Opinion

Does Your Vote Matter (in AMS Elections)?

Last week my eleven-year-old nephew emailed to ask "about a debate that Mama and I have been having, the question does it matter whether or not you vote? I'm not talking about it ethically since I know it's good to vote and exercise your right as an American citizen; I'm talking about it mathematically. I think that unless the election comes down to 1 vote it really doesn't matter whether you vote or not because you're not changing the outcome of the election." Since this issue of the *Notices* contains information about the annual AMS election (see page 965), I decided to write my reply in the form of this editorial column.

There are certainly examples in history in which a change of one vote could have reversed the outcome. In the 1868 trial of President Andrew Johnson in the U.S. Senate, 35 of 54 senators voted for conviction (one vote fewer than the two-thirds required to remove the president from office). Turning to more recent events, I note that about one-quarter of the cases heard by the U.S. Supreme Court over the past three years were decided by a vote of 5 to 4.

One expects that in an election in which many votes are cast, the likelihood of the outcome depending on a single vote is small. Consider the AMS election, in which typically a few thousand votes are cast. A combinatorial counting argument shows that if voters were to mark their ballots randomly for one or the other of two candidates, the probability that one extra vote could change the outcome is on the order of 1 or 2 percent.

An individual voter does not know ahead of time, though, what the margin of victory will be in the election, so it is impossible to predict whether one's vote would make a difference to the result. Even in an election with millions of eligible voters, where the mathematical probability of a result exactly on the borderline is a fraction of 1 percent, the margin of victory could well be within the margin of error of the vote-counting mechanism. If the citizens registered to vote in Florida had known in advance how close the year 2000 U.S. presidential election would be, no doubt many more of them would have showed up at the polls on election day.

When the result of an election is not in doubt, an individual's vote may nonetheless be valuable. A candidate who wins an election by a large margin is in a strong political position to implement changes, while a losing candidate who makes a strong showing has a good chance of promoting a minority agenda.

Of course one may question whether winning an election by 1,000 votes is significantly different from winning by 1,001 votes. The question may be recast as a version of the ancient *sorites* (or heap) puzzle. A winning margin of one vote is not a landslide, nor is a margin of two votes, nor …; yet winning an AMS election by 1,000 votes is a landslide. At what size does a margin of victory become a landslide?

Since the sorites paradox has exercised philosophers for two thousand years, I will not try to resolve the puzzle here. I will, however, suggest an analogy to the idea that one individual's vote is valueless in a decisive election. One might just as well say that in a basketball game between unequal teams, there is no motivation for either side to try to score, because any one particular basket is almost certainly not going to decide the game. I say that if there is value in playing the game, there is equally value in voting in the election.

Why then is the turnout low in many elections? According to the Secretary, about 12 percent of the members typically vote in the AMS election. Apparently most members have felt that participating in the affairs of the Society has less value to them than a few minutes of their time and a three-bit stamp. Although this year's new electronic voting option may increase the turnout, I would be surprised if a majority of members cast ballots.

A possible explanation is implicit in the famous 1968 essay by Garrett Hardin, titled "The tragedy of the commons". The essay suggests that if individuals are motivated solely by immediate direct payoffs, then a catastrophe inexorably follows for the whole society, as when overfishing of the oceans results from many individuals seeking to maximize their take of an ever-diminishing resource. The idea is similar to the "prisoner's dilemma" in game theory, in which two persons hoping to maximize their individual returns end up with a result disastrous to both.

The happy news is that (in view of the Archimedean principle) many small actions directed toward the betterment of a Society can collectively achieve a great common good. In my view, this is the significant reason to be involved in the Society's affairs, if only by voting in the election.

—Harold P. Boas, Editor

Letters to the Editor

On Calculus Texts

In all of the U.S. universities where I have taught calculus so far, students are expected to buy what is called a required calculus text. Some of the students (usually the best students in their classes) try to study mathematics, reading the required calculus texts. As far as I know, such reading is a painful experience for those who have tried. I explain this phenomenon in the following way.

To be interesting and useful reading for mathematically oriented students, a calculus text should contain:

1. Accurate, written-for-beginners proofs of the results of the course which do not belong to Foundations of Mathematics, in their logical order. The most well-known examples of the style that I mean are the books: G. H. Hardy, *A Course of Pure Mathematics*; and G. H. Hardy, J. E. Littlewood, and G. Polya, *Inequalities.*

2. Interesting, nonroutine problems of different levels of difficulty, with hints and possibly solutions.

On the other hand, to have a good chance to be adopted as required, a calculus text should contain as much as possible of the following:

3. Understandable-for-an-average-student discussions of all typical problems from all topics usually included in calculus courses at universities of the U.S., pictures and tables that can help average students to use the standard calculus algorithms and to solve typical exam problems, and attractive printing and layout. The discussions are to be written in such a way that one can read them without reading the theoretical part of the text. (The majority of students never reads the theoretical part.)

4. Routine exercises of all natural types and correct answers to odd-numbered exercises at the end of the text. The selection of exercises requires a lot of work, because randomly selected they can lead to very lengthy and complicated (and not so useful and instructive) computations. Also, it is highly desirable to publish a *Student Solution Manual* with properly written correct solutions of all odd-numbered exercises.

It does not seem impossible in principle to produce a calculus text satisfying all of the conditions listed above. I can even imagine a text containing separate theoretical sections, discussion sections, exercise sections, and problem sections. However, I do not know any examples of calculus texts satisfying all of the listed conditions. One of the reasons for this I see in the fact that different tasks (among those listed above) require quite different skills from the author(s). Another reason is that a high level in each of the directions can be achieved only after many "successive approximations" and only if the authors systematically use ideas of their predecessors.

I think that a calculus text satisfying the conditions listed above can be written only as a result of collective efforts of many mathematicians. Also, I think that such a text should not have a fixed, definite list of authors. Instead, the authorship should belong to an authoritative organization of mathematicians, such as the AMS, MAA (Mathematical Association of America), or SIAM (Society for Industrial and Applied Mathematics), or to a funding organization, such as the NSF (National Science Foundation). Since the topic list in calculus is in a state of constant change, the main advantage of collective authorship is that the text could be changed on a regular basis over an indefinite period of time. Also, collective authorship will help to remove (eventually) misprints and errors from the text. To produce a high-quality calculus text, it is important to encourage all users to participate in the text improvement. In this connection it is necessary to develop different forms of sharing some part of the income from the sales of the text with those people whose suggestions are used to improve the text.

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Peer Review

In the thoughtful series of essays on peer review in the June/July 2003 issue, Steven G. Krantz writes, "A dean whose pedigree is in Celtic history is probably something of a Luddite and is (in my experience) likely to be most comfortable with traditional, hard-copy, refereed journals." The stereotype of the technophobic humanist compared to the e-adept scientist is oversimplified and no longer valid, if indeed it ever was. Humanists have been making electronic texts for nearly as long as there have been computers, and have been publishing peer-reviewed electronic materials since before the creation of the Web (`http://ccat.sas.upenn.edu/bmcr/about.html` notes two journals that started near the end of 1990).

The humanist deans of my experience respect good online work in refereed journals or at refereed publication sites like the Stoa Consortium (`http://www.stoa.org`). While we may not know the relative reputation ranking of journals in mathematics (or any other field but our own), humanists will not immediately assume that all online journals are inferior to all print journals.

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