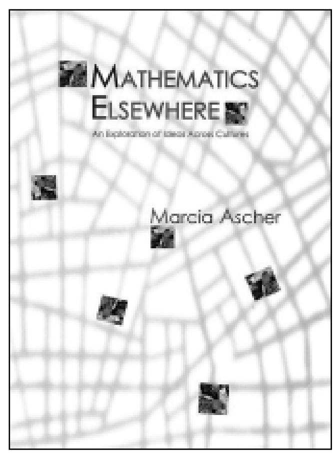
Mathematics Elsewhere: An Exploration of Ideas Across Cultures

Reviewed by Victor J. Katz

Mathematics Elsewhere: An Exploration of Ideas Across Cultures
Marcia Ascher
Princeton University Press, 2002
224 pp., cloth, $24.95
ISBN 0-691-07020-2

Marcia Ascher's new book, *Mathematics Elsewhere*, is certain to stimulate discussion of the nature of mathematical ideas and how they find their expression. For many Westerners the term "mathematics" means the intellectual exercise begun by the Greeks of stating theorems and then proving them by beginning with explicitly stated axioms and using logical arguments. Of course, in ancient Greece the axioms were about either geometrical or arithmetical objects. Today the axioms can be about numerous other kinds of objects, but mathematicians are still proving theorems about objects, be they C^*-algebras or regular local rings. So to look at mathematics *elsewhere* might be interpreted to mean the finding of logical arguments from axioms about certain kinds of objects in cultures other than "Western".

Marcia Ascher, however, is not looking for mathematics in that sense. In fact, most of us use a working definition of mathematics much broader than the one implied above. It was "mathematics" when the Babylonians figured out how to solve what we know as quadratic equations. It was "mathematics" when Egyptian scribes determined the volume of the frustum of a pyramid. It was "mathematics" when the Chinese figured out how to solve simultaneous congruences by the Chinese remainder theorem. It was "mathematics" when Indian scholars solved the so-called Pell equation. It was "mathematics" when people from many cultures figured out that the sum of the squares on the legs of a right triangle equals the square on the hypotenuse, or that there was a regular method of determining the number of ways you could choose k objects out of a set of n. In none of these occurrences, at least originally, was there any notion of logical proof from explicit axioms.



These are examples of what Marcia Ascher calls "mathematical ideas"—ideas that she has sought in her research to find *elsewhere*. In particular, Ascher defines mathematical ideas to be those "involving number, logic, spatial configuration, and, more significant, the combination or organization of these into systems and structures." Today the contributions to mathematical ideas of the major literate cultures, including the Babylonian, Egyptian, Chinese, Indian, Islamic, and what we normally

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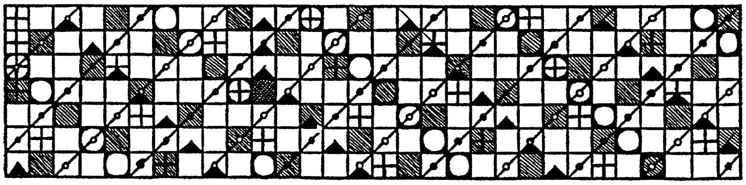
call "Western", are being studied in more and more detail by numerous scholars. Ascher takes a different path, exploring such ideas in "traditional" or "small-scale" cultures, generally nonliterate in our sense. In her book *Ethnomathematics: A Multicultural View of Mathematical Ideas* [Ascher, 1991], she explored mathematical ideas in certain cultures, and in the current book—really a sequel to the earlier one—she continues her study with other ideas in other cultures.

Ascher's works are examples of what has become known as ethnomathematics, a discipline that grew up beginning in the 1980s, although, of course, its roots go back much further. Initially, the impetus came from mathematics education in traditional societies. The colonial powers ruling many of these societies introduced Western mathematical education in the schools they established, frequently driving out indigenous educational practices. And even after the colonial powers departed, the educational systems frequently remained. But with independence, inhabitants of these countries began to realize that their own heritage was being ignored. All too frequently, the disconnect between the indigenous culture and the Western educational system was too broad for students to bridge. So projects began in some of these places to ferret out the mathematical ideas inherent in the societies, with the hope that these ideas would make mathematics, even "Western" mathematics, more palatable to students. Even in the West, students who traced their heritage to these traditional societies clamored for information about mathematics of their ancestors. Thus researchers began to search out this information. Among the first works of ethnomathematics was Claudia Zaslavsky's *Africa Counts: Number and Pattern in African Culture* [Zaslavsky, 1973], which grew out of the author's desire to satisfy the request of her African-American students in New York for information about mathematical ideas in Africa.

It can certainly be debated whether students learn mathematics better by knowing of the mathematical ideas of their own ancestors. Nevertheless, modern theories of education recognize that students need to construct their own knowledge based on what they already know. And if there are mathematical practices in the students' cultural heritage, it certainly behooves their teachers to be aware of them and to build on that knowledge rather than to tear it down. Thus, after the Brazilian mathematician and mathematics educator Ubiratan D'Ambrosio began to understand how attention to cultural background can improve mathematics education, he formulated his ethnomathematical program, introducing it publicly in a plenary address at the Fifth International Congress on Mathematical Education in Adelaide, Australia, in 1984 and elaborating on it in many writings thereafter [D'Ambrosio, 1985a, 1985b, 1986]. As he put it, not only are there mathematical ideas and practices in "small-scale" cultures, but certain ideas and practices are part of the culture of identifiable groups within Western societies, such as, for example, carpenters, well-diggers, musicians, and even engineers. Much of this more general ethnomathematics is not taught in schools but is certainly understood and used by practitioners. If one looks at the history of mathematics, it is probably true that much of "Western" mathematics originated in the ad hoc practices and solutions to problems developed by small groups in particular societies. So the theoretical questions posed by D'Ambrosio include: (1) how are these practices developed into methods? (2) how are the methods developed into theories? and (3) how are theories developed into scientific invention? Mainstream history of mathematics considers these questions for topics that have now become part of modern mathematics. However, many mathematical methods that arose in traditional societies never developed into theories but simply remained methods. Despite the lack of theoretical development, the methods work for the purposes for which they were intended and are therefore worthy of study. It is this study that Marcia Ascher has carried out in numerous instances in her earlier *Ethnomathematics* and in the book under review.

Marcia Ascher's own motivation for her studies of mathematics in "small-scale" societies was not primarily educational. She had a very traditional mathematics background but was married to an anthropologist. Wanting to embark on joint research, they decided to study the *quipus* of the Inca civilization in Peru. Quipus are knotted strings used by the Incas as a data collecting and recording device. The question Ascher wanted to answer was whether the ideas expressed on the quipus had mathematical significance. As her work and the parallel work of her husband on Inca culture progressed, she began to see that in fact many things could be "read" from these artifacts. But she also found that the artifacts alone were not enough to gain an understanding of the meaning of the quipus. As she wrote, "We were constantly amazed that the structural characteristics I was coming up with had resonance in other parts of the culture" [Ascher and D'Ambrosio, 1994, p. 36]. She came to believe that in other cases too the "implicit mathematics" in a society could be understood only by studying the ambient culture, not just the mathematical ideas or artifacts themselves. So, since her original study of the quipus [Ascher and Ascher, 1981], Ascher has studied the implicit mathematics and the cultures in which it is imbedded in societies around the globe.

In *Ethnomathematics* Ascher discussed number words and symbols in several societies, including the Inca. She looked at graph tracing among the Bushoong people of the Congo, the Tshokwe people of Angola,



Paper tika (Figure 3.6, page 78, *Mathematics Elsewhere*).

and the inhabitants of Malekula in the Republic of Vanuatu in the South Pacific. She considered kin relationships in Malekula and among the Warlpiri in Australia's Northern Territory, discussing how these people described these relationships logically. She wrote about games of chance in numerous societies, including a detailed study of the Maori game of *mu torere*. She looked at how various Native American groups described aspects of the space in which they lived. And she considered the symmetric strip decorations of both the Inca and the Maori, analyzing them through the seventeen-fold classification of those two-dimensional patterns.

In *Mathematics Elsewhere* Ascher discusses different mathematical concepts in other societies. In every case, the originators of the particular practice had to solve a certain problem. They did so by developing some mathematical idea. And often, as is so frequent in the history of Western mathematics, they did not stop at the answer to the problem they were solving. They followed the mathematical ideas in new directions, often led by aesthetic considerations. For each mathematical idea she discusses, Ascher describes the particular concept both in modern mathematical terms and, to the extent possible, in the terms understood by the culture itself. She also shows how the idea was imbedded in the ambient culture.

The book begins with a discussion of the logic of divination in three different cultures. Divination is "a decision-making process, utilizing, as part of the process, a randomizing mechanism. The decisions coming out of the process sometimes involve the determination of the cause of an event or, more often, how, when, or whether to carry out some future action" (p. 5). Divination has been a part of virtually every recorded culture, including modern Western ones. For Ascher the interesting part of the subject is the logic behind the randomizing mechanism, that is, the procedures by which a diviner puts together a particular arrangement from which he or she can make a decision.

The most fascinating divination system is that of *sikidy*, a system with a long history that is still used in Madagascar. In this system, the *ombiasy* (expert in sikidy) uses a randomizing procedure to lay out four columns of four entries each, where each entry contains either one or two dried seeds of a fano tree. This array is called the *mother-sikidy*. From the four columns, which we will call C_1 through C_4, and their associated rows, which we will call C_5 through C_8, Ascher shows how the ombiasy uses what we recognize as Boolean algebra to create eight more columns. (For the purposes of this algebra, we can think of one seed as representing "odd" or "1" and two seeds as representing "even" or "0".) For example, column 11, C_{11}, is created as $C_4 \oplus C_3$, where $\oplus$ represents the XOR (exclusive or) operation on the corresponding elements in each column, namely the operation that gives "even" when combining two elements of the same parity and "odd" when combining two elements of opposite parity. The final arrangement of the columns, together with additional manipulations of them, are the basis for the ombiasy's predictions or answers to his client's questions.

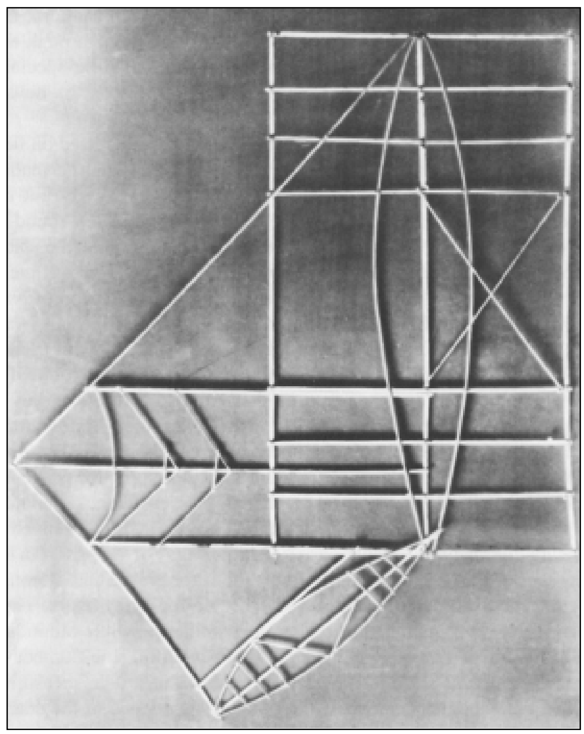
Of course, in performing his algorithm the ombiasy does not think he is doing Boolean algebra. But when he is done creating his 16 columns, he does several checks to see that he has not made any errors. In other words, he knows that certain relationships must be present in his final arrangement, assuming he carried out the algorithm correctly. For example, the final arrangement must always produce at least two identical columns, and Ascher gives a modern proof of this result. How did the originators of sikidy discover this result, and did they develop some sort of proof of it? Ascher does not address this question. But she does note the interest of some of the ombiasy in certain special final arrangements and therefore in determining what original layouts lead to these arrangements. That is, the ombiasy acted as "mathematicians".

Ascher next has two chapters dealing with time. The first shows how various peoples have organized the physical cycles of the day, the phases of the moon, and the yearly motion of the sun to create calendars, while the second highlights the use of more arbitrary cycles in marking time in the civilization. Although Ascher deals briefly with the luni-solar calendars of the Trobriand islanders and the Kodi people of the South Pacific, she gives the most detail on the development of the Jewish calendar. In particular, she discusses the rules for determining the first day, Tishri 1, of the Jewish year. In general, it is the day of the new moon of Tishri, with four exceptions, detailed in a diagram (which has an unfortunate typographical error). However, Ascher gives a reason for the first exception, that Tishri 1 cannot fall on a Wednesday, Friday, or Sunday. That would mean that either the Day of Atonement (Tishri 10) would fall on the day before or the day after the Sabbath—thus forcing a double Sabbath—or that Tishri 21 would fall on a Sabbath, forcing a contradiction to that day's observances, with activities forbidden on the Sabbath. The reasons for the other exceptions are that without them either the current year would be too long or the previous year would be too short, since a regular year must have either 353, 354, or 355 days, while a leap year must have 383, 384, or 385 days. Furthermore, all the exceptions together, along with the known inequalities in the Moon's motion, bring Tishri 1 and the first sighting of the crescent moon as close together as possible in a system based on the mean length of a lunation.

In her second chapter on time, Ascher deals primarily with the calendrical system of the Maya. This is based on a 260-day cycle, the Sacred Round, itself based on two independent 13- and 20-day cycles, and a 365-day cycle, the Vague Year, based on 18 months of 20 days each as well as 5 extra days at the end. These two cycles are joined in a larger cycle, the Calendar Round, equivalent to 52 Vague Years or 78 Sacred Rounds, as well as in a Great Cycle of 1,872,000 days, anchored at a fixed historical starting point. An important function of the Mayan priests was to do several types of elapsed time calculations. For example, they needed to find the number of days between two particular dates in the Calendar Round. Since the Calendar Round was made up of several cycles, this amounts to determining the solution of several simultaneous congruences, that is, solving a Chinese Remainder problem. We unfortunately have no records of the methods that the Mayan priests used to solve this problem. But we do know that they calculated correctly. Thus somehow they developed techniques for solving the Chinese Remainder problem. Whether their techniques resembled those originally used in China or those used today to solve such problems is unknown.

Another fascinating calendar is that of the island of Bali, in Indonesia. This calendar is based on 10 arbitrary cycles of 10 days, 9 days, 8 days, and so on, down to 1 day. A year in this calendar is 210 days, a number evenly divisible by all the cycle lengths except 4, 8, and 9. Special adjustments are made for those cycles so they fit reasonably into this calendar. As in the Mayan calendar, to do calculations in this calendar requires the solution of simultaneous congruences. In this case, we know that the Balinese use a wooden board known as the *tika*, on which an array of seven rows and thirty columns is carved or painted. Various symbols are placed in many of the boxes of the array representing important days in the calendar. Manipulations on the tika then enable one to find answers to typical calendrical questions. An unanswered question about the Balinese calendar is how the tika was originally constructed.

The most impressive chapter of *Mathematics Elsewhere* is the chapter on models and maps, which discusses the stick charts of the Marshall Islanders of the South Pacific. There are two types of these charts, made of palm ribs tied with coconut fibers, often with a few shells attached, and ranging in size from about 60 cm square to 120 cm square. The first type, known as *mattang*, was designed to introduce prospective Marshall Islands navigators to the interplay of oceanographic phenomena and land masses necessary for navigation through the islands. Thus, these charts represent wave fronts as they approach land and help navigators understand refraction, reflection, and diffraction of the wave fronts. After using the mattang under the guidance of an expert, an apprentice can move on to the second type of stick chart, the *rebbelith* and the *meddo*. These are maps of either the entire Marshall Islands archipelago or some smaller region within it, designed to help navigators wend their way among the islands through a knowledge of the typical wave patterns



Meddo (Figure 4.17, page 119, *Mathematics Elsewhere*).

in the ocean and certain physical features of the islands. Interestingly, these stick charts are all intended for study on land. A navigator does not carry one with him on his boat but relies on his memory. Ascher gives many details of both kinds of stick charts, with numerous examples, and shows how they fit into the culture of the Marshall Islanders. As before, we do not know exactly how the islanders organized their sea-faring knowledge into these charts, but we do know that they were able to create these models of a most important aspect of their culture.

Ascher deals with "systems of relationships" in her fifth chapter, considering these among the Basque people of Sainte-Engrâce, France, near the Spanish border; among the Tonga of Polynesia; and among the Borana of Ethiopia, near the Kenyan border. Her discussion revolves around the formal relationships that enable the people to understand their roles in the society. Ascher is careful to note that the people of these communities have "articulated the properties of the relations" in a way easily translatable into our formal terms. Thus she feels comfortable in making that translation, even though the people themselves did not do so. Nevertheless, certain formal consequences of the initial form of the relationships were discovered by the peoples involved, consequences that we can derive "logically". For example, the Borana historian who discussed the father-son classes in Borana society noted that the consecutive appearances of particular class names occurred in alternate son-to-father lines, while a modern proof of this result requires the algebra of congruences.

Finally, Ascher considers the *kolam* designs of southern India. The women of Tamil Nadu draw these designs each morning at the threshold of the house using a thin stream of rice flour. These designs, which serve both to welcome guests and to avert misfortunes and illness, are mathematically interesting because their creation involves transforming and superimposing several basic units. There are different types of designs, some for daily use and some for special occasions, but the tradition has been passed on from mother to daughter for centuries. Sometimes the kolam are constructed around and within an initial grid of dots, but others are made up of one or a few continuous curves. Throughout the centuries, the women of Tamil Nadu have elaborated these designs well beyond any practical necessity, evidently impressed by the beauty of their creations. Interestingly, the creation of these kolam designs has now become part of the literature in computer science, inspiring those working in formal language theory and with picture languages. Thus, the mathematical ideas inherent in this traditional culture have influenced the development of modern computer science theory.

In *Mathematics Elsewhere*, as in her earlier book, Marcia Ascher has made a significant contribution toward a global perspective on the history of mathematics. Her work has made it increasingly clear that mathematization—that is, the combination and organization of ideas about number, logic, and space into systems and structures—is not the province of a limited number of cultures but has in fact been accomplished by numerous cultures around the world. People have always "mathematized" and have even acted as "mathematicians" by following their ideas well beyond the solution of their initial problems. Anyone who enjoys this process of mathematization will enjoy seeing how it was accomplished in the diverse societies about which Ascher writes, and those who teach mathematics will be tempted to use her examples in relevant classes. These examples will help us demonstrate to our students the universality of mathematical thinking. And since our students, both at the undergraduate and graduate levels, are increasingly coming from countries outside the West, Ascher's ideas will prove useful in helping us meet their needs as we teach them the modern mathematics so important to understanding our world.

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