# 2004 Steele Prizes

The 2004 Leroy P. Steele Prizes were awarded at the 110th Annual Meeting of the AMS in Phoenix in January 2004.

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–06, and Birkhoff served in that capacity during 1925–26. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository-research paper; (3) Seminal Contribution to Research (limited for 2004 to analysis): for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field or a model of important research. Each Steele Prize carries a cash award of $5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2004 prizes the members of the selection committee were: M. Salah Baouendi, Andreas R. Blass, Sun-Yung Alice Chang, Michael G. Crandall (chair), Craig L. Huneke, Daniel J. Kleitman, Tsit-Yuen Lam, Robert D. MacPherson, and Lou P. Van den Dries.

The list of previous recipients of the Steele Prize may be found in the November 2003 issue of the *Notices*, pages 1294–8, or on the World Wide Web, `http://www.ams.org/prizes-awards`.

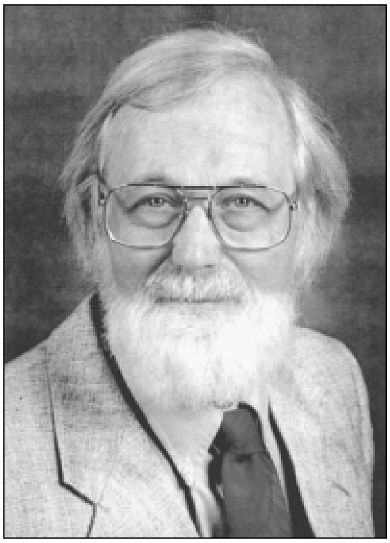
The 2004 Steele Prizes were awarded to John W. Milnor for Mathematical Exposition, to Lawrence C. Evans and Nicolai V. Krylov for a Seminal Contribution to Research, and to Cathleen Synge Morawetz for Lifetime Achievement. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

## Mathematical Exposition: John W. Milnor

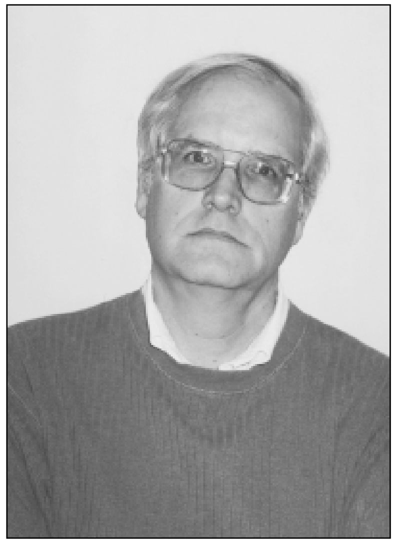
### Citation

The Leroy P. Steele Prize for Mathematical Exposition is awarded to John W. Milnor in recognition of a lifetime of expository contributions ranging across a wide spectrum of disciplines including topology, symmetric bilinear forms, characteristic classes, Morse theory, game theory, algebraic K-theory, iterated rational maps…and the list goes on. The phrase "sublime elegance" is rarely associated with mathematical exposition, but it applies to all of Milnor's writings, whether they be research or expository. Reading his books, one is struck with the ease with which the subject is unfolding, and it only becomes apparent after reflection that this ease is the mark of a master. Improvement of Milnor's treatments often seems impossible.

A portion of Kauffman's review of *Symmetric Bilinear Forms* by Milnor and Husemoller conveys the beauty evident in all of Milnor's expository work: "…Appendix 4, where this result is proved, is alone worth the price of the book. It contains Milnor's proof of a Gauss sum formula (due to R. J. Milgram) that uses elegant combinatorics and Fourier analysis to produce an argument whose



John W. Milnor



Lawrence C. Evans

Nicolai V. Krylov

corollaries include the divisibility theorem, the law of quadratic reciprocity and its equivalent in the language of forms over $Z$: the Weil reciprocity theorem. The proof is short, beautiful, and mysterious."

Milnor's many expository contributions to the mathematical literature have influenced more than one generation of mathematicians. Moreover, the examples that they provide have set a standard of clarity, elegance, and beauty for which every mathematician should strive.

### Biographical Sketch

John Milnor was born in Orange, New Jersey, in 1931. He spent his undergraduate and graduate student years at Princeton, working on knot theory under the supervision of Ralph Fox, and also dabbling in game theory with his fellow students John Nash and Lloyd Shapley. However, like his mathematical grandfather, Solomon Lefschetz, he had great difficulty sticking to one subject. Under the inspiration of Norman Steenrod and later John Moore, he branched out into algebraic and differential topology. This led to problems in pure algebra, including algebraic K-theory and the study of quadratic forms. More recently, conversations with William Thurston and Adrien Douady led to studies in real and complex dynamical systems, which have occupied him for the last twenty years. But he is still restless: one current activity is an attempted exposition of problems of complexity in the life sciences.

After many years at Princeton at the university and also at the Institute for Advanced Study, and after shorter stays at the University of California, Los Angeles, and the Massachusetts Institute of Technology, Milnor moved to the State University of New York at Stony Brook, where he has been the director of the Institute for Mathematical Sciences since 1989.

### Response

It is a great pleasure to receive this award, and I certainly want to thank the members of the Selection Committee for their consideration. It is of course also a tribute to my many coauthors: let me mention Dale Husemoller, Larry Siebenmann, Jonathan Sondow, Mike Spivak, Jim Stasheff, and Robert Wells.

I have always suspected that the key to the most interesting exposition is the choice of a subject that the author doesn't understand too well. I have the unfortunate difficulty that it is almost impossible for me to understand a complicated argument unless I try to write it down. Over the years I have run into a great many difficult bits of mathematics, and thus I keep finding myself writing things down. (And also rewriting, since I never get things right the first few times. Years ago, I was the despair of secretaries who would produce beautifully typed manuscripts, only to have them repeatedly cut, pasted, and scribbled over. Computers have eliminated this particular problem, but it still makes life difficult for coauthors.)

I am very happy to report that as mathematics keeps growing, there are more and more subjects that I have to fight to understand.

## Seminal Contribution to Research: Lawrence C. Evans and Nicolai V. Krylov

### Citation

The Steele Prize for Seminal Research is awarded to Lawrence C. Evans and Nicolai V. Krylov for the "Evans-Krylov theorem" as first established in the papers:

Lawrence C. Evans, "Classical solutions of fully nonlinear convex, second order elliptic equations", *Communications in Pure and Applied Mathematics* **35** (1982), no. 3, 333–363; and

N. V. Krylov, "Boundedly inhomogeneous elliptic and parabolic equations", *Izvestiya Akad. Nauk*

*SSSR*, ser. mat. **46** (1982), no. 3, 487–523; and translated in *Mathematics of the USSR, Izvestiya* **20** (1983), no. 3, 459–492.

Fully nonlinear elliptic equations are of interest in many subjects, including the theory of controlled diffusion processes and differential geometry. It is therefore of great interest to understand when these equations have classical solutions. The first results of any generality exhibiting classical solutions of the subclass of uniformly elliptic equations under suitable convexity conditions are due to the recipients in the cited works. These authors, independently and with different arguments, established the Hölder continuity of second derivatives in the interior, via a priori estimates, a result now known as the Evans-Krylov theorem. The Evans-Krylov theorem was both a capstone on fundamental contributions of the recipients and others and a harbinger of things to follow from the community.

While the Steele Prize for Seminal Research is explicitly awarded for the named works, it is noted that both recipients have made a variety of distinguished contributions to the theory of nonlinear partial differential equations.

**Biographical Sketch: Lawrence C. Evans**

Lawrence C. Evans was born November 1, 1949, in Atlanta, Georgia. He received his B.A. from Vanderbilt University in 1971 and his Ph.D. from the University of California, Los Angeles, in 1975; his advisor at UCLA was M. G. Crandall. Evans held positions at the University of Kentucky from 1975 to 1980, at the University of Maryland from 1980 to 1989, and is currently professor of mathematics at the University of California at Berkeley, a position he has held since 1989. He has been a visiting professor at Northwestern University (1977–78) and at the Institute for Advanced Study (1988). Noteworthy publications include *Weak Convergence Methods for Nonlinear Partial Differential Equations* (CBMS Regional Conference Series in Mathematics, volume 74, AMS, 1990), *Measure Theory and Fine Properties of Functions*, coauthored with R. F. Gariepy (Studies in Advanced Mathematics, CRC Press, 1992), and *Partial Differential Equations* (Graduate Studies in Mathematics, volume 19, AMS, 1998).

**Response: Lawrence C. Evans**

It is a wonderful honor to share with Nick Krylov this year's Steele Prize for a Seminal Contribution to Research. When I was Mike Crandall's graduate student at UCLA and at Wisconsin over thirty years ago, I learned from him the then startling lesson that nonlinear analysis need not be solely based upon linearization, meaning small perturbation theory from linear approximations. Brezis, Browder, Crandall, J.-L. Lions, and many others in the 1970s pioneered the analysis of various sorts of strongly nonlinear operators, a theory in which linearity played little role at all. I think this was why I was not especially afraid to look at so-called "fully nonlinear" elliptic equations in the late 1970s and early 1980s.

These are important PDEs, examples of which are the Monge-Ampère equation and Hamilton-Jacobi-Bellman equations in stochastic optimal control theory. And they are really, really nonlinear. But their solutions satisfy maximum principles, and this was a clue. It turns out that (i) when the nonlinearity is convex, we can get "one-sided" control on second derivatives; and that then (ii) the PDE itself provides a functional relationship among the various second derivatives, yielding thereby "two-sided" control. (Earlier Calabi had derived third derivative bounds for the Monge-Ampère equation, and Brezis and I had treated the very special case of the maximum of two linear elliptic operators.)

All success in mathematics turns largely upon persistence and luck; and while I can take some credit for the persistence, the luck was, well, luck—chiefly in that, quite unknown to me, one N. V. Krylov in the Soviet Union had turned his attention to these same problems at about the same time. And Nick's contributions to the subject have been extraordinary, including not only the interior Hölder second derivative estimates, for which independent discovery we are being honored, but also his previous, and great, work with Mikhail Safonov on Hölder bounds and the Harnack inequality for nondivergence structure second-order elliptic equations with discontinuous coefficients. We needed these to carry out step (i) mentioned above. Nick also later derived boundary second derivative estimates, something at which I completely failed.

So it is really an honor to share this prize with Nick and to have seen over the past twenty years the magnificent work of Caffarelli, Guan, Li, P.-L. Lions, Nirenberg, Spruck, Trudinger, Urbas, Wang, and many other researchers vastly extending these ideas.

**Biographical Sketch: Nicolai V. Krylov**

Nicolai Vladimirovich Krylov was born in Soudogda, the region of Vladimir, Russia, on June 5, 1941. He received his Ph.D. in 1966 and his doctorate of science in 1973 from Moscow State University; his scientific advisor was E. B. Dynkin. Krylov taught at Moscow State University from 1966 to 1990; he has taught at the University of Minnesota since 1990, and currently holds the position of Samuel G. Ordway Professor of Mathematics. He has supervised the graduate degrees of fifteen students.

Krylov has given invited addresses at the International Congress of Mathematicians in Helsinki (1978) and Berkeley (1986) and has given fifty-eight invited lectures, has written nearly two hundred research articles, and has published five monographs. A member of many journal editorial boards, Krylov was elected a Fellow of the American Academy of Arts and Sciences in 1993, received a Humboldt Research Award for Senior U.S. Scientists in 2001, and has been a recipient of numerous National Science Foundation grants.

**Response: Nicolai V. Krylov**

It is a great honor to share with Craig Evans this year's Steele Prize for a Seminal Contribution to Research.

In the times when I was an undergraduate student in Moscow State University, all kinds of control theory became popular. My scientific advisor, E. B. Dynkin, became interested in stochastic control theory, and being a brilliant lecturer, he easily attracted many people, including me, into it.

As often happens in probability theory, it was very easy to understand why certain probabilistic quantities should satisfy Bellman equations, but discouragingly for quite a while there were no ideas on how to prove this. Bellman equations are fully nonlinear possibly degenerate second-order partial differential equations with convex nonlinearity, of which the Monge-Ampère equation is the most famous example. When in about 1963 I asked O. A. Oleinik what was known about such equations, the answer was very short: "Nothing." This boosted even further my desire to prove the solvability of Bellman equations by probabilistic means. However, it took seven years before I realized how to prove a basic estimate, and after that the theory was completed in 1971–72.

It took even longer to develop an analytic approach. Working on some very natural questions from stochastic control theory, M. Safonov and I were lucky enough to obtain in 1978 Hölder norm estimates for solutions of linear equations with possibly rough coefficients. These estimates prove, in particular, the continuity of harmonic functions corresponding to diffusion processes with measurable coefficients. An automatic consequence of this fact is the lower semicontinuity of superharmonics. On the other hand, it is trivial to see that the second order directional derivatives of solutions of Bellman equations are superharmonics for certain diffusions. Thus they should be upper semicontinuous. But the equation itself says that a certain function of these directional derivatives is continuous. In addition, the function is monotone, and this yields the continuity of second-order derivatives.

Remarkably, Craig Evans obtained similar results at about the same time. Since then I have become a great admirer of Craig's talent, and I am very honored to share the prize with him.

Our results opened up the area to analytic treatment, and since then very many mathematicians have made amazing contributions. I want to mention only one directly related to our prize. A weak point in Craig's and my argument is that we need to differentiate the equation twice, which led to extra smoothness assumptions on the data. A major step forward in this respect was achieved by M. Safonov in 1984 when he showed that the estimate holds only under "natural" conditions.

## Lifetime Achievement: Cathleen Synge Morawetz

**Citation**

Cathleen Morawetz has greatly influenced mathematics in the broad sense throughout her long and distinguished career. Her fundamental research has resulted in seminal contributions to a number of areas. These contributions include her early work on equations of mixed type, with its striking consequences for the theory of flow around airfoils, her work on local energy decay for waves in the complement of an obstacle, and her results concerning the existence of transonic flow with shocks. Throughout Professor Morawetz's work one finds the theme of deep, creative mathematics used in the treatment of problems selected because of their interest in applied areas. She has not only contributed greatly to mathematics but also to the vitality of the interaction between mathematics and its applications.

Cathleen Morawetz's influence on mathematics extends well beyond her research contributions. In residence at the Courant Institute of Mathematical Sciences for almost all of her career, she provided guidance and inspiration to the stream of visitors and postdoctoral appointees, as well as to her own students. Her works include a number of influential contributions written in collaboration with younger mathematicians.

Beyond these mathematical contributions, commanding in themselves, Cathleen Morawetz has provided strong leadership for and representation of the mathematical community via her remarkable and generous service. The AMS has benefited from her membership on many committees, from her ten years of service as a Trustee of the Society, and her service as President of the Society. She dispatched her duties in these roles with excellence and did not merely serve; she provided leadership. The larger community benefited from her wisdom in positions such as that of a Trustee of Princeton University and a Trustee of the Sloan Foundation; mathematics also benefited from being represented by her in these roles. Among her pioneering "firsts", one notes that she was the first woman to direct an institute of mathematics in the U.S. and she was the first woman to receive the National Medal of Science for work in mathematics.

Thank you, Cathleen, for all you have done.

**Biographical Sketch**

Cathleen Synge Morawetz was born in Toronto, Canada, on May 5, 1923. She received a B.A. in applied mathematics from the University of Toronto in 1945, an M.Sc. from the Massachusetts Institute of Technology in 1946, and a Ph.D. from New York University in 1951. From 1950 to 1951 she was a research associate at MIT working on hydrodynamic stability with C. C. Lin. From 1951 on she worked with the group at NYU that became the Courant Institute, mainly at first with L. Bers, K. O. Friedrichs, and H. Grad.

Bers and Friedrichs introduced her to the fascinating problems of transonic flow; Harold Grad introduced her to problems in magnetohydrodynamics, especially the mathematical problem associated with very thin plasmas; and from Joe Keller she learned the open problems of wave propagation.

She became an assistant professor at the institute in 1958. Always involved in some administration, she eventually served as director of the Courant Institute from 1984 to 1988. She retired in 1993.

Cathleen Morawetz gave the AMS Gibbs Lecture in 1981. During much of her career she received support from the Office of Naval Research.

She served the Society as a member of the Council from 1973 to 1975, as a member of the Executive Committee in 1975 and from 1994 to 1998, as a trustee from 1975 to 1985, and was the second woman president of the Society from 1995 to 1997. She is still a member of two committees. She received the National Medal of Science in 1998.

Cathleen Morawetz was a trustee of Princeton University, a trustee of the Sloan Foundation, a member of the board of NCR, and a founding director of JSTOR (1995–98). In addition, she served on the board of the Mathematical Sciences Research Institute and chaired the board for theoretical physics of the Dublin Institute for Advanced Studies. She has received numerous honorary degrees.

She first studied the nonlinear wave propagation of shock wave theory as a student and later, at the suggestion of I. Segal, of semilinear equations. This resulted in fundamental work with Walter Strauss. Both her transonic theories and her work in wave propagation involved finding special identities and inequalities for the relevant equations.

**Response**

Receiving the Steele Prize for Lifetime Achievement is not only a huge honor but a stunning surprise for which I am very grateful. But I can never be quite as grateful as I am to those people who mentored and encouraged me in a lifetime of mathematics which, somewhat to my surprise, still goes on. The person to whom I am most grateful is Richard Courant, who steadfastly employed me in real research as I struggled to get a Ph.D. and to bear and raise four children between 1946 and 1958. He claimed it was Kurt Friedrichs who constantly recommended me to him, but Courant was surely the only person with the authority to follow this nonstandard path. Before that time I wavered a great deal in my career ideas, working as a chronographer during World War II, seriously contemplating teaching in India (a chance meeting with Cecilia Krieger sent me off to graduate school instead), trying out and failing at electrical engineering at MIT. There was also a considerable amount of external social pressure to abandon my career, but such ideas did not enter the minds of Courant and his colleagues—nor for that matter of my husband, Herbert.

Cathleen Synge Morawetz

Among the many people at the Courant Institute who educated, mentored, and helped me in the vast literature of mathematics (I have a bad memory) were not only Friedrichs but Lipman Bers, Joe Keller, Harold Grad, Fritz John, Paul Garabedian, Peter Lax, and Louis Nirenberg. Let me add the names of my collaborators who taught me so much: Walter Strauss, Jim Ralston, and Ralph Phillips.

Lastly, and by no means least, I am forever indebted to my mother for instilling in me the idea of ambition (then very unladylike) and to my father for the idea of intellectual achievement (not to mention the introduction to Courant).