Research Mathematicians and Mathematics Education: A Critique

Anthony Ralston

> Not since the New Math period of the 1960s had university mathematicians played such important roles in K–12 education as in California during the 1990s.
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> —David Klein, "A brief history of American K–12 mathematics education in the 20th century" [1]

Since at least the publication of *A Nation at Risk* [2] in 1983, there has been ferment about precollege mathematics education in the U.S. Since then, but particularly since 1993, research mathematicians have been more active on the precollege mathematics scene than at any time since the days of the New Math in the 1960s. Indeed, the pages of the *Notices* have regularly had articles, opinion pieces, and letters on the subject of school mathematics. This seems, therefore, a good time to review the impact of research mathematicians on school mathematics over the past ten years. In this article I will consider where the intervention of research mathematicians in school mathematics has had favorable results and where the results have been less than favorable.

Just about everyone agrees that research mathematicians have the knowledge and expertise to make important contributions to the improvement of school mathematics in the U.S. Indeed, it has been stated by a prominent mathematics educator that "American mathematics education has benefited from a virtually continual stream of support from prominent research mathematicians" [3]. Equally, just about everyone believes that school mathematics is in great, some would say dire, need of improvement. International comparisons, such as those in the Third International Mathematics Study [4], as well as scores on various tests, together with a plethora of anecdotal evidence, suggest that far from achieving (the first) President Bush's aim that U.S. mathematics education should be second to none by 2000, mathematics education in the U.S. is still nowhere near "second to none". Thus, the efforts of research mathematicians, working together with the other constituencies in math education, will be needed if the current situation is to show improvement. As noted in [5], "one of the most important ways mathematicians can be socially responsible [is] by working to improve precollege math education."

But instead of cooperation, we have had for the past decade, although recently at a lower decibel level, the Math Wars [6], [7], which pit (mainly) research mathematicians against (mainly) college and university mathematics educators and school mathematics teachers. No matter which side, if either, of these wars you are on, it is clear that they have, at least, prevented more improvement in U.S. school mathematics education than might otherwise have been achieved. Throughout this article I will use the terms "traditional" and "progressive" (or "reform") to designate the two sides in the Math Wars, because, whether you like these terms or not, they have become traditional (!) in the literature.

My aim here is not to refight or continue to fight the Math Wars, at least insofar as their mathematical substance is concerned. However, I will not hesitate to criticize the *tactics* of the math warriors when I think these have been counterproductive.

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On the Plus Side

Why is there agreement among most university mathematicians and mathematics educators that the potential contribution of university mathematicians to school mathematics education is great? Is it because their knowledge of mathematics is superior to that of mathematics educators? Partly, even though most mathematics educators, although they know far less mathematics than research mathematicians, are knowledgeable enough about school mathematics to design and implement curricula for school mathematics. More important, I think, is the generally deep knowledge that university mathematicians have of mathematics, because this gives them useful insight into what topics are particularly important in school mathematics and sometimes into good approaches for teaching these topics. With that said, here is a list of areas where research mathematicians can make and have made useful contributions to the debate about school mathematics.

Topics

The division of fractions was not included in the original version of one of the more popular reform curricula (Connected Mathematics Program). Whatever the reason for this omission, knowledge of how to divide fractions, while of limited value when the fractions consist entirely of numbers, is clearly important in much of more advanced mathematics. When this omission was noticed by a well-known university mathematician, the developers of the curriculum in question promised to remedy this in the next version of the curriculum and did so [8].

A less clear-cut example was the postponement of teaching the quadratic formula until the twelfth grade in another popular curriculum (Interactive Mathematics Program) [9], [10]. Surely this is idiosyncratic, and even if this delay is defensible, it is useful to have university mathematicians question choices of this kind.

In general, it is valuable for curriculum developers to have advice and criticism from university mathematicians about what to include or omit, if only because it is all too easy to make some bad decisions when immersed in the details of a curriculum to be used for all of elementary or all of secondary school mathematics.

Errors

Lengthy curriculum documents like books or long papers are almost certain to contain mathematical errors, no matter how competent the authors. This is particularly true when the documents have many authors, as is almost always the case for curriculum documents. Review of curriculum documents by research mathematicians *before publication* would always be salutary. Instead, the authors of documents such as the *NCTM Standards* [11] have made their lives considerably more difficult because of the errors, sometimes considerable numbers of them, that have been found by research mathematicians after publication. The errors found are often of a trivial and easily fixable nature, but their existence must serve to weaken the effect of a curriculum proposal.

Teaching Methods

On the *mathed* listserv of which I am a member, I am often impressed by the insight of other members, often research mathematicians, about how particular topics of school mathematics might be approached in novel and useful ways. Sometimes the ideas may be impractical because of aspects of school mathematics not fully understood by university mathematicians, but even when this is the case, the ideas may suggest changes in approach to mathematics educators and teachers. Indeed, research mathematicians should be used much more than they have been as a source of ideas for teaching the content of school mathematics.

Another aspect of this concerns methods that have become popular in school mathematics because they are easy to teach but are counterproductive to the understanding of the underlying mathematics. One such is the "infamous" [12, p. 7] FOIL algorithm to multiply two linear polynomials by bypassing the distributive law entirely. Research mathematicians have an important role to play in apprising mathematics educators and teachers of when the methods they teach, although perhaps narrowly effective and efficient, give the wrong mathematical lesson and may, therefore, ill serve the student.

In-Service Education of Teachers

Although there may be doubts about whether university mathematicians understand enough about school pedagogical issues for them to contribute much about these, there can be no doubt that they can contribute to improving the mathematical understanding of elementary and secondary school teachers. Even if it is true that in-service courses for teachers in summers and on weekends will probably never reach a significant fraction of teachers and often reach just those teachers who are already among the most competent, these courses are an important way of getting mathematical knowledge to teachers that their preservice education did not. Even if the subject matter of such courses is not directly applicable to the grade taught by a teacher, it may nevertheless provide breadth and insight that will improve teaching immediately or in subsequent years. Considerable numbers of research mathematicians, but surely not enough, have been involved in in-service courses for teachers in recent years (e.g. [13, p. 535]). While such activities can only scratch the surface of the problem of mathematics teaching in the U.S., they are nevertheless valuable and deserving of applause.

Preservice Education of Teachers

Another matter on which there appears to be almost universal agreement is that the mathematics education of prospective teachers of school mathematics is, with plenty of exceptions of course, woefully inadequate. Indeed, in some elementary education programs, prospective teachers take no mathematics at all. Moreover, even when mathematics courses for prospective teachers are taught in mathematics departments, they are normally taken much less seriously by the instructors than even the standard lower-division mathematics courses. There will probably not be much disagreement with the claim [13, p. 535] that "university mathematics departments must do a better job of teaching their students" or with the stronger statement that both mathematicians and mathematics educators have [14, p. 127] "largely failed to help teachers learn the mathematics they need in pre-service." A recent attempt at redress of this situation is [15] a collaboration of mathematics educators and mathematicians. In any case, more—preferably many more—research mathematicians need to get involved with the mathematics education of prospective elementary and secondary teachers.

On the Minus Side

Relations between the research mathematics community and the university mathematics education community have never been worse than they were in the late 1990s, although they appear to be less vicious now than they were then. The fault is surely not all on one side. However, my belief is that much of the fault—and most of the bitterness that has resulted—is indeed the fault of the research mathematics community, almost entirely the pure mathematics research community. In the Math Wars the research mathematics community has departed from its own high intellectual standards for research and has displayed an arrogance that has made things much worse than they need have been. Of course, neither of these strictures applies to every research mathematician who has been involved with the Math Wars, but it applies to too many and particularly to many of those who have been most vocal. Note that these remarks are not meant to imply that the positions taken by research mathematicians have necessarily been wrong or unwise, but just that they have often been expressed in most unfortunate ways.

The Riley Letter

If there is one single incident that epitomizes the Math Wars, it is the open letter to then Secretary of Education Richard Riley published in the *Washington Post* on 18 November 1999 [16]. The letter was signed by 6 mathematicians and "endorsed" by 219 others, almost all of whom were research mathematicians, among them many eminent ones. Among the handful of nonmathematicians were some eminent scientists, including several Nobel prizewinners. (It was noted by one observer [7, p. 201] that while the letter listed affiliations, chairs held, and prizes won, there was no mention of any teaching awards.)

The gist of the letter was a criticism of the Department of Education for designating five reform mathematics curricula as "exemplary" and five others as "promising". (While of course criticism of the designations of any of these ten programs was quite appropriate, the Department of Education had been mandated to make such designations by Congress, which also chose the "exemplary" and "promising" categories.) The six signers were, I assume, familiar with all ten programs, but it is nearly certain that at most a handful of the endorsers were familiar with all ten and highly probable that most were familiar with none.

No doubt many readers of this article have at one time or another signed advertisements in newspapers on strictly political matters whose text was distributed to you by someone well known to you. In such cases, the text of what you are signing is the whole story, and you are not endorsing opinions about documents not read nor about issues you know nothing about directly.

The Riley letter was quite different. Most of the endorsers were expressing an opinion about documents they had not seen. No doubt they decided to do so because they trusted the opinions of those who distributed the letter, and perhaps they were influenced by the inclusion in the letter of an inflammatory quote written five years previously by someone who served on the expert panel (of fifteen members) that made the exemplary and promising recommendations.

No reader of the *Notices* would, I think, express an opinion about a research paper he or she had not read, even if the paper was by a trusted colleague and even if it was also vouched for by other trusted colleagues. This would not be intellectually respectable. (Yes, there was a caveat in the open letter to the effect, "While we do not necessarily agree with each of the criticisms of the programs described above…," but that does not negate my point.)

An interesting contrast to the Riley letter was a letter from sixty-five mathematicians published in the *American Mathematical Monthly* and *The Mathematics Teacher* in 1962 [17] in which they expressed concern about the New Math ("Mathematicians, reacting to the dominance of education by professional educators who may have stressed pedagogy at the expense of content, may now stress content at the expense of pedagogy and be equally ineffective"). The signers of this letter knew whereof they spoke, but also their language was restrained, as evidenced by the two appearances of "may" in the quotation above.

Note that I express no opinion whatever about the quality of the ten exemplary/promising programs. How good or bad they may be is not my point here. It is certainly quite appropriate for mathematicians to involve themselves with the politics of mathematics education, but when they do so using the techniques of the average politician, we are all worse off.

Test Scores

In late 2002 I wrote an op ed piece [18] that suggested that one result of the No Child Left Behind Act [19] would be an epidemic of rising test scores in the U.S. and that this would mask a continuing decline in school math education in the U.S. In response, one of the most zealous of the math warriors wrote a letter to the newspaper attacking my claim but in effect supporting it, because his argument consisted of giving tables of increasing test scores in Sacramento. In a similar vein, when I visited another prominent math warrior a couple of years ago and expressed doubts about the 1999 California Framework [20], his response was to give me a file of rising test scores in California schools.

I am constantly amazed that research mathematicians place any faith whatever in the results of standardized tests, much less make them the arbiter of success or failure of a curriculum ("if the test scores in these programs don't go up, California isn't interested" [21]). True, mathematicians are not statisticians, but surely they generally know that experiments (i.e. standardized tests) with a plethora of uncontrolled variables cannot possibly yield meaningful results. (I suppose the uncontrolled variables are pretty obvious: classroom preparation time for the test, teaching to the test generally, accountability pressures on teachers to get them to show results, year-to-year experience with the test and the kind of questions asked, to name but a few. And there is the accumulating evidence that schools in some states are manipulating their dropout rates to assure that students who would get low scores on tests do not take the tests. See also [22, p. 645].)

The number of states where so-called "high-stakes" tests are being used is increasing rapidly. In Massachusetts the MCAS (Massachusetts Comprehensive Assessment System) is a continuing source of controversy. It has, claims the 1998 Massachusetts Teacher of the Year, put teachers under "enormous pressure" to prepare students for the MCAS with the result that "teachers are doing things that are developmentally inappropriate with students" [23]. Now I do not suppose readers of this article know much more than I do about what is "developmentally appropriate" for schoolchildren, but perhaps you will agree that good elementary and secondary school teachers are likely to know more than we do about how to represent ideas so that they make sense to their students.

The folly of using standardized tests to assess the value of a curriculum is best illustrated (to me) by a story that Steve Willoughby tells in his book *The Other End of the Log* [24]. When he was jointly appointed to the Department of Mathematics and the School of Education at the University of Wisconsin in Madison in 1960, he simultaneously became head of the mathematics department at Wisconsin High School, the university's laboratory school. After his first year as the high school department head, a faculty member in the university department of mathematics noted to his colleagues that the Wisconsin High School scores in a "statewide school mathematics test had deteriorated seriously from the previous year." Willoughby, who believed that the test was "hogwash," said he would arrange it so that the scores in the high school "will be the highest in the state this year." And that was what happened, because Willoughby knew how to give the students the kind of test-taking skills that assured much improved performance on the statewide test.

Research mathematicians do their reputation as trenchant thinkers no good whatsoever when they use sloppy standards to judge whether a given curriculum is improving math education or not. The fact is that judging how well or how poorly a new curriculum is faring is damnably difficult and except in very rare cases impossible, except over a considerable number of years. In particular, those who tout test scores as a measure of how well or how badly a new curriculum is performing do a disservice to the entire mathematics education enterprise.

Calculators

Probably no issue in math education has generated as much heat and as little light over the past two decades as that of the use of calculators in mathematics education. Research mathematicians, particularly those who oppose the so-called reform curricula, are generally opposed to the use of calculators in elementary school and wary about their use in secondary school (and, for that matter, in university mathematics). Still, despite the claim that "a clear majority [of academic mathematicians] oppose the new trends in math education" [25], of which the use of calculators is perhaps the most prominent, there is precious little evidence about how the university mathematics community as a whole feels about this issue.

Mathematics educators, even those who favor the use of calculators in schools, would generally admit that there is no *conclusive* evidence about the effect of using them in school mathematics. However, there are numerous studies that purport to show that the use of calculators in schools at worst does no harm to the learning of traditional mathematics and may at best enhance that learning. Only a very few studies seem to contradict these results. Few of the studies on either side are compelling, but there is no question about where the weight of the evidence lies [26].

Then there is the large amount of anecdotal evidence ("horror stories" [12, p. 9]) and oracular pronouncements that support the position that calculator use in school is likely to rot the brain. This evidence is not compelling at all, not just because anecdotal evidence seldom can be, but also because it is almost always used to support a predetermined position.

There is certainly a widespread belief, which I share, that students arriving at American colleges and universities have been steadily more poorly prepared for college mathematics over a period extending back now at least a quarter of a century. This belief is consonant with the scores of American students on international comparisons and on college entrance examinations such as the SATs. Some of this poor preparation of incoming college and university students may perhaps be attributable to their use of calculators in elementary or secondary school. But at most a miniscule amount could be from this cause, since the problem predates the time when there was any substantial use of calculators in American schools, and even today such usage is far, far from universal. Nevertheless, calculators are the standard scapegoat for the poor preparation of students in basic mathematics (e.g., "beguiled by ever fancier calculators and computers, teachers appear less and less able to produce students who are masters of these basic topics" [27, p. 868]). Too often in the debate about calculators, as in much of the debate about mathematics education, research mathematicians are wont to use *post hoc ergo propter hoc* reasoning (e.g., "many are ignorant due to a miseducation which involves heavy use of calculators" [28, p. 459]).

It may be that the teaching of pencil-and-paper arithmetic, which has been the gateway to the study of school mathematics for more than a century, is as important as it has ever been. But this position can be supported only if there is recognition that the terms of the debate, although not necessarily the conclusions, about what is important to teach in mathematics have been changed forever by calculators. Thus

> Even if everything had been fine with U.S. math education, we would have to pay attention now to how the availability of sophisticated calculational tools changes what is important to teach [29, p. 244].

Before calculators became ubiquitous and cheap, it was easy to argue that some skill in pencil-and-paper arithmetic was necessary for just about all adults. No longer. This skill itself is of essentially no value any more. Devoting a considerable amount of the instruction in elementary school to pencil-and-paper arithmetic can only be justified in the twenty-first century by arguing its value in instilling essential understanding of numbers themselves (e.g. place value) or for its value in preparing students for the further study of mathematics. If you wish to argue that something like traditional instruction in pencil-and-paper arithmetic is a necessary part of elementary school mathematics and also that the use of calculators should be banned from elementary school classrooms, you need to argue that the use of calculators *at all* will inhibit sufficient learning of pencil-and-paper arithmetic. Or you need to argue that teachers will so misuse calculators if they are present that adequate learning of pencil-and-paper arithmetic will not ensue.

However, although there is no plausible evidence that in the hands of good teachers calculators produce bad effects in elementary school classrooms, this seems to have had no effect whatever on the research mathematics community.[1] Oh yes, in the hands of poor teachers or in classrooms in schools where very little learning of any subject takes place, children may become totally dependent on calculators, resulting in what has been called "Computer-Assisted Mathematical Incompetence" [31]. But it seems to me to be fantasy to believe that banning calculators in such classrooms or such schools will have any noticeable effect on the arithmetic and mathematical abilities of students.

The *ex cathedra* statements of research mathematicians about school arithmetic are particularly unhelpful, because even when the arguments they use are plausible in themselves, they often try to bias the debate in ways that can only antagonize the mathematics education community. Long division provides a case in point. At least since the publication of the Cockcroft Report in 1983 [32, p. 114], which stated a belief "that it is not profitable for pupils to spend time practising the traditional method of setting out long division on paper, but that they should normally use a calculator," there has been an ongoing debate about whether the traditional long division algorithm (LDA hereafter) should be taught at all and if it is taught, what level of proficiency pupils should attain.

This is an important debate, because it gets to the heart of the question of what is still important

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[1] *What appears to be almost visceral opposition to technology sometimes leads otherwise reasonable people to make ridiculous claims such as "A computer cannot teach any more effectively than an oscilloscope can bring about world peace" [30, p. 991]. I am not an enthusiast for computers in classrooms or in teaching, but this comment is rubbish and evinces no understanding whatever for what has been accomplished in computer-related teaching in the past quarter century.*

in school mathematics in the calculator age. A variety of positions, from attaining traditional proficiency in LDA to not teaching it at all, can be reasonably argued, but what is not reasonable is to argue, as research mathematicians are wont to do [33], that there is only one right answer, namely teaching the traditional LDA. In [33] a variety of benefits of teaching LDA are argued. All are defensible, but the impression that only LDA can achieve these benefits is, frankly, nonsense. One example of this is the claimed value of LDA for converting proper fractions to decimals. Actually this conversion is more easily accomplished by an algorithm equivalent to LDA that writes the quotient as A/B = .abcdef... and computes the successive digits of the quotient by multiplying by 10 and subtracting. Not only is this method easier to represent in algorithmic notation than LDA, but in addition it suggests immediately the well-known algorithm for converting repeating decimals to fractions. The importance of LDA is linked in [33], as it often is, to the similar problem of the division of polynomials. But division of polynomials is in fact a much simpler computation than LDA and is easily expressed in algorithmic notation by a simpler algorithm than LDA.[2]

The penchant of research mathematicians for claiming that there is one right way to teach a particular aspect of school mathematics is virtually never correct. Another example is the reported claim that there is "one right way to teach odd and even numbers" [7, p. 199].

More generally, harking back to the papers of Benezet in the 1930s [34], it has been suggested by a prominent research mathematician that perhaps the teaching of the algorithms of arithmetic "should be postponed…until grade 6" [35]. Indeed, a useful exercise for all mathematicians and mathematics educators would be the gedanken experiment of wondering what would happen if mathematics education did not exist now and you had to invent it. How much pencil-and-paper arithmetic would there be? How much mental arithmetic? How much use of calculators [36]?

Fuzzy Math and the One Right Answer Syndrome

Those who advocate the reform of school mathematics curricula are often accused by their detractors—almost always research mathematicians or journalists influenced by research mathematicians or parents influenced by journalists—of advocating "fuzzy math". What is fuzzy math? It is itself a fuzzy concept, meaning whatever the critics of new school curricula want it to mean at a given time. Of course, even if the term bears no relation at all to any truth, it is a wonderful club with which to beat the proponents of reform curricula. (See the next section.)

At one extreme it means anything whatever to do with any nontraditional school mathematics curriculum [37]. Almost as extreme is the charge that nontraditional curricula tolerate wrong answers or at least make no attempt to correct wrong answers. This charge is of course false [14] and cannot be laid correctly at the door of any developer of a nontraditional curriculum. Less extreme but also false is the claim that there are some (who?) who believe that "there is always more than one correct answer to a math problem" [38, p. 869]. Least extreme but probably most numerous are those who reject the notion that *some* math problems can have more than one correct answer. This last position is not "fuzzy" in any sense; it is true. Here are two examples: (1) Given two parallel lines and a segment longer than the distance between the two lines, marked out on one of them, draw an isosceles triangle with the segment as one side and the third vertex on the other parallel line. (2) Discover and explain all you can about numbers that can be written as the sum of consecutive natural numbers; e.g. $9 = 4 + 5, 9 = 2 + 3 + 4; 15 = 1 + 2 + 3 + 4 + 5$. Now of course each of these could be transformed into a problem with a single right answer. But to do so would be to miss the point that for the first problem it is instructive for students to see that other students have solved the problem in different ways. The second, more advanced, problem illustrates the general lesson that mathematics is about investigation, conjecture, refutation or proof, and explanation.

Of course, *precision* is crucial in mathematics, but an emphasis on precision does not require that all problems have only one correct answer. This notion, which seems obvious to me, appears to be anathema to many research mathematicians. It carries over all too easily to the notion that curriculum should be rigidly specified ("one right curriculum") and that the method of teaching that curriculum should also be rigidly specified.

Arrogance

The "fuzzy math" epithet implies that the traditional math side of the Math Wars has not made much attempt at the "civil, constructive discourse" that Suzanne Wilson [7, pp. 216–29] thinks is the way to end the Math Wars (see also [39, p. 488]). Indeed, if there is one reason more than any other why the Math Wars may be expected to rumble on into the future, it is because all too many "mathematical scientists have tended to look upon education professionals with doubts bordering on ill-disguised contempt" [40, p. 21]. Indeed,"sarcasm and ridicule" [41] and "caricatures" [14, pp. 127–9], while noticeable on both sides of the Math Wars, appear to be much more prevalent on the traditional side espoused by many research mathematicians.

[2] *The implicit assumption here, of course, is that the coefficient arithmetic in the division of polynomials is, in virtually all cases of interest, easily done mentally.*

The arrogance of mathematicians towards mathematics education manifests itself in various ways. Here are two:

1. *Research in Mathematics Education.* At the extremist end are those research mathematicians who believe that research in math education is (virtually?) an oxymoron (e.g., "mathematics education research as described in this book is, in fact, inferior to descriptive discussion because it *is* descriptive discussion without humility" [42, p. 282]).[3] This conclusion is reached by defining research as a word that can be used only when applied to theorem/proof mathematics research or the scientific method paradigm of the physical sciences. From this perspective just about everything done in education or the social sciences is not research. Even those who take a less extreme view often subscribe to the dictum that "teaching must be an art and not a science" [42, p. 284]. This is very convenient, because if teaching is an art, then anyone's opinion is equally valid ("I know what I like"). This automatically gives the opinions of research mathematicians about math education the same standing as those of mathematics educators. But although some research mathematicians have thought long and hard and effectively about mathematics education, too many are in the grips of "the questionable belief that, just because mathematicians are good at mathematics, they should also be able to contribute to the effective presentation of elementary mathematics to an often unmotivated and unresponsive public" [44, p. 180].
2. *The Mathematics Credentials of Mathematics Educators.* Many mathematics educators who have been involved with the development of reform mathematics programs have Ph.D.'s in mathematics and thus might a priori be considered competent in school mathematics. But, no: "There is a distinction between math educators who are primarily interested in questions involving education, and mathematicians who know about mathematics" [45]. This is, of course, ironic, because more often than not the research mathematics critics of mathematics educators have no credentials whatever in school mathematics education.

Not all research mathematicians are so arrogant with respect to their abilities as math educators. For example, Wilfrid Schmid has written, "We [i.e. mathematicians] are not qualified to do their [i.e. mathematics educators'] work" [25]. And he quite correctly goes on, "Yet we are qualified as *critics* (emphasis added) of reforms in math education."

Often the attitude of research mathematicians about the mathematical abilities of mathematics educators is manifested in their tendency to nitpick the documents, such as [11], of reform mathematics curricula. Of course finding errors and communicating them to the authors of these documents is a valuable service, even when the errors are little more than typographical. But to use such nit-picking as a club to try to discredit these documents and their authors is unworthy. I dare say that most of the readers of this article have written books and/or long papers in which inevitably minor errors remained. But they surely did not expect to be pilloried for these. The publication in 2000 of *Principles and Standards of School Mathematics* [47], the revision of the *NCTM Standards*, has damped down the criticism of [11] to some extent but has by no means done away with it.

If research mathematicians would engage in "civil, constructive" criticism rather than, more often than not, arrogant putdowns, the result of the Math Wars would not be an endless battle to the detriment of school mathematics education in the U.S. (Of course, many (most?) mathematicians are not arrogant, but those who doubt the prevalence of intellectual arrogance among mathematicians might care to look at a recent study of seventy mathematicians in Britain [46].)

My conclusion is that although a number of research mathematicians have contributed positively to school mathematics education in recent years (see, for example, [40, p. 18]), the research mathematics community has largely squandered an opportunity to have a significant positive impact on American mathematics education. Too many have used a "scattershot approach" that often takes the form of "unsubstantiated claims and random anecdotes" [3]. Too often the result has been that when they have become active in mathematics education, research mathematicians have not lived up to the high standards that they normally bring to their own professional work.

How could research mathematicians make a more positive contribution to school mathematics education in the future? The most important way would be for research mathematicians to see their role as colleagues of mathematics educators and *constructive* critics of work in mathematics

[3] *If humility is needed in the Math Wars, a bit from the research mathematics community might be in order. As Igor Sharygin [43] has noted: "…professionals in pure mathematics usually consider school mathematics as an integral part of mathematical science.…And it is a grave fallacy. Having indisputable authority and enormous knowledge in their own field, these prominent scientists have frequently invaded a domain of public education in which they are not always competent. (Another false belief is that it is quite enough to be a good mathematician to solve the problems of mathematical education.)" Although Sharygin was referring to Russia, this quote is equally applicable to the U.S.*

education. In addition, (almost all) research mathematicians, who have had little or no experience in elementary or secondary classrooms, should learn to be rather more humble about school mathematics, since their experience in college and university classrooms is seldom relevant to elementary and secondary education.

Another way for research mathematicians to make a positive contribution to school mathematics education would be to help in recruiting more intellectually able people to teach mathematics rather than throwing brickbats about curriculum matters. This is a crucial problem [48] that badly needs to be addressed, and it is one where both mathematicians and mathematics educators should find it easy to sing from the same hymn book. It is past time that the intellectual leaders in American mathematics started to make the case that those we attract to elementary and secondary school teaching need to be the "brightest and best" [24, p. 57], as intellectually able as those attracted to professions such as law and medicine and, yes, to the academic world.

Pending such an epiphany by research mathematicians, the Math Wars, which once were avoidable, will continue to be part of the school mathematics landscape. The research mathematics community, through its hubris, has by and large contributed—and continues to contribute—to a worsening situation in school mathematics in the U.S., a situation that shows no signs of getting much better in the foreseeable future. The lesson of the New Math has not been learned.

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About the Cover

The Internet on April 1, 2003

This month's cover was contributed by Bill Cheswick of the Lumeta Corporation and diagrams the Internet as it was on April 1 of last year. It nicely fits in with the topic of this year's theme for "Math Awareness Month", which is networks. Color coding in the figure indicates distance from the scanning host. Layout in the diagram was determined by an algorithm that considers the graph as a physical system of springs and determines its minimal energy configuration. Thus, in a sense the graph interprets itself. More information on the Lumeta Internet mapping project can be found at `http://research.lumeta.com/ches/map/` and also in the paper "Mapping and visualizing the Internet" by Cheswick, Hal Burch, and Steve Branigan in the proceedings of the 2000 USENIX Annual Technical Conference. Designing graph layout is an interesting problem. A good place to start in the huge literature on this topic might be the proceedings publication "Graph Drawing", *Lecture Notes in Computer Science,* Springer 1984.

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