Conferences

AMS Short Courses

Trends in Optimization–2004

Phoenix, Arizona
January 5–6, 2004

It is planned that lecture notes will be available to those who register for this course. Advance registration fees are \$80 for AMS/MAA members, \$110 for nonmembers, and \$35 for students/unemployed/emeritus; on-site registration fees are \$100 AMS/MAA members, \$130 for nonmembers, and \$50 for students/unemployed/emeritus. Registration and housing information can be found in this issue of the *Notices*; see the section "Registering in Advance and Hotel Accommodations" in the announcement for the meetings in Phoenix. The registration form is at the back of this issue.

The titles of the talks, the names of the speakers, and the abstracts are as follows:

Synopses

Graphs and Combinatorial Optimization

Gérard Cornuéjols, Graduate School of Industrial Administration, Carnegie Mellon University

The integer programming models known as set packing and set covering have a wide range of applications, many of which arise in the context of graph theory. Sometimes, because of the special structure of the constraint matrix, the natural linear programming relaxation yields an optimal solution that is integral, thus solving the problem. Sometimes both the linear programming relaxation and its dual have integral optimal solutions. Under which conditions do such integrality properties hold? This question is of both theoretical and practical interest. Min-max theorems, polyhedral combinatorics, and graph theory all come together in this rich area of discrete mathematics. In addition to min-max and polyhedral results, some of the deepest results in this area come in two flavors: "excluded minor" results and "decomposition" results. This minicourse will introduce this area. In particular, it will survey the celebrated Strong Perfect Graph Conjecture and its recent solution by Chudnovsky, Robertson, Seymour, and Thomas.

References

GERARD CORNUÉJOLS, *Combinatorial Optimization: Packing and Covering*, CBMS, vol. 74, SIAM, 2001.

Polyhedral Methods in Optimization

Alper Atamturk, Department of Industrial Engineering and Operations Research, UC Berkeley

In the last decade our capability of solving integer programming problems has increased dramatically due to the effectiveness of cutting plane methods based on polyhedral investigations. Polyhedral cutting planes have become the central features in optimization software packages for integer programming. In this lecture we will survey some fundamental results that led to the development of polyhedral analysis and methods in optimization. We will present applications in graph problems, mixed integer programming, and robust optimization.

References

[1] G. L. NEMHAUSER and L. A. WOLSEY, *Integer and Combinatorial Optimization*, Wiley-Interscience Ser. Discrete Math. Optim., 1988.

[2] A. SCHRIJVER, *Combinatorial Optimization: Polyhedra and Efficiency*, Springer-Verlag, 2002.

Integer Programming Duality

Jean-Bernard Lasserre, LAAS-CNRS, Toulouse

We consider the integer programming problem $P : \max\{c'x \mid Ax = b; x \text{ nonnegatif integer}\}$. A formal parallel between linear programming (LP) and continuous integration on one side and discrete summation on the other side shows that a natural duality for integer programs can be derived from generating functions and Brion and Vergne's counting formula. One may thus relate P with its (continuous) LP analogue and provide discrete analogues of some usual (continuous) duality concepts. In addition, we also provide a discrete Farkas lemma and an LP, strictly equivalent to P.

References

[1] M. BRION and M. VERGNE, Residue formulae, vector partition functions and lattice points in rational polytopes, *J. Amer. Math. Soc.* **10** (1997), 797–833.

[2] A. SCHRIJVER, *Theory of Linear and Integer Programming*, Wiley, Chichester, 1986.

[3] L. A. WOLSEY, *Integer Programming*, Wiley, New York, 1998.

Nonlinear and Semidefinite Programming

Stephen J. Wright, Computer Science Department, University of Wisconsin

Following the discovery of linear programming (its broad range of applications and the simplex method) in the 1940s, obvious questions of generalization were raised. Could the range of applications be broadened significantly by allowing the objective function and constraints to be nonlinear? And could effective methods be devised for the solution of such problems? The field of nonlinear programming was founded as a result of these questions. In all its aspects—applications, fundamental study of optimality conditions, algorithms, and software—it remains a very active area of study today.

Semidefinite programming concerns problems in which the variables include a symmetric matrix that is constrained to be positive semidefinite. Additional linear constraints and a linear objective are also present. Such problems arise in applications such as control and in approximation schemes for discrete optimization problems. Research in semidefinite programming was given a boost in the early 1990s with the discovery that interior-point methods could be used to solve these problems efficiently.

Nonlinear and semidefinite programming draw extensively on pure mathematics—chiefly, different flavors of analysis. The two areas continue to inform each other through, for example, the fields of variational and nonsmooth analysis.

In this talk we start by sketching the development of the two areas and their current range of applications. We then discuss optimality conditions, which are needed in order to recognize solutions and as the basis of algorithm design. We then outline current activity in algorithmic design, an area of continuing vitality and surprising developments.

References

[1] J. Nocedal and S. J. Wright, *Numerical Optimization*, Springer, 1999.

[2] Michael J. Todd, Semidefinite optimization, Acta Numerica, vol. 10, Cambridge University Press, 2001, pp. 515–560.

Approximation Algorithms for Discrete Optimization Problems

David B. Shmoys, School of Operations Research and Industrial Engineering, Cornell University

Most discrete optimization problems are NP-hard and hence unlikely to admit a polynomial-time algorithm that always finds optimal solutions. Instead, one can focus on polyonmial-time algorithms that compute near-optimal solutions and try to analyze the quality of the solutions that are computed. In particular, an r-approximation algorithm is a polynomial-time algorithm for an optimization problem that is guaranteed to find a feasible solution of objective function value within a factor of r of the optimum; the value of r is often called the performance guarantee of such an algorithm.

There has been a great deal of recent progress in research on the design and analysis of approximation algorithms for NP-hard problems, thereby expanding the breadth and depth of techniques used in this area. We shall examine several different algorithmic paradigms, including linear programming-based methods, such as rounding and primal-dual algorithms, as well as local search procedures and variants of more traditional greedy-type heuristics. We shall discuss how these methods can all yield approximation algorithms with specific performance guarantees. We shall focus primarily on just two (closely related) discrete optimization problems—the k-median problem and the uncapacitated facility location problem—and through recent results in this problem domain, we shall illustrate the gamut of the algorithmic techniques listed above.

References

[1] D. B. Shmoys, Computing near-optimal solutions to combinatorial optimization problems, *Advances in Combinatorial Optimization* (W. Cook, L. Lovász, and P. Seymour, eds.), Amer. Math. Soc., 1995, pp. 355–397.

[2] V. V. Vazirani, *Approximation Algorithms*, Springer-Verlag, 2001.

Algebraic Methods in Integer Programming

Bernd Sturmfels, Department of Mathematics, University of California, Berkeley

Integer programming is the problem of solving a linear system of equations over the nonnegative integers. The solutions are the lattice points in a convex polyhedron; and among all of these points, one wishes to locate one solution that minimizes a given linear cost function. In this lecture we demonstrate how algorithms from commutative algebra provide practical tools for various questions in integer programming. The basic idea is to encode lattice points by monomials, the cost function by a term order, and the linear constraints by a toric ideal. Gröbner basis methods can perform parametric computations with families of integer programs where the cost function or the right-hand side is allowed to vary.

Recently there have been two exciting advances in this field. On the theory side, there is the work of Barvinok and Woods which implies that, in fixed dimension, the Gröbner basis of a toric ideal and the relevant normal form reductions can be computed in polynomial time. On the practical side, there are two new software packages, LattE and 4ti2, both developed at UC Davis, which allow us to perform previously prohibitive computations. We give an introduction to these developments, and we present some concrete applications to problems from statistics.

References

[1] Bernd Sturmfels, *Gröbner Bases and Convex Polytopes*, University Lecture Series, vol. 8, Amer. Math. Soc., 1996.

Lattice Basis Reduction in Optimization

Karen Aardal, School of Industrial and Systems Engineering, Georgia Institute of Technology

Synopsis not available.