Math through the Ages: A Gentle History for Teachers and Others

Reviewed by Philip C. Curtis Jr.

Math through the Ages: A Gentle History for Teachers and Others
William Berlinghoff and Fernando Gouvêa
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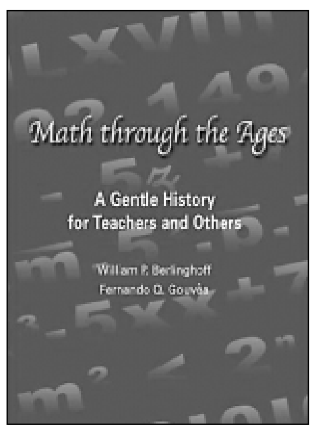
The state of school mathematics education in the United States and the training of school mathematics teachers both have been of constant interest to the professional mathematics community in recent years. The content of school mathematics has changed dramatically. An emphasis on discrete mathematics, probability, and statistics has emerged that was completely absent not long ago. Beginning algebra has often been moved to the eighth grade, and single variable calculus is the acknowledged goal in the 12th grade. Computers and sophisticated handheld graphing calculators have greatly enlarged the scope and type of problems that can be considered.

This change in content and emphasis has great implications for the training of elementary and high school teachers. It has been long acknowledged that prospective elementary school teachers need much more than just the mathematics they learned in school to be effective teachers. A mathematics major or the equivalent is recommended, and in many cases required, for high school teachers. However, recent studies have shown that more than advanced course work is necessary if the teaching is to be effective.



The report of the Conference Board of the Mathematical Sciences, *The Mathematical Education of Teachers*, published in 2001, makes this quite explicit: "Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach." At the high school level, the report recommends "Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics." This course should bring together all the strands of school mathematics, algebra, number theory, geometry, analysis, and probability and statistics, considering the basic ideas involved from an advanced standpoint, and "explicitly tracing the historical development of key ideas, identifying questions that were challenging for mathematicians and will be difficult for students."

The book under review, *Math through the Ages*, by William Berlinghoff and Fernando Gouvêa, satisfies this objective brilliantly. The authors first give

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a gentle history or, as they put it, a "History of Mathematics in a Large Nutshell," which covers the development of mathematics from its earliest days to the present, emphasizing the development of fundamental ideas that are central to school mathematics.

This account begins with a discussion of the type of measurement problems considered in ancient Egypt and the Near East. This is followed by a brief discussion of Greek, Indian, and Arabic contributions that then stimulated development in medieval Europe. Progress in algebra and geometry during the fifteenth and sixteenth centuries leads to the development of the calculus and applications during the seventeenth and eighteenth centuries. A rigorous foundation for analysis was developed in the nineteenth century. The history concludes with a brief discussion of modern mathematics and the role played by technology. The discussion is low key, and the mathematical demands on the reader are minimal. It provides an appropriate overview of mathematics that ideally should be in the background of mathematics teachers at all levels.

This is followed by twenty-five detailed historical sketches of how various mathematical ideas developed, each with examples and problems to illustrate how the ideas emerged. These begin with a discussion of arithmetic, the writing of numbers and the various symbols used, fractions, negative numbers, the decimal system, and the concept of zero. It may come as a surprise to the reader that the standard technique of solving polynomial equations by factoring and setting each factor equal to zero was a revolutionary step when it was first proposed by Thomas Harriot in the seventeenth century. An important thread in the discussion is the simplification of notation and the emergence of the Hindu-Arabic number system based on powers of ten.

Following a discussion of plane and coordinate geometry, there are projects devoted to the development of probability and statistics. The discussion ends with projects devoted to computers and the concept of infinity. Each sketch contains well-chosen suggestions for further reading. The bibliography at the end is a marvel, containing a very extensive selection of historical references both recent and older.

Each of the sketches is designed to illuminate the historical development of the ideas in such a way as to deepen the reader's understanding of the mathematics involved. The exercises accomplish this objective admirably both from the point of view of a prospective or practicing teacher and, in many cases, from the point of view of a student struggling to understand the material for the first time.

As an example, young students often have difficulty with the concept of negative numbers when they first meet them. I remember conversations with my granddaughter, then in the sixth grade, who was convinced that negative numbers just did not exist. It is important for the teacher to be aware that mathematicians in the fifteenth and sixteenth centuries had the same problem. As the authors point out in the sketch on negative numbers, mathematicians could manipulate with negative numbers well before they were clear about the concept. It was not until the middle of the nineteenth century that negative numbers were completely accepted. It also took a long time for mathematicians to be comfortable with the statement that Cardano's solution to the cubic equation

$$x^3 - 15x - 4 = 0$$

was

$$x = \sqrt[3]{2+\sqrt{-121}} + \sqrt[3]{2-\sqrt{-121}},$$

and that this was indeed $x = 4$. It wasn't until Gauss, Argand, and Hamilton in the nineteenth century showed how complex numbers could be represented as points in the plane and how the arithmetic operations could be interpreted that mathematicians completely accepted complex numbers. For teachers to have a deep understanding of complex numbers, this historical background would seem to be essential.

Another example concerns the discussion of the quadratic formula. In many introductory textbooks, this formula is introduced directly following a discussion of factoring of quadratic polynomials. Such an approach often leaves students completely at sea and encourages them just to memorize the formula and not to understand why it works. Historically, a geometric illustration of the process of completing the square always accompanied a verbal description of the technique of solving the equation. This began with Al-Khwarizmi in the ninth century and was standard in the discussion of quadratic equations for hundreds of years. It should also be well known to all algebra teachers today.

Probability and statistics and dealing with data is another area in which knowledge of the historical development of the subject can be very useful. Elementary probability is now introduced in the middle grades, and students are encouraged to conduct experiments with real data and draw conclusions. None of this was in the curriculum a generation ago. What were the problems that gave rise to the study of probability? The discussion of the Chevalier de Méré's problem of points by Pascal and Fermat is no doubt unfamiliar to most teachers. However, it provides an illuminating introduction to the study of games of chance. Statistics had its origins in the analysis by John Graunt of weekly burial records in London in the beginning of the seventeenth century. What questions did he ask, and how reliable were his answers?

That mathematics is a vital and growing creative enterprise is important for all teachers to know. The discussion of Andrew Wiles's solution to Fermat's Last Theorem is especially pertinent. What are some other elementary problems that are easy to state but seem to be very difficult to solve? The direction of modern mathematics, the development of computer technology, and its impact on mathematics are all made accessible to the teacher via the historical discussion and associated sketches and problems contained in *Math through the Ages*.

This book has a role to play in the mathematics education of prospective teachers at all levels. For teachers in the elementary and middle grades, the sketches dealing with arithmetic and the introduction to algebra and geometry should be part of the course offerings provided by mathematics departments for these students. For mathematics majors contemplating high school teaching, this book would form an ideal base for the recommended capstone course at the senior level. The standard major designed to prepare students for graduate school or careers using mathematics does not in general provide an in-depth look at these more elementary ideas. In fact, such a course would be useful for all math majors.

Practicing teachers, just as much as new teachers, need an in-depth understanding and historical perspective of the mathematics they teach. Thus this book has an important role to play in the in-service courses provided to practicing teachers. Even if the training of new teachers were ideal, the rate of replacement of retiring teachers by fully trained new teachers is painfully slow. In California, for example, in 2002 it was projected that 2,131 new mathematics teachers were to be hired for 2003–2004. However, the number of mathematics teaching credentials awarded to holders of undergraduate degrees in mathematics was only 422. That year there were 1,389 bachelor's degrees in mathematics awarded in California. The need for the insight provided by *Math through the Ages* and similar books will be with us for a long time.

The authors invite readers to make suggestions for additional sketches to be added to later editions. I would like to see a sketch devoted to a discussion of the well-ordering principle of the positive integers and the principle of mathematical induction. The applications of induction are of course well known. That every nonempty collection of positive integers has a smallest member is the fundamental assumption implying that every positive integer is either prime or has a prime divisor, and therefore there are infinitely many primes. It is implicit in the geometric proof of the irrationality of the square root of 2. In the essay, "Irrational numbers", Dedekind uses it explicitly in his short proof of the irrationality of square roots of integers that are not perfect squares (c.f. James R. Newman, *The World of Mathematics*, vol. 1, p. 531).

Math through the Ages is both clearly and accurately written and is a joy to read. The exercises, an important tool in developing the ideas, are well chosen and often quite thought-provoking. The only historical inaccuracy that I uncovered was the listing of Niels Henrik Abel as Danish rather than Norwegian (p. 50). This book should have considerable impact on the mathematical education of teachers. I hope that it does.