# Mathematicians Are from Mars, Math Educators Are from Venus: The Story of a Successful Collaboration

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A mathematician, Alan, is teaching his calculus course at a rapid pace. He is content in believing that he does his job well. Suddenly a mathematics educator, Alice, gets in his path. "You mathematicians are all the same" she says to Alan. "Chalk and talk at 90 miles per hour! Do you ever think of what the students are learning?" Alan, somewhat taken aback, thinks to himself, "What is she talking about?" Alice, somewhat frustrated, continues, "We need secondary mathematics teachers, and we don't have any. Why? Because you mathematics professors make calculus so impossible! Students quit mathematics after their first year of college." By now, Alan is getting a bit agitated and thinks to himself, "This is college, lady." Controlling himself, he calmly replies, "If the kids can't pass calculus, they shouldn't be mathematics teachers!" Alice thinks to herself, "Another arrogant mathematician!"

### Problem Statement

Can these two people from different worlds get together and start a new program, whose goals are to increase substantially the number of secondary school mathematics teachers through recruitment and a change of curriculum and teaching style in beginning calculus?

Recent national reports have indicated the urgent need for the improvement of mathematics teacher preparation (Glenn Commission, 2000; Conference Board of the Mathematical Sciences, 2001; American Mathematical Society, 1999). Furthermore, these reports suggest that mathematics departments need to become more involved in this process. Specifically, one of the recommendations of the 2001 report from the Conference Board of the Mathematical Sciences (CBMS) on *The Mathematical Education of Teachers* (MET report) states that, "Mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching." The Mathematical Association of America has received funding from the National Science Foundation (NSF) for a project entitled, "Preparing Mathematicians to Educate Teachers" (PMET) (Katz and Tucker, 2003) that addresses this very problem. Prompted by the severe shortage of undergraduate mathematics teacher preparation candidates, several years prior to the aforementioned national reports, we secured a grant from the NSF entitled Teaching Improvements through Mathematics Education (TIME, 2000), in which we recruited mathematically talented high school seniors into a newly designed mathematics teacher preparation program. Recognizing that recruitment was only the first step, we focused our

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efforts on the retention of these teacher candidates through the redesign of the freshman year mathematics courses. Our promise to NSF was that the project mathematicians and mathematics educator would work together in the design and implementation of these courses. Such collaboration, we promised, would lead to reformed curricula and instructional strategies, as well as our mutual professional development. Having undergone what was an extremely challenging, yet enriching process, we believe that we have learned much about what it takes to have the kind of collaboration that is needed if national recommendations for the mathematical preparation of teachers are to be realized. We share our story in the hope that it will be of help to others who may soon engage in the process. Here is our story, in interview style, to enable us to reflect our own personal perspectives on how the collaboration evolved.

### Motivation for Creating the Program Together

**Alan:** As you can tell from the setting described above, when Alice and I met we were worlds apart in personality as well as in philosophy. The meeting described above is a slightly modified version of the one I remember. I was running to class and Alice stopped me to say, "Alan, we have to do something about the shortage of mathematics teachers." I was taken by surprise and thought, "What do I know about high school teaching?" Nevertheless, I liked the thought of getting involved in a project and said, "Sure," not knowing what I was really getting myself into. "How do we do it?" I asked.

***Alice:*** I then told Alan about my plan to apply to the National Science Foundation for a grant to recruit high school seniors who were good in mathematics and offer them a strong preparatory program, different from any other currently in existence. I remembered that Alan had once told me that he had an interest in mathematics education, so I thought he might be interested. I was thrilled when he agreed to come on board.

At the time, students in the undergraduate mathematics education program did not take their methods course until their senior year. That is the first time that I would meet any of them. This particular year I had reached an all-time high enrollment of five students. I kept thinking that there might have been more students who entered college thinking about becoming mathematics teachers, but dropped out along the way because they either could not pass or did not like their college mathematics classes. At the same time, I was receiving daily calls from local high school mathematics chairs asking for graduating teacher candidates. I knew that in order to attract and retain more preservice mathematics teachers, a big change had to be made.

**Alan:** The number of mathematics majors in general had greatly dwindled, and many of the upper-level courses that used to be the bread and butter of our department were no longer offered. Things were not looking good for us, and people in the administration were doing analyses of how much each of us was worth on the dollar. It was a discouraging time to say the least.

### Promised Collaborative Efforts: Our Worries

***Alice:*** When we received notice that NSF had agreed to fund our program, I had mixed emotions. I was elated, but also worried about some of the things we had promised to do. Specifically, one of the key elements of the project was that Alan's calculus class would be taught in a "learner-centered manner" and "incorporate real-life applications". To ensure that this would occur, Alan and I agreed to meet in regular planning and debriefing sessions and sit in on each other's classes. In the initial writing of the proposal, Alan had described many incredible applications he was planning to use. I was excited to sit in on his class. He had an excellent reputation as a teacher, and I had not taken a course in calculus for more years than I would care to admit. However, I was well aware that changing Alan's strict lecture approach would be extremely difficult.

**Alan:** As far as I was concerned at the time, there was really no reason to change. I tried as hard as I could and was good at what I did. I accepted the fact that no matter how good a teacher one is, there will always be students who fail, and it is something that one can do very little about. Students fail for many reasons, and that is just a fact of life. But there was something else I was feeling. Although I was good at what I did, and loved the mathematics and the teaching of it, when the fifty minutes of class were up, I was gone, and so were the students. I could be enthusiastic, arouse their interest, but I never saw them again after the semester was over. My eighth-grade mathematics teacher turned me on to mathematics, and until today, I still write to her, telling her how much of an influence she had on my life. I wanted to have that effect on my students. I wanted more of a connection with them. After all, part of the reason I became a teacher was to help people and become part of their lives. So, when Alice suggested the idea of making changes to my instructional strategies I wasn't totally against it. I jokingly told her, "Change? Calculus is the study of change. I teach it. Surely I can do it." I said those words, but wondered deep down, "Could I?"

## The Project Begins: Successes and Frustrations

***Alice:*** When classes started in September, we fulfilled our promise to NSF and enrolled thirty-three students in the program. However, since recruitment turned out to be far more difficult than we had anticipated, we ended up accepting many students who were not really prepared to take the first semester of calculus (as determined by their scores on the mathematics placement examination). Alan and I spent hours discussing the different instructional strategies he would try and the topics he would include in the course. My plan was that while sitting in on each of Alan's classes I would ask the questions that I thought students had on their minds but were too reluctant to ask. In return, Alan sat in on the new education class I was teaching to the same group of students, called "The Psychology of Learning Mathematics". The point of this course was for students to reflect on their experiences learning mathematics in Alan's class as the context for thinking about the educational theories they were learning in my class. In fairness, Alan was also given the liberty to speak up as desired in my class.

**Alan:** The students felt special that they were in this program and were very enthusiastic about beginning. I, on the other hand, was really quite nervous about this new method of instruction. It was to be student based. I was to try to let the students discover the concepts, and my main role was to be a facilitator. I was not supposed to give them all the answers. They had to do all of the discovery, and I had to make the mathematics relevant. And Alice would be sitting in on all my classes to help me do it.

I decided that for the first lesson I would use a problem from the calculus reform curricula:

"A ball is being dropped from a building 550 feet tall. Draw a graph of the speed of the ball as it progresses downward."

There were seven possible graphs that the students could draw: (1) a line with a positive slope; (2) a line with a negative slope; (3) a line with zero slope; (4) a curve that was increasing and concave up; (5) a curve that was increasing and concave down; (6) a curve that was decreasing and concave up and; (7) a curve that was decreasing and concave down.

So, how did I get 10 different graphs on the board? (And yes, I did get a vertical line!) It was astounding to me. Some of the graphs that were put on the board really showed that the students were thinking, but not in the limited way I was thinking. For example, if the $y$-axis represented the speed of the ball and the $x$-axis the height of the ball above ground, what would the graph look like? Because I knew what the answer "should be" I had a bit of tunnel vision. One by one we discussed the graphs, and of course, what I immediately saw was that some people really had no idea of what a graph meant. I expected someone to draw a decreasing graph (when the $x$-axis represented time) and to realize the error immediately. "The ball is going down," the student reasoned, "so the picture must be a graph going down." "But we are graphing the speed," someone said. "The speed is not decreasing." "The speed is constant," someone else called out. Some agreed, many didn't, and what ensued was the first argument I had ever seen in my calculus class. And even though I don't really think the student who drew a line with a negative slope saw why he/she was wrong, he/she finally agreed to go with the class majority. What happened to me was somewhat of a revelation. I thought the question was straightforward. When I saw the variety of answers, it suddenly dawned on me, that when I normally taught, I pretty much had in mind that most of the class had one sense of things: my sense! To me only answers (1), (3) and (4) above were reasonable, as surely they were to everyone else. That I got so many different answers opened my eyes. At the end, we had to decide which of the above three answers (that I thought were reasonable) was correct. This was even more interesting. Gravity had to be taken into account and questions like "Why is it that when an eraser is thrown straight up, it slows down, stops for an instant, and then comes back down?" I threw the eraser up just for some drama. This led to a discussion of gravity, and does gravity cause a body to accelerate? Of course some knew the answer to that question. Others, however, did not. Is that so surprising? After all, Newton at first believed that a body fell with constant velocity.

The two hours of class flew by. We finally agreed on the correct graph, the graph I wanted, naturally! The discussion was exciting, and the class was exhilarating. I loved this new way of doing things. It is true that I did not get to teach all that I had planned, but look at the quality of the lesson! To me it was the best lesson I ever taught. And I said almost nothing! I couldn't wait to get back and meet them again. Only this time, I was going to get something done. I was ready to lecture.

***Alice:*** Alan's lesson on graphing a ball dropping was amazing. I could not believe that he was able to change his style so quickly. His thrill with the lesson meant even more to me. This was going to be easier than I thought. But the next day, in his lesson on functions and domains and then onto limits, I saw that Alan was ready to go back to his old ways. I suggested that he change his statements into a series of questions. I knew that many of the students probably had some notions of functions, and I wanted him to draw on that. So, rather than giving the definition, Alan began the class by asking students to write down what they thought a function was. Through a rich class discussion, he was able to elicit what a function was, and just as importantly, what a function was not.

**Alan:** The answers that I got troubled me a lot. Although I had put the intuitive definition of function up on the blackboard (a rule that associates with each element in a first set one and only one element in the second set), it was clear to me that even after several examples, many students still did not understand the function concept, especially when it was put in the context of an application.

***Alice:*** During one part of the lesson, Alan asked the students a question to which he got no response. He looked over to me as if to ask what to do (by now, he was reluctant to give them any answers). I suggested that he let them discuss the answer to this question with their neighbors. They did this for approximately three minutes, and when he called them together as a class, many students raised their hands. I was pleased about how that worked and thought Alan would be pleased as well.

**Alan:** There was a lot of discussion and that was good, but the results at the end of the lesson were the same as the previous class. Two hours went by, and I did not get to the other topics that needed to be taught. It is true that we had extra hours built into the course for this student-centered instruction, but at the rate we were going, we would use these hours up in no time. I went home that day, a bit depressed and I started worrying that I would never teach them what they needed to know to be prepared for their next calculus class. I also realized how little of what I thought I was teaching in my traditional lectures was really coming across. At the same time, I began to question myself, wondering, "Do I ever really teach anything except how to do standard problems?"

***Alice:*** I felt bad for Alan. I saw how much he was trying and how distressed he seemed. I shared with him what one of my professors once told me. "Learning is what you remember after you forget all the facts." I assured him that the whole point of this program was for all of us, students and professors alike, to become more reflective mathematics teachers and that the fact that he was beginning to question the ways he had always taught was a good sign.

**Alan:** I heard her, but not really. Three weeks passed and the same thing kept happening. I lost more and more time and became more anxious by the day. But at the same time, some good things were happening. I saw the benefits of first developing a need to learn a topic before teaching students a topic. For example, before we introduced limits, Alice suggested that I give the students a problem to work on that would require them to discover the concept of limit and establish a need for it. The question I asked was a logical one. A ball is dropped from a height of 550 feet. What is its speed two seconds after it is dropped? This is a formidable problem for a student who sees it for the first time and, of course, is related to the problem I discussed earlier about graphing the speed of the ball. I just told them that in physics, one learns that the distance the ball drops after $t$ seconds is $d = 16t^2$ (and that if they stay with the course, they will learn why). Every student who sees this for the first time wonders, "How do you even begin this problem?"

We discussed the idea of average speed over a time interval. I asked the question: does the average speed of the ball, during the time interval [2,5] (two seconds after it is dropped to five seconds after it is dropped) give any information about the actual speed at time 2? Almost everyone said "No." Then I made the interval [2,4] and asked the same question. Again the answer was "No." I kept making the time interval smaller and smaller and kept asking the same question using the smaller time intervals, each time getting the answer "No." When they finally got the idea that the answer was "No," I asked, "But if the time interval is small, there is very little time for the speed to change. So shouldn't the average speed during a small time interval beginning at $t = 2$ be close to the actual speed at 2?" This was a revelation, and suddenly everyone said, "Yes." ( I just love the lightbulbs that go on when they finally see the point!) We took out our calculators and started calculating the average speeds over smaller and smaller time intervals. How exciting it was to see that the average speeds were stabilizing as the time interval shrank. I then said, "Let's see if we can find a formula for the average speed during the time interval $[2, 2+h]$." We did this. "Now, how can we describe that the time interval is getting smaller? What does that say about $h$?" It was this kind of reasoning that led to the concept of limit, and then I asked them to do the same kind of thing for the speed at three seconds. Of course to do it analytically and let $h$ approach zero was easiest and enabled us to get a formula for the speed at any time, and we did that too. In this way we were able to verify that our guess about the correct graph for the speed discussed earlier was, in fact, correct. Having a reason to do limits, the lesson on limits went a bit smoother, until of course someone said, "Why bother with all these analytic methods? We can always use the calculator to get the answer." And then I gave them some problems for which the calculator could not compute the answer accurately. So again, they saw the need to use analytic methods. All the students seemed involved and they were certainly engaging in worthwhile mathematics. But it took me eight hours to do what I normally do in four.

***Alice:*** Although I was enjoying watching the student involvement created by the way Alan motivated them to learn, the clock was ticking, and I knew Alan would be upset. Because of the time loss, Alan seemed anxious to get back to his lecture style. But since, in their education class, the students were studying about how active involvement contributes to increased understanding and better retention, they made it uncomfortable for Alan to return to his lecture approach. Besides that, expressing their ideas in a mathematics class made it more interesting for them.

**Alan:** For the first time in my life I had a minimutiny in my class! The students complained to both Alice and me if I dared to try to lecture. But I had to "cover" the course work. So I kept slipping back into lecture mode. From the back of the class Alice would try to focus my attention on the students, "Uh, Alan, why don't you stop for a minute. Some students have another idea about how to approach the problem." "Sure," I would say, as more minutes would tick away.

***Alice:*** I felt that if Alan kept an open mind and took my suggestions, the students would get a better understanding of the material. The students, I naïvely thought, would not fail the exams with these new teaching methods. After all, isn't that what the research showed?

**Alan:** The results of the first exam were very disappointing. We had spent all these extra hours discussing lesson strategies with each other and numerous hours holding review sessions and giving the students extra help. We took a lot of classroom time slowly developing the concepts, and still, many of them failed.

***Alice:*** After seeing how poorly some students did on the exams, I thought that it would be good if the students did more work in small groups. We also changed some of the group formations to enable more productive helping relationships to develop. But, no matter what we did, and no matter how many changes we made, some students just could not do the work. This frustrated me even more. I wanted desperately to show Alan the benefits of cooperative learning. After all, I had used it successfully for many years.

**Alan:** The more I saw, the more entrenched I became in my belief that these new strategies were not sufficiently tested. Our friendship was being strained. Alice genuinely wanted me to change. While I did see some real benefits to some of the things she recommended, the more I incorporated these strategies, the further behind I got with each lesson.

***Alice:*** I started feeling very deflated and stopped interrupting Alan in class. I knew Alan had to "cover some ground", and I reluctantly sat quietly while he resumed his previous lecture approach.

**Alan:** Actually, I felt something was missing without her interruptions, because I genuinely enjoyed doing some of the lessons in this manner, but I kept in the back of my mind that my job was to prepare the students for future courses. Coverage of the content was important. And I could do that well with my traditional approach. So we compromised. I would try as much as was practical to make the course student centered, but she had to trust my judgment that when it was time to move on, I be allowed to move on. This seemed to be a fair compromise.

Despite my frustrations with the time problem, I must admit that I was happier teaching than before. Why? I found the student involvement stimulating. I was actually learning from some of their comments. Often in the past I had complained about students looking like they were bored or even asleep. Having students jumping out of their seats wanting to express their opinions made it very difficult to revert back to a lecture approach. I could no longer do it. Because of the time loss, I reluctantly cut some content out that year. It was a difficult compromise to make, but did I hurt them? Did I lower the standard? I don't think so. Some of the methods of integration, while fun for me to do when I was a student, are grueling and boring for many. Students get more interested in and more out of solving a simple differential equation to answer a question like, "Why does a satellite go into orbit?" or "Is this painting a forgery?" than "How do we do this integral?" After five months of teaching in the program, I softened, and Alice was well aware of it.

***Alice:*** By now, Alan was astounding me with his natural use of student-centered approaches. In the daily journal I kept at the time, I wrote: Alan began teaching his class in a completely different way. He had the kids put up their work, and then said, "instead of me asking you questions about the problems, I want you to ask each other questions about the problems." He allowed a long wait time for this. Some students started asking him questions, and he referred them to the person who put it up (the problem). It went beautifully. As the lesson progressed Alan made a point of calling on each and every student in the class. It was a real change and I admire him for doing it.

## The Results of Our Collaboration

***Alice:*** So where are we now? Of the original thirty-three students, seventeen dropped out of the program. That is a harsh statistic. But the other sixteen are now in the classroom teaching their own classes, in jobs that can only be described as enviable. They are proving themselves over and over by their professionalism and innovative teaching strategies. They claim that they are just using the strategies that they experienced in our program. They also tell stories of how they try to work with other teachers as they saw us work together.

As a result, our reputation for producing excellent mathematics teachers has spread, and our students are in great demand. Indeed, some of them have been invited to speak at local and national conferences. As of April 2004, our third cohort of graduating students have all been offered teaching positions. One school hired three of our students to work together to build a program! We also have eighty-seven students in the pipeline now, and the program is growing. Other schools in the area are trying to work with us to get similar programs started, and a local community college has received a grant to emulate the first two years of our program so that we may allow their students easy transition to our program, if they meet our standards.

**Alan:** So other than preparing significant numbers of teachers, what else have we accomplished? Well, as one person in the mathematics department recently told us, "You have revitalized the department." As a result of the large number of students we have, we are now offering courses that have not been offered for many years. We have also been developing new courses to meet the changing needs and requirements of our program.

And, what of my teaching? I still need to "cover the content". That is a reality. But I am now much more flexible in allowing students to get into long and interesting discussions, if I feel the results are worthwhile. And in courses with less prescriptive syllabi, I can be the teacher Alice wanted me to be, and that I now want to be. For example, in a new problem-solving class that I teach, I let the students work in groups on interesting problems. They then come before the class and explain their different solutions. I let them find each other's errors. I really play a very small role in the classroom, spending most of the time on the sidelines.

***Alice:*** And what has become of our collaboration now that the program is running more smoothly? Alan and I continue to attend conferences together, do research together, write papers together, solve students' personal problems together, team teach, design new courses, and constantly stay vigilant of changes that need to be made in our program to make it better. And best of all, we are still good friends!

## Conclusion

So why write this article in the first place? And what of the title, "Mathematicians Are from Mars, Mathematics Educators Are from Venus"? Mathematicians and mathematics educators seem to think completely differently. In some sense this is surprising, since we all love mathematics and wish everyone could see the beauty of it the way we do. But we are different. There is no question that mathematicians are going to be called on to teach teachers. The tide is already turning in this direction, and programs are already in place to prepare mathematicians to meet this need (e.g., PMET). This will require changes not only in how we teach but in what we teach. And this requires collaboration. If there is one thing that we have learned it is that collaboration is a complex process, and the key word is "process". We have to be willing to learn from each other, we have to respect each other, and we have to be willing to change. We have to be committed to hanging in there even through the difficult times. If we are committed to making it work, it will work.

We would like to end this article with what we believe are four necessary conditions for a successful collaboration:

M: Motivation to collaborate

A: Acknowledgement of the strengths of each collaborator

T: Trust that the motives of each collaborator involve improving student learning

H: Helpfulness of both collaborators in reaching the mutual goals

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