Book Review

The Constants of Nature and *Just Six Numbers*

Reviewed by Brian E. Blank

The Constants of Nature
John D. Barrow
Pantheon Books, 2003
368 pages, $26.00
ISBN 0-375-42221-8

Just Six Numbers
Martin Rees
Basic Books, 2001
208 pages, $14.95 (softcover)
ISBN 0-465-03673-2 (softcover)
ISBN 0-465-03672-4 (hardcover)

Consider the two constants G and G. The first G arises frequently when we manipulate special functions. We represent this G by dozens of series and integrals, we study its continued fraction expansion, and we calculate millions of its digits. Any mathematician who sees its definition *knows* that it is an interesting number. This first G, Catalan's constant, is an exemplar of the mathematical constant: we are not surprised to see it appear in disparate problems in combinatorics and analysis, but we do not expect to ever learn that it has anything to do with the price of tea in China. How different it is from the second constant G. This second G is, for now, of little interest to the mathematician. We do not ask whether it is irrational. We do not, in fact, give it a second thought. This G, Newton's gravitational constant, is an exemplar of the constant of nature, and it has everything to do with the price of tea in China.

The contrast between the constants of mathematics and the constants of nature brings to mind "The Road" that physicist Leon Lederman talks about [6]. In Lederman's metaphor, The Road branches off into tempting side streets down which the chemist, the electrical engineer, and other unnamed vagabonds wander. But "Those who stay with The Road find that it is clearly marked all the way with the same sign: 'How does the universe work?'" It is not a sign that mathematicians always heed. We go where we please, gratified nonetheless by the frequency with which our meandering side streets cross The Road. The physicists who have not strayed tell us that the workings of the universe, as they are now understood, involve a small number of matter particles, or fermions; a small number of interactions among the fermions; and a small number of carrier particles, or bosons, that affect the interactions. The particles, as well as the equations that describe their interactions, entail certain constants, quantities that might be called fundamental constants of nature.

One or two at a time, the constants of physics make their appearance in many of the popular books that cram the cosmology shelves. In the last few years two nontechnical works entirely devoted to these constants have been added to the shelves. Each is written by a leading English astrophysicist with a long track record of superb expository writing. In *Just Six Numbers*, Sir Martin Rees discusses how his six titular numbers shape the universe. His theme is that these numbers are *fine-tuned*: if at an early instant of the universe the value of one of these numbers had been changed by more than a bit, then a sterile, lifeless universe would have resulted. John Barrow's *The Constants of Nature* discusses this theme as well, but with very little overlap. Whereas Rees is concerned with hypothetical change, Barrow concentrates on the real thing.

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Just Six Numbers lives up to the brevity of its title. By the end of the third page Rees has described his six numbers and set out his program for the reader. Within another 160 pages he has neatly wrapped it up. There is scarcely an equation in his book, but it turns out that a good deal of physics can be explained by concepts rather than mathematics. Many physicists excel at this type of exposition, but Rees must surely be among the best.

We can get a good idea of Rees's book by restricting the discussion to just one number, the nuclear fusion number $\mathcal{E} \approx 0.007$ that measures the strong nuclear force. It is this number $\mathcal{E}$ that determines the lifetime of a star and the elements of the atomic table. When hydrogen atoms fuse to form helium in a stellar reaction, 0.7% of their mass is converted into energy. This fusion is a multistep process, the second step of which is the formation of deuterium. If the strong nuclear force were weaker so that $\mathcal{E} = 0.006$, then deuterium would be an unstable isotope and the process would not continue to the formation of helium. If the strong nuclear force were stronger so that $\mathcal{E} = 0.008$, then two protons would be able to bind directly, with no need of neutrons to overcome the electrical repulsion. In this scenario, no hydrogen would have survived from the big bang, and without stellar fuel there would be no life.

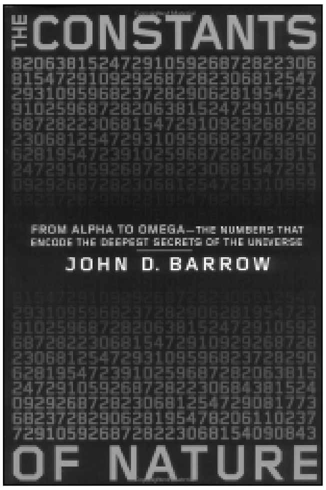
This analysis of $\mathcal{E}$ is typical of the fine-tuning argument. In the case of $\mathcal{E}$, additional requirements may further limit the range in which $\mathcal{E}$ must lie. For example, in order for life to exist, carbon must be synthesized, a process that also goes forward in stages. First, two helium atoms come together to form the isotope beryllium 8. This isotope then captures another helium atom to form a radioactive isotope that decays to stable carbon. The problem is that beryllium 8 is itself unstable and apt to disintegrate before a carbon nucleus has been formed. Without further fine-tuning of $\mathcal{E}$, an essential step in the creation of carbon would be most unlikely and carbon-based life forms would not exist.

One by one, Rees constrains the values of his numbers most persuasively. He is like an expert magician who induces us to keep our eyes on his right hand while his left hand is busy pushing unwanted complications from our view. Few who read Rees's proton-proton fusion discussion will pause to wonder, "Hey, wait a minute! Where does that deuterium neutron come from?" In the end, readers will have to rely on Rees's considerable authority. However, not everything he asserts is without controversy. In an essay titled "A designer universe?" [13, p. 236], Steven Weinberg also considers the implications of carbon synthesis, referring to precisely the same research that Rees cites. But where Rees sees fine-tuning, Weinberg concludes, "Looked at more closely, the fine-tuning of the constants of nature here does not seem so fine."

Rees closes his book with the sort of chapter that has become commonplace in popular cosmology writing: science fiction with academic cachet. As he has done in a few previous cosmology books, Rees advances a conjectural but "compellingly attractive" theory of the multiverse, a theory that allows for multiple big bangs. In the multiverse, "Separate universes may have cooled down differently, ending up governed by different laws and defined by different numbers." At one time a theory postulating multiple universes would have been dismissed as inordinately extravagant. Nowadays, however, the paradigm of simplicity is not an implicit principle of physical law. As Weinberg states the matter [12, p. 224], "The experience of the last three-quarters of a century has taught us to distrust [Ockham's razor]; we generally find that any complication in our theories that is not forbidden by some symmetry or some other fundamental principle actually occurs. Simplicity, like everything else, must be explained."

One basic simplifying assumption, the hypothesis that the constants of nature do not change from place to place in our universe or from epoch to epoch, is currently under intense scrutiny. The possibility that this assumption might be refuted is the basis of Barrow's book on the constants of nature. Although Barrow once took a stab at defining these fundamental numbers [2, p. 358], he shies away from a precise definition in his new monograph. Instead, he relies on imprecise prose. We read that the constants of nature "give the Universe its distinctive character," that they are the "bedrock ingredients of our Universe," that they "lie at the root of sameliness in the Universe," that they "encode the deepest secrets of the Universe," that "they define the fabric of all that is," and that "they are the barcodes of ultimate reality, the pin numbers that will unlock the secrets of the Universe—one day." For those of us not following The Road, these descriptions may elicit a pang of aimlessness, but they do not allow us, for example, to look at the numbers that constitute Rees's list and say, "This one is a constant of nature and this one is not."

In fact, at the time of this writing there is no canonical list of fundamental constants of nature. There is not even agreement on what sorts of constants should go on such a list [4]. To begin with, physics has two kinds of constants. One type of constant consists of what you and I call numbers. Physicists call them *pure numbers* or *dimensionless numbers*. The constant π is an example: we measure the circumference and diameter of a circle using some sort of units, and then when we form the ratio, the units cancel and the dimensionless number π results. In this example, the numerator



and denominator are what the physicists call *dimensionful numbers*. So are the speed c of light in a vacuum and the masses m_p and m_e of, respectively, the proton and electron. Although a dimensionful number is a numerical quantity, the pure number used to describe it depends on the choice of units.

There are three basic physical dimensions that require units: mass M, length L, and time T. These are necessary and sufficient for describing the dimension of any physical quantity. From this point of view, Boltzmann's constant k, though important as a conversion factor between energy and temperature, is not fundamental. In the 1870s the Irish physicist George Johnstone Stoney postulated the basic carrier of electric charge, the electron (as he later named it). Stoney also predicted the charge e of the electron and proposed that G, c, and e be used to create fundamental units of measurement. The dimensions of G, c, and e are $L^3M^{-1}T^{-2}$, LT^{-1}, and $L^{3/2}M^{1/2}T^{-1}$ respectively. Simple algebra shows that powers of G, c, and e cannot combine to produce a dimensionless number, but combine in exactly one way to yield each of the three basic dimensions. Thus it was that Stoney introduced the fundamental units $M_S = e/\sqrt{G}$, $L_S = e\sqrt{G}/c^2$, and $T_S = e\sqrt{G}/c^3$.

Stoney's units were neither adopted nor even much noticed. In 1899 Max Planck rediscovered them in a slightly different guise. Using the quantum of action h instead of e, Planck advocated the natural units $M_P = \sqrt{hc/G}$, $L_P = \sqrt{Gh/c^3}$, and $T_P = \sqrt{Gh/c^5}$. As he argued, "All systems of physical units, including the so-called absolute C.G.S.-system, have appeared up to now due to accidental circumstances...from the needs of our earthly culture." The proposed Planck units "would not depend on the choice of special bodies or substances and would be valid for all epochs and all cultures including extraterrestrial and extrahuman ones and could therefore serve as 'natural units of measurements.'"

If we let $\alpha = 2\pi e^2/(hc) = e^2/(\hbar c)$, then we find $M_S/M_P = L_S/L_P = T_S/T_P = \sqrt{\alpha/(2\pi)}$. The dimensionless constant α that appears in these ratios is the *fine structure constant*, which Arnold Sommerfeld introduced in 1916. One place in which the fine structure constant shows up is quantum electrodynamics (QED), where $\sqrt{\alpha}$ is the amplitude for an electron to emit a photon. In his book *QED* [5, p. 129], Feynman says of $1/\alpha \approx 137.036$, "It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from. Nobody knows. It is one of the *greatest* damn mysteries of physics: a *magic number* that comes to us with no understanding by man."

One physicist who dearly wished to understand $1/\alpha$ was Wolfgang Pauli. Abraham Païs relates how Pauli, defeated in his efforts to get to the bottom of $1/\alpha$, took it as a bad omen that he was assigned Room 137 when he entered Zurich's Red Cross Hospital in December 1958. Pauli died ten days later [10].

Whereas Pauli did not understand $1/\alpha$ and knew it, the distinguished astrophysicist Sir Arthur Eddington did not understand $1/\alpha$ but deluded himself into thinking that he did. Eddington came to international prominence in 1919 when he confirmed Einstein's theory of general relativity by measuring the deflection of starlight as it passed through our sun's gravity field. Eddington is also remembered for discovering the role of nuclear fusion in stellar evolution. More than any other physicist of his time, Eddington was obsessed with explaining the constants of nature. To the detriment of his reputation, this pursuit brought out a mystical side of his character. In 1946 Eddington's final thoughts on the constants of nature were published posthumously under the title *Fundamental Theory*. It is a work that Eddington's most respectful critics call "an exceedingly obscure, annoying book." More frequently, the book is dismissed as pseudoscience or numerology or just plain "nuts".

In *Fundamental Theory* Eddington lists seven basic constants of physics, including an 80-digit integer, N_{Edd}, claimed to be the exact number of protons in the universe. After eliminating three of the constants through a choice of units, Eddington settles on m_p/m_e, $1/\alpha$, G, and N_{Edd} as the "constants of nature." He then carries out a program in which "all four constants are obtained by purely theoretical calculation." In particular, Eddington believed he could prove that $1/\alpha$ is exactly 137. At the time, only three significant digits of α were known, permitting Eddington to say, "So far as I can make out, the values of the constants given by this theory are in full agreement with observation." Although the contents of *Fundamental Theory* have received plenty of ridicule, every mathematician can sympathize with Eddington's attempt to bring Newton's constant G into the mathematical fold.

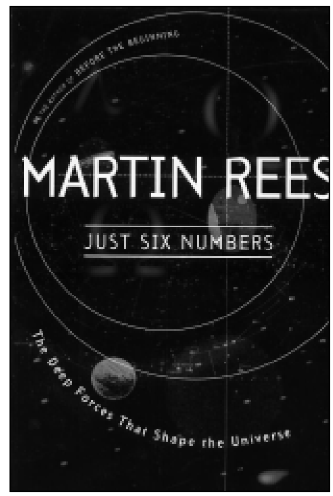
Another list of the constants of nature was compiled by Freeman Dyson in 1972. In addition to the constants m_p, c, e, $\hbar$, and G that we have already encountered, Dyson included Fermi's constant g of weak interactions, Hubble's constant H, and the average mass density ρ of the universe. The eight constants in Dyson's list combine to form five pure numbers: α, $\beta = \left(g m_p^2 c\right) / \hbar^3$, $\gamma = \left(G m_p^2\right) / (\hbar c)$, $\delta = (H\hbar) / \left(m_p c^2\right)$, and $\epsilon = G\rho / H^2$. The presence of the cosmological *variables* H and ρ in Dyson's tally points to an unfortunate awkwardness in the terminology of physics: not only does the term *constant* refer to dimensionful numbers that are not impervious to a change of units, the term is also used to describe quantities that vary with time. Until the 1930s nobody seriously questioned whether the quantities m_p, c, e, $\hbar$, and G were truly constant. That changed quickly when the English physicists Edward Arthur Milne and Paul Dirac pointed out that the beliefs that then prevailed amounted to untested hypotheses. In particular, Dirac conjectured that Newton's gravitational constant obeys the proportionality law $G \propto 1/t$.

The inspiration for Dirac's suspicion was a seemingly untenable coincidence among three astoundingly large dimensionless numbers: N_1, the ratio of the radii of the observable universe and the electron; N_2, the electromagnetic-to-gravitational force ratio between the proton and electron; and N_p, the number of protons in the observable universe. Dirac noticed that $N_1 \approx N_2 \approx \sqrt{N_p} \approx 10^{40}$. On the basis of this evidence, Dirac conceived the Large Numbers Hypothesis (LNH), which in 1938 he stated as: "All very large dimensionless numbers which can be constructed from the important natural constants of cosmology and atomic theory are connected by simple mathematical relations involving coefficients of the order of magnitude unity." Given the relationships $N_1 \propto t$ and $N_2 = e^2 / \left(G m_e m_p\right)$, Dirac's LNH seems to require at least one of the constituent constants of N_2 to be changing. Thus it was that Dirac proposed the proportionality $G \propto 1/t$.

It was not long before Dirac's suggestion was dismissed. Nevertheless, the idea that the fundamental constants can change is still very much with us. It should be noted that physicists do not agree on whether a change in a dimensionful constant such as G can be regarded as anything more than a change in units [4]. Refutations and counterrefutations have been flying [3], [9]; this controversy has even been the subject of a book-length tirade [7]. What is not contentious is that any change in a dimensionless constant such as α must be significant. Think of it this way: If your girth measurement is greater than last year's, then you may attribute it to a contraction in your tape measure. But if your girth-to-height ratio has increased, then you must accept that you have changed.

In 2001 a research team to which Barrow belonged announced statistically significant evidence of a time-variation of α. It was a news item with a headline too good to ignore. Though the idea of a varying fine structure constant is not revolutionary among physicists, news of an inconstant constant is about as close to a "man bites dog" story as the popular press can anticipate from The Road. You may remember newspaper and magazine articles that proclaimed "The Cosmos Becomes More Fickle" or "As Constant as the Stars?" or "Speed of Light May Not Have Been Constant after All" or "Challenge to a 'Constant' Shakes Modern Physics" or "Anything Can Change, It Seems, Even an Immutable Law of Nature". The purpose of Barrow's new book is to explain the physics behind these headlines at greater depth than a newspaper or magazine article can afford.

In Barrow's hands, the constants of nature make for a pretty fair science yarn with a natural beginning, middle, and end. His first six chapters introduce the reader to the most basic of the fundamental constants and develop the contributions of Stoney, Planck, Eddington, and Dirac. These chapters conclude with the final rebuttal of Dirac's LNH by the American physicist Robert Dicke in the late 1950s. To Dicke, the large numbers that intrigued Dirac were exactly what were to be expected *given the presence of sentient life in our universe.* We have seen this line of thought at work in the discussion of the nuclear fusion number $\mathcal{E}$: we cannot explain the value of $\mathcal{E}$ from first principles, but, given that we are here at a time in cosmic history when heavy elements have formed, intelligent life has evolved, and the stars have not all died, we can deduce that $\mathcal{E}$ must lie in a narrow interval. This type of reasoning has given rise to a number of axioms that are now called *anthropic cosmological principles.* Barrow devotes the middle portion of his book to these ideas.



In the final third of *The Constants of Nature*, Barrow describes some of the efforts that have been made to detect any changes that α may have had over time. We learn, for instance, how a natural nuclear reactor at Oklo, Gabon, has allowed physicists to measure the value α had on Earth some two billion years ago. In the recent research that resulted in the announcement of a time-variation in α, Barrow's team used the ancient light of quasars to determine the value that α had 11 billion years ago. If their measurements hold up, they will have accomplished quite a feat!

The Constants of Nature describes striking research on a topic of fundamental importance. It is a first-hand account written by a prominent physicist. Clearly, it is a book that has a lot going for it. Unfortunately, it is also an extremely exasperating book. The first harbinger of systemic trouble is found on the second page of the preface, where Barrow refers to a line of reasoning that he asserts he set out in 1981 in his "first book, *The Anthropic Cosmological Principle*." *The Constants of Nature* does not have a bibliography, but twice in the end-of-book notes Barrow gives the correct year, 1986, for *The Anthropic Cosmological Principle*. The discrepancy in the preface cannot be a simple matter of getting one digit in the year wrong: elsewhere Barrow cites *The Left Hand of Creation*, a book with a 1983 copyright, as his first book. These are inconsequential details, noteworthy only for being the first of a relentless series of bewildering errors and inaccuracies.

Let us consider an example that illustrates how much carelessness can be inserted into a very small space. On page 30 Barrow discusses the late nineteenth-century perception of physics as a dead subject. He states, "Caricaturing this hubris, Albert Michelson wrote in 1894 that there was a view abroad that 'The more important fundamental laws and facts of physical science have all been discovered.... Our future discoveries must be looked for in the sixth place of decimals.'" In fact, in the write-up of an address that he gave at a University of Chicago laboratory dedication in 1894, Michelson states, "An eminent physicist has remarked that the future truths of physical science are to be looked for in the sixth place of decimals." Since there is quite a bit of folklore associated with Michelson's attribution to an unnamed eminent physicist [12, p. 13], an attribution that does not appear in Barrow's quotation, I thought it worthwhile to track down Barrow's reference. My trip to the physics library turned out to be a waste of time: Barrow has misquoted his source. Moreover, my waste of time was greater than it should have been: Barrow's citation is itself inaccurate. It would be confusing enough that Barrow ascribes words to Michelson that Michelson himself ascribed to someone else. But Barrow compounds the confusion by prefacing his misquotation with a grammatical lapse that misrepresents the *intent* of Michelson's remarks. Because Barrow's participial phrase "Caricaturing this hubris" modifies Michelson, the conscientious reader can only infer that Michelson meant his remarks to be taken as a caricature. To the contrary, Michelson sincerely believed that the future of physics would lie in better measurement, a position he reiterated several years later, even though Planck's discovery of the quantum nature of energy had occurred in the interim [8, p. 24].

The following sample will suffice to illustrate the numerical and logical errors that abound in *The Constants of Nature*. On page 45 a plot of computer processing speed as a function of time begins with the year 1900. On page 86 the formula for $1/\alpha$ contains two mistakes. On page 206 the sentence "If you walk at random in three (or more) dimensions of space you will never return to your starting point" badly misstates Pólya's Theorem. On page 252 it is asserted that sunlight reaches us in "about 3 seconds." In the very next sentence, Alpha Centauri is said to be 4.1 light years away from us, a figure that is at least one trillion kilometers off the mark. On page 236 we encounter a passage that surely invites a second reading: "When the Earth formed about 4.5 billion years ago...After about 2.5 billion years, when the Earth was 2 billion years old...". Well, these things happen. More seriously, only seven pages later Barrow asserts that the Earth is 4.6 billion years old. This inconsistency mirrors a similar discrepancy with the stated age of the universe: 13 billion years on page 129, "about 14 billion years" on page 243. One billion years here, one trillion kilometers there—when an author is so nonchalant about numerical accuracy, how excited can the reader become about a possible change in a far-flung significant digit of α?

The problems that infest *The Constants of Nature* are not characteristic of Barrow's previous work. Perhaps haste is to blame here. The stuff of headlines come and go; the attention of the public is known to be fickle. Certainly one can understand the desire to publish a book while its subject is still hot. One can even imagine a desire to publish research findings before contradictory results arrive to muddle the story. So far as I have been able to determine, at the time of this writing the existence of a time-variation in α has not been definitively refuted or confirmed. I can point to a paper that raises concerns with the methodology of Barrow's team [1], but browsing the abstracts that are available on the physics preprint server, I find no overwhelming body of opinion that justifies the statement "The consensus is that there are subtle problems with the observations—the fine-structure constant is probably not changing after all" [11, p. 222].

There is surely a great book about the constants of nature yet to be written. In the meantime, Barrow's *The Constants of Nature* may be regarded as an interesting, albeit flawed, progress report. To the reader who desires only a brief, nontechnical look at how the constants of nature are fine-tuned for life in the universe, Rees's *Just Six Numbers* can be confidently recommended. An earlier book written by Rees, *Before the Beginning*, may be suggested to the reader who prefers a less narrowly focused introduction to cosmology that covers similar ground. All of these books prepare their readers for the next front in the science curriculum wars, the theory of "intelligent design". During the past decade, the battle over biological evolution has escalated into a heated debate that encompasses the evolution of our universe. The fine-tuning of the basic physical constants has been cited as evidence that the universe is the result of intelligent design. As I write this review, several initiatives placed before school boards and state legislatures seek to mandate instruction in the theory of intelligent design. Whether or not we as mathematicians find the constants of nature interesting, we may as citizens wish to learn more about them.

Physicists tell a joke that emphasizes the stubborn mystery of the constants of nature. According to the story, God grants the spirit of Pauli one question. When Pauli asks why α is approximately $1/137$, God fills a blackboard with equations and when done turns to Pauli. The frustrated scientist shrugs off the explanation and exclaims, "Baloney!" That is where we remain. There is still a fleeting hope that a theory of everything will one day explain the constants of nature. However, more and more physicists are coming to believe that the best we can do is use anthropic principles to further refine the bounds that constrain the constants of nature. Could Michelson have had the right thought, wrong century? It might just be that the numbers so fundamental to our universe are mathematical duds.

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