

Postmodern at an Early Age

Reviewed by Michael Harris

Mathematics and the Roots of Postmodern Thought

Vladimir Tasić

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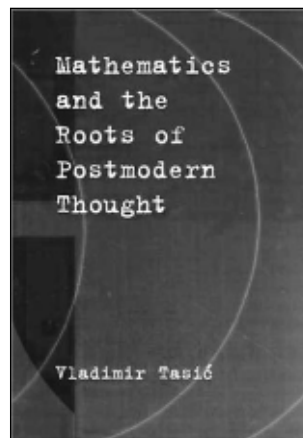
Chairs, Tables, Beer Mugs

Three number theorists—one American, one French, one German—were sitting in a restaurant, and after a few beers they began to debate the ontological status of the continuum. The Frenchman (who was actually drinking wine, not beer) opted for the radical position that the real numbers do not exist, though he confessed that this belief had never stopped him from using them when convenient. The German allowed that real numbers might not exist individually, but claimed that the set of real numbers nevertheless existed as a “totality”—for emphasis he also used the German word *Gesamtheit*. The American asserted that real numbers not only exist but that he could and often did compute them on his computer to as many digits as you like.

To illustrate his thesis, the Frenchman proposed a Diophantine variant of Gregory Chaitin’s “maximally unknowable” real number Ω , the probability that a randomly generated program will halt on a universal Turing machine. The American cheerfully replied that this was not the kind of real number he had in mind.

The punch lines, if any, will be a long time coming. The three mathematicians, all distinguished

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specialists, actually held this conversation in my presence during a conference in Münster. My sole contribution¹ was to ask whether the “existence” of which they spoke had anything to do with the axioms of set theory, axioms which all three participants claimed not to know. What the incident seems to demonstrate is that, under

the right circumstances, long dormant memories of the “crisis of foundations”, which agitated mathematicians during the period roughly bounded by Russell’s paradox (1901) and Gödel’s theorem (1931), can resurface to surprising effect. The right circumstances must be relatively rare: two of the participants, colleagues in close proximity for the better part of two decades, had never before discussed ontology. But the claim by Bourbaki co-founder Jean Dieudonné that “the young mathematicians said to be ‘formalist’ no longer even know that there had once been a “crisis of foundations”² is patently false, as Dieudonné himself would have realized if he had given his students

¹ That evening I was mainly engrossed in another discussion, concerning a controversial program to prove a famous conjecture in number theory using nonstandard analysis, among other ingredients.

² Pour l’honneur de l’esprit humain, Hachette, 1987, p. 246; published in English as *Mathematics: The Music of Reason*.

enough to drink. “No longer even care” would be more accurate, though who can say what repressed anxieties were seeking an unexpected outlet under an apparently untroubled expression of opinion? The most one can say is that differences of opinion of this sort do not seem to matter, at least from the viewpoint of the mathematics itself, if one can imagine such a thing, abstracted from the activities of the mathematicians in creating and in understanding each other’s work.

The basic premise of Vladimir Tasić’s unusual book *Mathematics and the Roots of Postmodern Thought* (MRPT) is that the “crisis of foundations”, more or less distorted, does continue to matter in the most unlikely setting of so-called postmodern thought, as practiced mainly in France. Tasić, seeking an alternative to the rhetorical violence of the “science wars”,³ wants to answer the following question:

Why would postmodern intellectuals bother invoking mathematics in their theories at all? (Tasić, p. 3)

His thoroughly original solution, obtained at the end of a breathless excursion through the history of Western philosophy and much of the history of mathematics, is

... to view postmodern theory as a deeply divided edifice: first, as a revival, or a reinvention in somewhat different terms, of a challenge that mathematicians who were influenced by romanticism once issued to logical reductionism; and second, as an extraordinary radical dismissal of romantic humanism, a dismissal whose roots can in part be traced to mathematics, and which in its postmodern edition becomes a rather extreme form of formalism (Tasić, pp. 5–6).

Or, more diagnostically, the “postmodern Oedipus...has a formalist daddy and an intuitionist mommy” (p. 99).

There is no doubt in my mind that Tasić’s question is the right one, the only genuinely interesting question to arise from *Fashionable Nonsense*, the book by physicists Alan Sokal and Jean Bricmont that revealed to the world the proclivity of some (mainly) French philosophers for ill-advised use of mathematical terminology. Jacques Bouveresse, a no-nonsense French philosopher and supporter of Sokal and Bricmont, can find no better explanation than to evoke the “oligarchic, hierarchical, clannish, even ... Mafia-esque” nature of

the current French literary scene and to ask “If we were deprived of the right to say more or less whatever we like about things that are difficult to understand and that we don’t want to understand, where would be the pleasure and interest?”⁴ Tasić emphasizes, rightly, that “Mathematics has been part of the Western tradition, inseparable from its culture and philosophy” (p. 4), and that the roots of the “nonsense”, if that’s what it is, must be sought in the history of that tradition.

The answers Tasić proposes are more problematic than the question, as the author is himself aware. The reader of MRPT, and of this review, should bear in mind that Tasić, in addition to being a mathematics professor (at the University of New Brunswick) and an active researcher in a core branch of mathematics (group theory), is a highly regarded author of fiction in his native Serbia; his novel *Oprostajni dar (koncerto)* won the Radio Belgrade 2 prize in 2001. Recognizing that the full ramifications of the problem he poses vastly exceed the scope of his book, he invites the reader to “think of this book as a story—a speculative reconstruction of a story—and as an invitation to a polemic” (p. 6). Not surprisingly, Tasić tells a good story, which I will try to re-reconstruct. “Let us, as philosophers, welcome fantasy,” Ian Hacking once wrote. Philosophically minded mathematicians arguably need fantasy even more. The polemic can wait.

Dramatis Personae

In a narrative extending over a period of several centuries, it is natural that there will be abstract protagonists alongside the merely human characters, limited by their “relatively short shelf life,” as Tasić puts it. We have already encountered some of the former, conveniently introduced in binary oppositions whose borders, however, they rapidly overflow. The couple “intuitionism/formalism” is familiar from the early twentieth century crisis of foundations. For Tasić it is important to view this couple as a peculiarly mathematical species of the larger opposition between “romanticism” and what romantics viewed as the excesses of determinism, variously identified with Kantian categories, with objectivity, with logic, and with science in general. Tasić’s list of romanticists in the sciences contains a few surprises. Of course there are Poincaré and Brouwer, cast as the lucid Jekyll and the radical Hyde of intuitionism, and Weyl. But there are also Einstein; Sapir and Whorf, famous for their thesis on the culture-dependency of language; and, perhaps inadvertently, Wittgenstein. Among other minor characters, Deleuze and Guattari, quoted to much humorous effect by Sokal and Bricmont, are here identified as romantics, though their relation

³Familiar to Notices readers from articles by Michael C. Sullivan and Evans M. Harrell II in the October 1996 issue.

⁴J. Bouveresse, “*Prodiges et vertiges de l’analogie*”, *Raisons d’Agir*, 1999, pp. 122, 63.

Living between Two Worlds—The Fiction of Vladimir Tasić

Tasić's fictional works include two books of short stories in Serbian—*Pseudologia Fantastica* (Matica Srpska, 1995) and *The Joy of a Shipwrecker* (Svetovi, 1997) (which were translated into English and published together as *Herbarium of Souls* (Broken Jaw Press, 2001))—and one novel, *The Farewell Gift* (Svetovi, 2000).

We mathematicians are particularly fond of duality theorems; translating mathematical statements from one category to another often gives us new and unexpected insight. Tasić the fiction writer explores duality, albeit of a different kind, the one between the rational world of science and logic and the mystical and metaphysical world of emotions, beliefs, and spirituality. He invites us to take an exhilarating journey to the plane where those worlds intersect, yielding sometimes wonderful but often fatal outcomes. His stories are carefully constructed, like a well-thought-out mathematical proof, and his style is glazed by a thin layer of the melancholy of one living in exile. It is therefore not surprising that Tasić is already considered one of the rising stars of Serbian literature.

Herbarium of Souls is a collection of four stories exploring the relation between myth and knowledge, the exoteric and esoteric, inspiration and delusion, and the search for the ineffable truth. They bear strong resemblance to Borges's *Ficciones* both in style and in subject and can be considered an homage to the great master. There is a story about a student struggling to publish the findings of his deceased mentor on Soloviev's eschatology and at the same time falling in love with the late professor's wife. In another one, a painter, whose reputation rested on his mastery of color, is left color-blind after an accident and tries to remember the world. The allegorical tale "Secrets of an Argonaut" explores the limits of modern science in the search for the meaning of our existence. But the most interesting for a mathematician is "Herr Doktor's Wondrous Smile", in which the narrator, a mathemat-

ics professor, decides as a prank to give a talk at a joint symposium of scientists and occultists in search of the hidden meaning of numbers. In his lecture on Chaitin's number Ω , he mixes mathematical logic and mysticism, randomness and religion, computability and the cabala. It is all intended to be a parody, a sort of *reductio ad absurdum* of the occultists' project, but alas the joke goes awry, and he unwillingly gets drawn into the world of one of the mystics, who is in search of his own spiritual Ω through art and sculpture. The scientific community does not forgive such transgressions, and the narrator's academic career is ruined by his conspiring colleagues. In the end, having lost everything, he himself starts sliding into mysticism and numerology.

The novel *Farewell Gift*, which is written in the form of a concerto, is certainly Tasić's most accomplished work. It is a melancholic but also humorous and ironic story of a generation of Yugoslav immigrants who, fleeing the Balkan inferno, find themselves facing the impossibility of integrating into the New World. But the novel is also a counterpoint between two opposing views of the world: the scientific reductionist view of genetics and computers, and the holistic spiritual one of art and handicraft. The narrator, an engineer working in a medical institute, misplaces the ashes of his deceased brother, who is depicted as a perfect being, an angel. The ashes are left in the pottery studio of the narrator's wife, who mistakenly uses them as a glaze and in this way achieves the missing artistic touch in her ceramics. Tasić seems to be asking: Is our essence to be found in our ashes or in the complete recording of our genes on a computer? The title and indeed the novel itself is a metaphor about farewell both from one's home country and from life itself and about overcoming the horror of leaving by making the gift of our own essence to the world, be it our genes, our ashes, our literature, or our mathematics.

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to the German romantic tradition, exemplified for Tasić by Herder and Fichte, is unclear.

A second couple, difficult to articulate with the first and not really developed systematically in MRPT, opposes "anthropocentrism" to "logocentrism". Kant, who situates the act of knowing in the individual consciousness of the introspecting subject, transcendental or otherwise, is the chief anthropocentrist. But so are the intuitionists, especially Brouwer. Among philosophers Husserl and, presumably, Descartes join Kant as anthropocentrists. The logocentrists, perhaps with Spinoza and Leibniz as antecedents, include Frege, Russell,

Whitehead—for whom "the subject emerges from the world"—and Hilbert's formalist school; structuralism and the postmodernism of Michel Foucault represent logocentrism in a radicalized form.⁵

And these philosophers and mathematicians—what is it they are being romantic or objectivist, intuitionist or formalist, logocentric or anthropocentric

⁵ Derrida uses the term "logocentrism" in a different sense, as the "metaphysics of phonetic writing," so that Leibniz's project of a universal characteristic "had opened a breach within logocentric security." Derrida, *Of Grammatology*, Johns Hopkins, 1976, pp. 3, 98.

about? About identity, sometimes; about imaginary numbers, briefly. But especially about the continuum, which looms over these historic antitheses, a brooding presence, frequently disguised, unknowable, undecidable, source of paradox, and, according to Chaitin, irreducibly random.

Synopsis of the Plot

Tasić's text is structured by a variety of literary devices—foreshadowing and flashbacks, digressions and false leads, nested narratives—so that trying to summarize it feels...reductionist, as if the review had to be at least coextensive with the book. Most of Tasić's protagonists do, however, seem to be concerned with two themes, intimately related:

Postmodern thought is concerned with *discontinuity* and *difference*. Hence, it seems reasonable to ... [consider] what some mathematicians had to say about continuity (the continuum) and identity (p. 36).

The continuum is problematic for at least two reasons: it is too big (cf. the first antinomy of Kant's *Critique of Pure Reason*, where he does not use the word *Gesamtheit*), and its bits are too small (cf. Kant's second antinomy). These days one might conveniently define a mathematician as a person unworried by these two antinomies—both mentioned in MRPT—thanks to the Axiom of Infinity (number 7 on the Zermelo-Fraenkel list) and the $\epsilon - \delta$ definition, respectively, and more generally to the influence on the curriculum of people like Dieudonné. But one hundred years ago lots of people, mathematicians included, found these antinomies very worrisome. Dedekind and Cantor had hardly finished constructing the continuum when Brouwer set about its deconstruction. This meta-narrative has been told many times, usually focusing on the conflict with Hilbert. Tasić manifests his difference by stressing the parallels of Brouwer's positions with the philosophy of Heidegger, Nietzsche, Wittgenstein, and Derrida.

Most contemporary accounts of Brouwer, presented from the standpoint of current mathematical practice, stress the negative features of his philosophy, especially his rejection of the law of the excluded middle. Tasić's Brouwer is at heart a romantic, whose mathematics is "a continuous creative flow, 'free will,' some inner activity of the mind that cannot be reduced to, or deduced from, language" (p 46). Here is Brouwer, speaking in Vienna in 1928:

There is neither exactness nor certainty in will transmission, especially not in will transmission by language [...]. There is, therefore, also for pure mathematics *no certain language* (quoted on p. 46, and again on p. 120).

If this reminds you of Wittgenstein, you are on Tasić's wavelength: the Viennese philosopher was in Brouwer's audience, an encounter reportedly responsible for Wittgenstein's return to active philosophical work and ultimately for some of the concerns addressed in the *Philosophical Investigations*. More intriguing perhaps and certainly more slippery is Tasić's comparison of Brouwer's tempestuous version of the continuum with that of Heidegger. For Brouwer the intuition of the continuum is derived from time rather than space, and Brouwer's anthropocentric vision replaces the instant in the time continuum with the "falling apart of a life-moment": "the basic concept of all mathematics according to Brouwer" (p. 38), whose "points" (the scare quotes are in Tasić) are "open, indeterminate processes that actively involve the individual" (p. 39). It is not at all clear, for example, whether Brouwer would have invited Chaitin's Ω into his continuum. This description leads Tasić to propose a dramatic "thought experiment" in which he is "a being thrown into the river of Brouwer's time continuum" who "cannot know myself as I *am* because I conceive of the time continuum *itself* by 'looking ahead'" (p. 42). How much of this is really in Brouwer is not clear from the exposition, but "thrown" is a key Heideggerian term Tasić introduces to describe the "self" from *Being and Time*, which "is never fully 'present' to itself ... but is a ceaseless opening toward the continuum of its existential possibilities" (p. 42). Though the word "continuum" seems to be absent from *Being and Time*, "continuous" (*kontinuierlich*) appears in Heidegger's indictment of the "inauthentic temporality" of *Dasein*—i.e., you and me, more or less, in our better moments—when, refusing to acknowledge human finitude, it conceives of time as an "uninterrupted ... sequence of 'nows'" (*Being and Time*, par. 81). Tasić compares Heidegger's inauthentic "they" to "the people who," like the American in Münster but unlike Brouwer, "think they can know the square root of 2 because they can describe it in mathematical language and approximate it with any desired precision" (p. 43).

As Tasić recalls, Heidegger was aware of "the controversy between the formalists and the intuitionists," to which he alludes explicitly in the beginning of *Being and Time* in support of his thesis on "the ontological priority of the question of being."⁶ Tasić notes a curious similarity of Brouwer's continuum to what Heidegger once called "the mathematical...that evident aspect of things within which we are always already moving...the fundamental presupposition of the knowledge of things" (quoted on p. 43). Heidegger's notorious characterization of *Dasein* as the kind of being

⁶ One might view the successful nonresolution of the debate in Münster as a contemporary refutation of Heidegger's thesis.

that “in its Being, has a relationship towards that Being—a relationship which itself is one of Being” (*Being and Time*, par. 4) is, for Tasić, “the consequence of the very structure of the continuum” (p. 151).

Later avatars of Brouwer’s continuum include what Hermann Weyl, under Brouwer’s influence, called “a medium of free Becoming;” literary critic Julia Kristeva’s “the semiotic;” and especially Derrida’s “*différance*,” of which more later. The dialectic of change and continuity has an ancient pedigree in metaphysics;⁷ in twentieth-century France Bergson (cited by Tasić) privileged the duration, whereas Bachelard (not cited) considered the instant primary. In one way or another, all these authors want “objective knowledge of the continuum” (p. 50)—to know what *really* happens between time t and time $t + \epsilon$ —what Weyl apparently meant by “Becoming;” no need to invoke postmodernism. What the postmoderns have in common with each other, and with Brouwer’s intuitionism, is, for Tasić, a concern with the “decidedly romanticist notion” of “the inability of logic and language”—the Cantor-Dedekind continuum, for instance—“to capture the sense of continuity, in particular the continuity of inner time” (p. 31). In a discussion of Turing’s Halting Problem, Tasić calls the continuum “the other of any programming language” (p. 50), a postmodern play on words and an anticipation of his discussion of Gödel’s and Chaitin’s theorems, symptoms of perhaps the most diagonal paradox of all: that we know the continuum well enough to derive from our ignorance of it nearly everything we know, mathematically speaking, about the limits to our knowledge.

After the funhouse of Brouwer’s continuum and its variants due to Weyl and Heidegger, it comes as something of a relief to encounter Poincaré’s earlier version of intuitionism. As philosopher, Poincaré seems to be enjoying something of a revival. Elie Zahar’s recent *Poincaré’s Philosophy* offers a “rational reconstruction” of positions Poincaré the scientist never developed as systematic philosophy. Here “rational” in large measure means making Poincaré safe for analytic philosophers; Tasić prefers to view Poincaré as a precursor of postmodernism, a “dialogical partner” of Derrida and others. From Poincaré Tasić chooses to concentrate on the problem of identity—on what remains constant between time t and time $t + \epsilon$, for example. Tasić uses Poincaré’s familiar critique of Euclidean geometry’s hidden assumptions regarding rigid motion as a springboard to the postmodern problematic of identity and difference, or *différance*:

Poincaré maintained that the identity of objects can only be continually *motivated*

⁷ Cf. Plato’s *Parmenides*, 156, and Aristotle’s *Physics*, Book VI.

by intuition, in the sense that the invariance of an object under some group of transformations is ceaselessly reestablished perceptually...The very concept of identity of an object depends on the possibility of it being different...given a chance (flow of time, “history”) to change. A familiar corollary follows: Identity can be established with absolute certainty only from beyond history...[T]his reflection...seems to be the starting point of parts of postmodern theory (p. 62).

This “diachronic” version of Poincaré’s critique of identity abruptly gives way to a “synchronic” version. Suppose, Tasić asks, the

identity of an object...is granted through its relationship with all the other objects from which it is different ...a view...usually associated with the work of the Swiss linguist Ferdinand de Saussure (p. 63).

If “granting identity” is conflated with mathematical definition, the result is an *impredicative definition*, one in which “an element is defined by an appeal to the *totality* to which it belongs” (p. 142; emphasis added). An impredicative definition particularly handy for freshman calculus is that of a least upper bound in the continuum. Less judicious impredicative definitions give rise to Russell’s paradox and the like, one reason Poincaré rejected them and presumably would have had little enthusiasm for the German’s suggestion in Münster regarding the existence of the continuum. For Tasić the danger is greatest in “structures that are in some sense ‘generative,’ for example the generative grammar of a language, which produces new elements over time” (p. 64). If you stop to ask what generative grammar has to do with the least upper bound axiom, you will miss the connection with what Tasić calls “Derrida’s basic theorem”, that “Identity is not ‘present to itself’”:

[The] totality cannot be ‘totalized’ because ‘writing’ [a basic Derridean term] induces change and the appearance of new units...the identity that can be assigned to structural units by means of structural differentiation alone is always subject to *possible* changes. It cannot a priori be guaranteed to have achieved its full signification...simply because we cannot a priori know the (finite but unbounded) totality through which it achieves its signification (p. 144).

Optimistically pegging his text at a nonspecialist public, Tasić includes a chapter on Hilbert's formalism and its unintended corollaries due to Gödel, Turing, and Chaitin, covering ground that most mathematicians will find familiar.⁸ But the title of this chapter, "The Expired Subject", leads directly to postmodernism, specifically the postmodernism of Michel Foucault. As missing link between Hilbert's logocentrism and that of Foucault—and thus between the "crisis of foundations" in mathematics and postmodernism—Tasić proposes French philosopher Jean Cavailles. Readers in English-speaking countries, where his work is practically unknown, can hardly imagine Cavailles's centrality for postwar French philosophy of mathematics. Tasić is to be thanked for making this compelling figure more widely known outside France—a French resistance leader, executed by the Nazis in 1944 for sabotage, of whom historian and philosopher Georges Canguilhem, his schoolmate at the École Normale Supérieure, wrote the year after May 1968: "A philosopher-mathematician loaded with explosives, lucid and reckless, resolute without optimism. If that's not a hero, what is a hero?" Canguilhem was speaking at the dedication of one of several philosophy lecture halls named in Cavailles's honor. Cavailles himself wrote that "the mathematician is embarked on an adventure which he can only stop in an arbitrary way and which at every instant brings a radical novelty."⁹

Tasić links Cavailles to Foucault through Canguilhem, the later philosopher's teacher, and indeed Canguilhem wrote in 1967, in a clear reference to Foucault and the structuralists, that Cavailles was twenty years ahead of his time in calling for "substituting for the primacy of lived or reflected consciousness the primacy of the concept, the system, or the structure." There are more links, not mentioned by Tasić: Cavailles visited Hilbert's Göttingen in 1931, where he collaborated with Emmy Noether on a translation of the correspondence between Cantor and Dedekind; at the École Normale he was a contemporary of the founders of Bourbaki and close friend of Chevalley in particular; and posthumous editions of his works contain prefaces by H. Cartan and Ehresmann, among others.

Alluding to an earlier subplot, Tasić sees Cavailles as "doing a little Hegel on Hilbert's 'Kant'" (p. 84) when he wrote that "the true meaning of a theory is not in what is understood by the scientist

as essentially provisional, but in a conceptual becoming that cannot be halted."¹⁰ It doesn't matter that this reading is probably wrong, insofar as Cavailles saw himself as a Spinozist rather than a Hegelian, and wrote that the fact that the development of a mathematical concept "doesn't happen all at once has nothing to do with history but is the characteristic of the intelligible."¹¹ Tasić wants to rescue a "hope" for an invariant mathematical truth¹² from the following notion he reads into Cavailles:

The meaning of mathematics is *in* the endless historical process of its changes. Since mathematical truth *always changes*, it remains beyond the reach of individuals (p. 89).

For Tasić this is a "dangerous bit of formalistic ideology" that "denies human beings any creative role" (p. 88). Again, it is not important that Cavailles, brought up as a devout Protestant, who saw his engagement in the resistance as following a necessity strictly analogous to that of the development of mathematics, would not have understood matters this way, that indeed he was convinced, in spite of Gödel's theorems, of the objectivity of mathematical truth and spent his short professional life looking for a philosophically sound way to express that conviction.¹³ Cavailles' role in MRPT is to pave the way to the danger that actually came to pass: Foucault's rejection of any notion of continuity and his denial "that people...can contribute to discourse in some innovative and irreducible manner" (p. 95).

The historical moment that saw the rise of Foucault's philosophy is not exactly virgin territory. The path Tasić draws from Cavailles to Foucault is slender, while French and American commentators alike agree on the influence of Bachelard on Foucault, in the ideas they shared about discontinuity, for example.¹⁴ But it is only a story, and with

¹⁰ *Ibid.*, p. 505. This text was written in a Vichy prison—from which Cavailles soon escaped—and is forgivably obscure.

¹¹ *Ibid.*, pp. 517–518.

¹² A hope he attributes to Gödel, Husserl, Brouwer, Weyl, and Poincaré.

¹³ See, for example, Cavailles, *op. cit.*, pp. 180–181. Cavailles's objectivity is immanent, rather than transcendent, in a Platonist or Kantian fashion. For a bit of variety, one can also read Cavailles as an intuitionist; cf. Emmanuel Barot, *Cahiers Alfred Binet*, vol. 666, Nancy, March 2001.

¹⁴ Cf. François Dosse, *Histoire du structuralisme, I, La Découverte*, 1991, p. 114. Dosse's comprehensive history mentions Cavailles briefly but does not attribute to him the "formative influence" on structuralism which Tasić claims to see. Here and elsewhere Tasić comes across like an investigative reporter protecting his sources.

⁸ In particular, he mentions Hilbert's purported intention to axiomatize geometry so thoroughly that the terms "points, lines, planes" could be replaced by "chairs, tables, beer mugs" without any change in meaning.

⁹ Jean Cavailles, *Oeuvres complètes de Philosophie des Sciences*, pp. 678, 594.

Foucault we have finally come home to postmodernism. Tasić, whose tone has mostly been playful up to this point, does not conceal his irritation with Foucault's antihumanism, arguing memorably that "the existence of a postmodern subject" in Foucault's sense "appears to be logically equivalent to the existence of an intelligent Turing machine" (pp. 95–96) and that "Foucault's conclusions" that there is no continuity in history, just a succession of "archeological layers," "are an easy corollary of Chaitin's theorem" (p. 97).

Tasić's pugnacity persists in his confrontation with structuralism, exonerating Saussure of the excesses structuralism committed in his name:

The problem is that the "meaning" of each element of some formal structure is identified with its role with respect to all other structural units. Thus the "identity" ...assign[ed] to the units of the structure...is...open to the same objections [i.e., to impredicative definitions] that Poincaré addressed to logicians and formalists, back in the days when mathematicians were in the business of "deconstructing identity" (p. 118).

Tasić scoffs at Artificial Intelligence theorists and their "thinking thermostats," classed along with this (non-Saussurian) structuralist tradition, confusingly, as "functionalists." Wittgenstein (who did use the word *Gesamtheit*, in the *Tractatus*) fares little better, his philosophy haunted by an ambivalence between "something unpleasantly close to cultural determinism" (p. 120)—his Brouwerian side that involves "little more than a reformulation of one of the fundamental principles of romantic language philosophy" (p. 122)—and "ordinary language formalism" in which "the I, or some deeper self,...ultimately 'divides out' from the language games" (p. 131).

And then we reach the end of the book and discover that Derrida, whose vision of a continuum of meanings as *différance* retrospectively illuminates Tasić's project, was always already the protagonist of MRPT. Derrida's neologism for "the origin or production of differences and the differences between differences," referring simultaneously to the verbs "differ" and "defer," designating a "sameness which is not identical," is "neither a word nor a concept" and therefore naturally resists definition.¹⁵ Derrida himself has not been stingy with characterizations of *différance*, calling it elsewhere "the formation of form," "the history of life," among others. Non-belief, practiced by the Frenchman in Münster, is not a real option for Derrida: "To risk

¹⁵ Quotations from Derrida, "La différence", translated in *Speech and Phenomena*, Northwestern University Press, 1973.

not meaning anything is to enter the game, and in the first place the game of *différance*".¹⁶

Tasić finds *différance* "somehow akin to the indivisible flow of Brouwer's continuum" (p. 147). Tying together his themes of identity and continuity, Tasić's final chapter argues that Derrida, claiming (1) that "I am not in a position to 'totalize' the entirety" of the "text-in-general" which is the source of meaning because (2) "anything that is written anywhere instantly causes a change in meaning" therefore concludes that (3) any attempt to imagine "writing-in-general" leads to a vision of "endless sequences interspersed with presently unknowable spontaneous acts", i.e., (4) "something like Brouwer's continuum" as Tasić has presented it (pp. 146–147). More concisely:

If the semantic identity of some unit can only be conceived of in terms of its "location" in the continuum of possible meanings—due to the changes incurred by generating choice sequences through rules and spontaneous acts of writing—then this identity is "not present to itself." In Brouwer's continuum, there is no present (p. 148).

Whether or not this is what Derrida had in mind, I found this hypothesis and the entire final chapter extremely helpful: a handle for me, as a mathematician, on the role of Derrida's philosophy in the (mainly literary) settings in which it is invoked, as well as on the possible role of mathematics in Derrida's thought.¹⁷

And so identity and difference, nonlinguistic and the linguistic, are somehow intertwined...woven in the continuous tapestry that Derrida calls *différance*. One would certainly like to know how Derrida understands the relevance of his position to science...What sort of mathematics would fit his philosophical position? What would be Derrida's philosophy of mathematics? (p. 153).

At the time of publication Tasić had not gotten around to asking Derrida these questions. For the sequel?

¹⁶ From *Positions*, *Minuit*, 1972, p. 23 ; quoted in *Dosse*, *op. cit.*, II, p. 50.

¹⁷ A role present from the beginning. Foucault's relation to Cavailles is mediated, but Derrida invokes him explicitly in his first publication—for which he was awarded the *Prix Cavailles* in 1962—an introduction to his translation of Husserl's fragmentary essay on *The Origin of Geometry*.

The Polemic

I had a lot of fun with this audacious and exasperating book. There is no shortage of recent non-specialist texts in English on the nature of mathematics, but the philosophical framework is almost invariably analytic and Anglo-American; those that even acknowledge the existence of post-Kantian continental philosophy can be counted on the fingers of one hand.¹⁸ Despite the unfamiliarity, not to say obscurity, of some continental concepts, Tasić vividly characterizes his protagonists, conceptual as well as human, and the book is a pleasure to read. Following Tasić's suggestion to think of MRPT as "a story," I went through it the first time with an eye for the unconventional connections, letting the details take care of themselves. On this reading MRPT looks more than a little like Greil Marcus's *Lipstick Traces*, a "secret history" tracing punk rock through May 1968 and French Situationism to utopian movements in various times and places in Europe (sixteenth-century Münster, for instance)—connections that, without a story, Marcus dismisses as merely "tradition as arithmetic" [p. 184].

On the second reading I began to worry about the details. Tasić's history is selective and often distressingly hypothetical, and his interpretations of philosophical texts are frequently outlandish. That doesn't bother me: he is hardly alone on either count. What interests me is whether or not his "speculative reconstruction" provides coherent and original insight into the problems motivating his main characters. But even if Tasić doesn't pretend MRPT is strictly faithful to history, one has to wonder what it means to say that influence could possibly have followed the course of his speculation. Mathematicians have not shown particular sensitivity to philosophical issues raised by the question of historical influence. Witness, for example, the dispiriting controversy over the intellectual authorship of the conjecture on modularity of elliptic curves over \mathbf{Q} , where different national mathematical communities have apparently adopted different solutions to the historical question without the slightest attempt to come to a methodological consensus. If specialists in the same branch of number theory are unable to agree on what is at stake, what can be meaningfully said about influence that transgresses disciplinary boundaries? While preparing this review, for example, I learned that mathematician Gregory Chaitin and performance artist Marina Abramovic discovered intellectual affinities in the course of a recent joint interview.¹⁹ Should we henceforth be looking for

¹⁸ *The situation in France is quite different, but there analytic philosophy is generally neglected.*

¹⁹ See <http://www.cs.auckland.ac.nz/CDMTCS/chaitin/abramovic.html>.

signs of mutual influence? How would we recognize them? In what terms could we judge their significance?

Tasić is absolutely right to remind us that mathematics did not have the first word on the continuum, or identity, and that mathematicians should not expect to have the last word either. There was a time of crisis, Tasić recounts, when Weyl quoted Nietzsche, Husserl discussed phenomenology with Hilbert, and Cavallès collaborated with Emmy Noether. In spite of catastrophes, chaos, and complexity, that time is past. Postmodernists and mathematicians may still be talking, separately, about continuity, but the evolution of the concept of the continuum since the crisis of foundations has apparently been...discontinuous. Which means the most rigorous historical work may well be beside the point. Even if Tasić had displayed an uninterrupted paper trail of direct person-to-person contact from Dedekind to Foucault, through Hilbert and Cavallès and as many links as you like, it would not mean very much if Dedekind's continuum and Foucault's discontinuity were intrinsically irrelevant to each other. MRPT would be much more effective if Tasić argued convincingly that there is some significant overlap, other than the name, between what Derrida and the number theorists in Münster have in mind by continuity.

But to be convincing, Tasić would have had to explore the contexts in which the continuum (or identity) appears relevant to postmodern philosophers and literary critics, as well as to contemporary mathematicians. Tasić did not set out to undertake the substantial scholarship such a project would require. Neither he nor anyone else is likely to do so in the future. Still, Tasić's silence on what contemporary mathematics and postmodernists are really after can lead to misunderstanding. Regarding impredicative definitions, for example, which have a precise meaning in logic, Tasić writes:

[I]t seems to be a general feature of impredicative definitions that...the identity of objects defined in such a manner is not guaranteed to remain immutable by the logical structure itself. Their identity is not present "in" them, but is granted by their structural relationships with other things. Therefore, introduction of new elements into the structure could very well make a difference (p. 64).

Someone whose experience of life is limited to mathematics, especially of the Bourbaki variety, may well wonder what on earth this could possibly mean.²⁰ To me it brought to mind Yoneda's Lemma on representable functors—that the functor, taking an object X of a category C to the contravariant functor from C to $\{\text{Sets}\}$ given by $Y \rightarrow$

Morphisms (Y, X) , is fully faithful—the basis of standard techniques in topology (classifying spaces) and algebraic geometry (moduli spaces). The danger that “new elements” might be introduced into a category seems minimal. In category theory the operational notion is that of *isomorphism* rather than identity. My Jussieu colleague Pierre Schapira recently had the opportunity to explain the difference between these notions to an audience of philosophers.²¹ For Schapira the set-theoretic point of view, which continues to dominate all writing about philosophy of mathematics for nonspecialists, Tasić’s book included, is merely “anecdotal”, having been “supplanted for nearly half a century by the categorical viewpoint” (outside of certain branches of logic, of course).²² Schapira mentions the influence of Grothendieck, but he also writes that “the appearance of categories is...more or less concomitant with the outbreak of structuralism in the human sciences.”²³

This is a connection Tasić does not pursue. Nor does he have much to say about the better documented interaction between the structuralists and Bourbaki. Two paragraphs in Tasić’s chapter on structuralism attest to his awareness of this interaction. But Tasić is content to quote Gian-Carlo Rota’s view of Bourbaki as an example of “the pernicious influence of mathematics on philosophy.” Tasić apparently agrees with this assessment, and he has a lot of company. Nevertheless, though Dieudonné may have been guilty of bravado in the passage quoted above, there is no sense of crisis in my field of number theory, for one, happily Bourbakist in most of its branches. In a book for nonspecialists, it is highly misleading to skip directly from the generation of the foundations crisis, through Gödel and Turing, to Chaitin, without mentioning that the rise of Bourbaki was surely one of the principal factors in the dissipation of the crisis.

Absence of a serious consideration of Bourbaki’s role or of the contemporary evolution of the foun-

²⁰I think Tasić’s discussion would have been clearer had he quoted Hilbert at this point: “When we increase the number of fundamental objects [in an axiomatic system] the axioms acquire...a new extension and must then be tested again and, if necessary, modified” (cf. Poincaré, *Science et Méthode*, p. 146).

²¹In Salle J. Cavallès, naturally; see <http://www.math.jussieu.fr/~schapira>.

²²Logician A. R. D. Mathias defends the opposite point of view in “The Ignorance of Bourbaki”, *Math. Intelligencer*, July 1992.

²³Some of these points, and many others besides, are made by Yu. I. Manin in “Georg Cantor and His Heritage”, at arXiv:math.AG/0209244. Manin’s talk, addressed to mathematicians rather than philosophers, features two quotations—of a literary nature—from MRPT!

datations crisis more generally is one omission that mars MRPT. Symmetrically, Tasić devotes little time to discussion of *nonmathematical* roots of postmodernism. The most glaring omission is perhaps the absence of any reference to politics. Apart from an allusion to the “little red book”, one would not suppose on Tasić’s account that the transformation of society was a major motivation for many of postmodernism’s central figures and even more so for their readers.²⁴ Tasić’s portrait of an anti-humanist Foucault who “calls for a formalistic ‘final solution’” to the “special status” of human beings (p. 93) is hard to square with the philosopher’s public role as what is usually called a human rights activist, who once described his books as “little toolboxes” to be used to “disqualify systems of power.”²⁵ As a direct result of the May 1968 strikes in Paris, Foucault spent the last period of his life as a prime example of the *intellectuel engagé*, and his work underwent a simultaneous shift in emphasis.²⁶ The most consistent theoretician of anti-humanism was not Foucault but rather Louis Althusser—nowhere mentioned by Tasić—whose notion of “structural causality” was designed to rescue Marxism from its Stalinist as well as its humanist associations by guaranteeing its status as “science” rather than “ideology.”²⁷ More recently, Bruno Latour’s retroactive abortion of postmodernism on the grounds that *We Have Never Been Modern* is explicitly directed against the theoretical separation of science and politics and is written under the sign of the events of 1989 in Eastern Europe.²⁸

Return to Reality

By chance I found myself sitting on a cross-country flight last year next to a well-known string theorist, and after lunch we began comparing our gripes about the continuum. In a host of problems in which it plays a supporting role, I find all those unknowable real (and complex) numbers an an-

²⁴*Nonpostmodern Bouveresse* saw his 1973 book on Wittgenstein as a “deliberate provocation” in the then-dominant ethos, for which “there could only be either political or psychoanalytical problems.” Quoted in Dosse, *op. cit.* II, p. 273.

²⁵Quoted in D. Eribon, Michel Foucault, *Flammarion*, 1991, p. 251.

²⁶See D. Eribon, *op. cit.* for the political activism, and H. Dreyfus and P. Rabinow, Michel Foucault : Beyond Structuralism and Hermeneutics, *The Harvester Press*, 1982, for an account of the philosophical shift. Tasić’s treatment of Foucault ends with his Archeology of Knowledge, published in French in 1969.

²⁷See Dosse, *op. cit.* I, especially chapters 30–32, for the role of Althusser.

²⁸Harvard University Press, 1993; the original French version was published in 1991.

noying distraction, concealing the really interesting ones (periods of algebraic integrals, zeros of the Riemann zeta function) in their midst. The physicist confided his hope that the universe could be modeled by a discrete dynamical system in which the real numbers' role would be merely anecdotal, a goal apparently shared by an increasing number of his colleagues.²⁹ Higher (much higher) up the amazon.com sales charts, one finds Stephen Wolfram in *A New Kind of Science* promoting a similar vision of "the whole history of the universe in spacetime...represented by a giant four-dimensional network." *Différance* would be a good name for the "three or four lines of code"³⁰ that Wolfram imagines might suffice to generate this network.

Successfully banishing the continuum from physics might alleviate the metaphysical tension latent in the Münster restaurant debate, inasmuch as the different positions would no longer have distinct material consequences. In the meantime, we mathematicians bear our responsibilities as best we can. If you are curious about the details of the "crisis of foundations", you should read a conventional history. If you are seeking a serious antidote to science wars doldrums, you should read Ian Hacking's *The Social Construction of What?*, which among its other merits actually managed to transform science war rhetoric into philosophically respectable debate, or *The One Culture?* by Jay Labinger and Harry Collins, in which leading warriors on both sides talk peace. On the other hand, if you are a mathematician mystified, as I think you should be, by the apparent fondness of (some) postmodern philosophers for mathematical metaphors, you will find MRPT stimulating, provided you follow the author's suggestion and read it as literature. And if your attempts to reconcile the set-theoretic continuum with something familiar and intuitive leave you prone to intermittent identity crises, Tasić's book will reassure you that you are not alone.

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²⁹ See, for instance, comments in the introduction to G. 't Hooft, "Quantum gravity as a dissipative deterministic system", *Classical Quantum Gravity* 16 (1999), 3263.

³⁰ Interview of Stephen Wolfram by Steven Levy in *Wired*, June 2002. See also reviews of Wolfram's book by Lawrence Gray (*Notices*, February 2003) and Steven Krantz (*Bulletin of the AMS*, January 2003), as well as Jordan Ellenberg's review on *Slate*, at <http://slate.msn.com/id/2067547>.

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