferences than I would have otherwise." The "graduate participating seminars" mentioned above were an important part of Garibaldi's VIGRE experience. The visit of Jean-Pierre Serre to UCLA in the winter of 2001 led to a three-way collaboration between Garibaldi, his research adviser Alexander Merkurjev, and Serre. They wrote a book to be published by the AMS. Garibaldi quipped in the recent *Science* article on the VIGRE meeting (see pages 1389–90 in the May 24, 2002 issue): "You can't get much above Serre, and you can't get much below me, so that's an example of vertical integration."

Dan Knopf was a VIGRE postdoc at University of Wisconsin, Madison (UW), for 1999–2002. He too says that he benefited from the reduced teaching load, travel funds, mentoring from senior faculty, and the department's VIGRE seminars. What is unique about his experience is that he designed and implemented the undergraduate research component of the UW VIGRE program: the UW Minimal Surfaces Lab. Readers can find more about this effort on the UW VIGRE Web page. This experience taught Knopf lessons that many of us have wrestled with in other contexts: "There is a delicate balance between the need to teach the students enough background, and the goal of giving them an opportunity for independent investigation. Too much structure risks turning the experience into a course; too little risks leaving the students floundering." Beginning in the fall of 2002, Dan Knopf will be at the University of Iowa.

VIGRE fellowships provide support for a large number of graduate students. Under most grants, students get two years of support in the first two or three years of graduate school when they are hard at work on their basic courses and later trying to find a thesis direction. At the undergraduate level, the additional 600 students supported by VIGRE have nearly tripled the number previously supported by the NSF; 300 are supported through REUs and sixty through individual grants. Statistics are not yet available to document the effect of the VIGRE program on the pipeline, but individual reports indicate significant increases both in the number of mathematics majors and the number of undergraduates continuing to graduate school to study mathematics.

Despite the impressive successes of the VIGRE program, not all members of the mathematics community agree with how it is being implemented. In a Saturday afternoon panel session, Benedict Gross, the VIGRE director at Harvard, which successfully completed its third year review, quipped: "Like the pill whose name it resembles, the purpose of the VIGRE grant is varied. Is it intended to increase the population, to restore strained relationships, or just to have more fun? In some cases it does have a side effect of raising blood pressure as departments try to decipher ambiguous signals from the NSF." One of the reasons for Gross's frustration can be seen in the response to Harvard's preceptor program, a new postdoctoral program for mathematicians especially interested in undergraduate teaching and advising developed in response to the VIGRE program. At the meeting he explained that the VIGRE site visit team thought that the preceptor program was a great idea, while an external committee reviewing his department had exactly the opposite opinion.

Calvin Moore is chair of the mathematics department at the University of California, Berkeley, whose VIGRE grant was terminated after three years. He also spoke in the panel discussion and has written an extensive analysis of the VIGRE program, which is a component of the workshop report. Here we present excerpts beginning with his comments on the Berkeley grant in particular and proceeding to the VIGRE program as a whole. Remarks within quotes are Moore's words. I have written the unquoted transitional material.

"It is clear that attempts to increase the number of students studying mathematics have to start at least at the undergraduate level, particularly in attracting students to major in mathematics. One of the small number of problems that I decided to focus on as chair was that of increasing the number of undergraduate majors in mathematics. Four years ago we had 167 majors (this counts only juniors and seniors), and as of this week (June 11, 2002) we had 556 declared junior and senior level majors."

As Moore explains in his write-up, this impressive growth in the number of majors came from improving the advising system, initiating systematic outreach activities, and encouraging double majors. "The introduction of our undergraduate research seminars, which was spurred by VIGRE, provided a significant incentive, especially to the very best students, to become math majors. In addition to providing stipends for students, we were able to use VIGRE funds to allow our majors to travel to conferences to present the results of their work both in our programs, and in other REU programs. We are only able to offer at present 15–20 percent of our majors enrollment in an undergraduate research seminar because of faculty resource

limitations. If VIGRE were to command that many or most majors have such an experience, we would not be able to comply, and it is not clear that every major would benefit from such an experience."

At the graduate level the Berkeley VIGRE grant also suffered because of the large size of the program. Moore described the situation as follows: "We have about $5M in all forms of financial support for our graduate students, and our VIGRE grant, which was defunded by NSF after three years, provided $250K, or about 5 percent of the total. This was an extraordinarily useful form of money, but ultimately its impact on the overall graduate program was not large. It provided fellowships for ten out of a total of about thirty first-year domestic graduate students, who would have otherwise been supported by a teaching appointment. This no doubt bothered the VIGRE management team that their money was not having a more visible impact."

When asked by the VIGRE site visit team, "What would produce a significant impact at Berkeley?", Moore's response was that "$1M in graduate fellowships would certainly have an impact—for instance, twenty first-year fellowships and twenty 'dissertation year' fellowships." In his essay he hastens to add: "But there are catches here—first of all, VIGRE would never give us this kind of money, and second of all, I would not want it. My problem with it would be that we have to do long-term planning of our graduate enrollments and that requires a reasonably steady base of resources. From recent experiences, we would face the prospect of having $1M pulled out of our graduate student support budget essentially on a moment's notice. That would leave us in an untenable position, and being so dependent on one source would open the whole program up to all manner of subtle and not so subtle influences from the funding agency as to program structure and educational philosophy."

Another issue surrounding graduate education, which has been raised throughout the country, is the restriction of VIGRE funds to U.S. citizens. In Moore's words, "The strongest and best-prepared graduate students who enter our program are either international students, or domestic students who come to us with NSF or other kinds of fellowships or are students whom we can propose in the campus-wide competition for university fellowships and who will be successful in that competition. The fact is, the students supported on VIGRE at Berkeley are generally, but not always, the students who are among the least well-prepared and the ones most at risk. Thus the VIGRE students are not an elite group of the best students, as the NSF had perhaps imagined they would be."

Finally on the subject of mentoring: "We arrange for several different kinds of mentoring of our graduate students—peer mentoring by more senior graduate students and mentoring by faculty advisors in the years before a student settles down with a dissertation supervisor. We also cultivate a degree of self-reliance in our students, a characteristic which we believe is essential for success as a mathematician. In the general spirit of our department which has few stated requirements, such mentoring is not required, but is recommended and is readily available. Students ultimately have to make a decision for themselves. As judged from the reviews of our VIGRE project, this kind of approach appears not to be the correct mold for VIGRE."

Turning to the VIGRE program as a whole, Moore has the following thoughts: "Let me conclude with some general observations flowing from these more specific comments. The VIGRE program, it seems to me, has created something of an adversarial relationship between the mathematics community and the Division of Mathematical Sciences at NSF. A small group of program officers in Washington have become, in effect, inspector generals of mathematics education and training. Is this the proper role for NSF to be playing?

"I take the fundamental goal of the VIGRE program to be the laudable one of increasing the number of students studying mathematics and pursuing careers based on this training. However, it seems that NSF has decided in advance how to achieve this goal and has constructed a program that does not give sufficient recognition to the diversity of departments and institutional goals, and the diversity of programs and of the students enrolled in them.

"A second and related problem is that VIGRE guidelines call for changes in departmental programs, even when there is no reason for change or when some significant changes have already been made. Finally, and most important, there has been an absence of any broad engagement by NSF with the mathematical sciences community in a discussion of the goals and implementation of the VIGRE program and of the resulting problems just described.

"One immediate step that should be taken by NSF is to conduct a thorough review of the VIGRE program. This review should engage the mathematical sciences community, should address questions about the proper role of NSF, issues of flexibility within general program goals, the balance of support for graduate students and postdoctoral fellows between VIGRE and other programs, and, most important, should address the mechanisms and process for review and evaluation of VIGRE projects. As the VIGRE program has stated goals concerning work-force issues, its effectiveness in advancing these goals should be assessed as part of the review."

In response to Calvin Moore's remarks and to this article in general, Philippe Tondeur, director of the Division of Mathematical Sciences[2], had the following to say: "The report of successful VIGRE activities at this meeting was extremely gratifying. The considerable space given above to expressions of dissatisfaction with the VIGRE activity seems out of proportion to the positive response from most participants in the meeting and in most parts of the mathematical sciences community. I would like to take this opportunity to review the context of this effort of the mathematical sciences community.

"VIGRE was designed to stimulate innovative educational projects that address declining enrollments in mathematics and statistics graduate programs. The idea was to give as much freedom as possible to the imagination of the proposing departments in formulating plans to address serious concerns about mathematical sciences training of students and postdoctoral fellows in the U.S. The long-range goal of the VIGRE activity is to increase the number of well-trained U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences, and to more systematically involve research mathematicians and statisticians in the training of the next generation of scientists. The restriction to funding U.S. citizens, nationals, and permanent residents in training programs such as IGERT (Integrative Graduate Education and Research Training), REU, NSF postdocs, NSF graduate fellowships, and VIGRE is an expression of NSF policy, as governed by the National Science Board. The evidence at hand so far is overwhelming that a significant proportion of the mathematics and statistics community is reassessing and improving its efforts to recruit, mentor, and retain students and postdocs. In recent years the career opportunities for mathematically trained scientists have become more diverse, and the mathematical sciences community has also increased activities to broaden mathematical education. The VIGRE activity in particular has attracted great interest both at the NSF leadership level and in the academic community in the U.S. as well as abroad.

"This initiative of DMS, developed with input from the mathematical sciences community, is one of the factors which have led NSF to declare the mathematical sciences a priority area. This has and will have profound implications on its current and future budgets. There is increasing recognition that the mathematical sciences are becoming crucial components of scientific training and research in many areas, and that in some cases they play a transformational role in the evolution of other science areas. DMS continues to expand its partnerships with other sciences at NSF, both in research and training. The educational broadening aspects of VIGRE form an integral part of this expansion. DMS has crafted programs that directly respond to the needs and interests of the mathematical sciences community in all its aspects, namely fundamental mathematical sciences advances, connections to the sciences and engineering, and the educational and training aspects of our professional activities. Manifestations of this include a significant growth in all core programs, an expanding Focused Research Group activity, and the funding of three new mathematical sciences institutes. It has been exhilarating to be able to serve the mathematical sciences in a period of such heightened attention to its fundamental pursuits."

[2]*Philippe Tondeur ended his term as director on July 31, 2002. His successor is William Rundell of Texas A&M University.*