New Publications Offered by the AMS

Algebra and Algebraic Geometry

The Classification of the Finite Simple Groups, Number 6

Part IV: The Special Odd Case

Daniel Gorenstein, Richard Lyons, Rutgers University, New Brunswick, NJ, and Ronald Solomon, Ohio State University, Columbus

This series of volumes ... is a model for all mathematicians of the standards and clarity that should be achieved.

—Mathematical Reviews

The classification of finite simple groups is a landmark result of modern mathematics. The original proof is spread over scores of articles by dozens of researchers. In this multivolume book, the authors are assembling the proof with explanations and references. It is a monumental task. The book, along with the previous five volumes (Surveys of Mathematical Monographs), provides the classification of finite simple groups of special odd type (Theorems C2 and C3, as stated in the first volume of the series).

The book is suitable for graduate students and researchers interested in group theory.

Contents: General introduction to the special odd case; General lemmas; Theorem C2: Stage 1; Theorem C2: Stage 2; Theorem C2: Stage 3; Theorem C2: Stage 4; Theorem C2: Stage 5; Theorem C3: Stage 1; Theorem C3: Stages 2 and 3; VI: Preliminary properties of K-groups; Background references; Expository references; Glossary; Index.

Mathematical Surveys and Monographs, Volume 40

Introduction to Quadratic Forms over Fields

T.Y. Lam, University of California, Berkeley

This new version of the author’s prizewinning book, Algebraic Theory of Quadratic Forms (W. A. Benjamin, Inc., 1973), gives a modern and self-contained introduction to the theory of quadratic forms over fields of characteristic different from two. Starting with few prerequisites beyond linear algebra, the author charts an expert course from Witt’s classical theory of quadratic forms, quaternion and Clifford algebras, Artin-Schreier theory of formally real fields, and structural theorems on Witt rings, to the theory of Pfister forms, function fields, and field invariants. These main developments are seamlessly interwoven with excursions into Brauer-Wall groups, local and global fields, trace forms, Galois theory, and elementary algebraic K-theory, to create a uniquely original treatment of quadratic form theory over fields. Two new chapters totaling more than 100 pages have been added to the earlier incarnation of this book to take into account some of the newer results and more recent viewpoints in the area.

As is characteristic of this author’s expository style, the presentation of the main material in this book is interspersed with a copious number of carefully chosen examples to illustrate the general theory. This feature, together with a rich stock of some 280 exercises for the thirteen chapters, greatly enhances the pedagogical value of this book, both as a graduate text and as a reference work for researchers in algebra, number theory, algebraic geometry, algebraic topology, and geometric topology.

Contents: Foundations; Introduction to Witt rings; Quaternion algebras and their norm forms; The Brauer-Wall group; Clifford algebras; Local fields and global fields; Quadratic forms under algebraic extensions; Formally real fields, real-closed fields, and pythagorean fields; Quadratic forms under transcendental extensions; Pfister forms and function fields; Field invariants; Special topics in quadratic forms; Special topics on invariants; Bibliography; Index.

Graduate Studies in Mathematics, Volume 67
Analysis

New Publications Offered by the AMS

Six Themes on Variation
Robert Hardt, Rice University, Houston, TX, Editor

The calculus of variations is a beautiful subject with a rich history and with origins in the minimization problems of calculus. Although it is now at the core of many modern mathematical fields, it does not have a well-defined place in most undergraduate curricula. This volume should nevertheless give the undergraduate reader a sense of its great character and importance.

Interesting functionals, such as area or energy, often give rise to problems whose most natural solution occurs by differentiating a one-parameter family of variations of some function. The critical points of the functional are related to the solutions of the associated Euler-Lagrange equation. These differential equations are at the heart of the calculus of variations and its applications to other subjects. Some of the topics addressed in this book are Morse theory, wave mechanics, minimal surfaces, soap bubbles, and modeling traffic flow. All are readily accessible to advanced undergraduates.

This book is derived from a workshop sponsored by Rice University. It is suitable for advanced undergraduates, graduate students and research mathematicians interested in the calculus of variations and its applications to other subjects.

This item will also be of interest to those working in geometry and topology and applications.

Contents: F. Jones, Calculus of variations: What does "variations" mean?; R. Forman, How many equilibria are there? An introduction to Morse theory; S. J. Cox, Aye, there’s the rub. An inquiry into why a plucked string comes to rest; F. Morgan, Proof of the double bubble conjecture; M. Wolf, Minimal surfaces, flat cone spheres and moduli spaces of staircases; B. L. Keyfitz, Hold that light! Modeling of traffic flow by differential equations.

Student Mathematical Library, Volume 26


Orthogonal Polynomials on the Unit Circle

Part 1: Classical Theory; Part 2: Spectral Theory

Barry Simon, California Institute of Technology, Pasadena

This two-part volume gives a comprehensive overview of the theory of probability measures on the unit circle, viewed especially in terms of the orthogonal polynomials defined by those measures. A major theme involves the connections between the Verblunsky coefficients (the coefficients of the recurrence equation for the orthogonal polynomials) and the measures, an analog of the spectral theory of one-dimensional Schrödinger operators.

Among the topics discussed along the way are the asymptotics of Toeplitz determinants (Szego's theorems), limit theorems for the density of the zeros of orthogonal polynomials, matrix representations for multiplication by z (CMV matrices), periodic Verblunsky coefficients from the point of view of meromorphic functions on hyperelliptic surfaces, and connections between the theories of orthogonal polynomials on the unit circle and on the real line.

The book is suitable for graduate students and researchers interested in analysis.

Contents: Part 1: The Basics; Szego's theorem; Tools for Geronimus' theorem; Matrix representations; Baxter's theorem; The strong Szego' theorem; Verblunsky coefficients with rapid decay; The density of zeros; Bibliography; Author index; Subject index; Part 2: Rakhmanov's theorem and related issues; Techniques of spectral analysis; Periodic Verblunsky coefficients; Spectral analysis of specific classes of Verblunsky coefficients; The connection to Jacobi matrices; Reader's guide: Topics and formulae; Perspectives; Twelve great papers; Conjectures and open questions; Bibliography; Author index; Subject index.

Colloquium Publications, Volume 54


Applications

Mathematical Modelling

A case studies approach
Reinhard Illner, C. Sean Bohun, Samantha McCollum, and Thea van Roode,
University of Victoria, BC, Canada

Mathematical modelling is a subject without boundaries. It is the means by which mathematics becomes useful to virtually any subject. Moreover, modelling has been and continues to be a driving force for the development of mathematics itself. This book explains the process of modelling real situations to obtain mathematical problems that can be analyzed, thus solving the original problem.

The presentation is in the form of case studies, which are developed much as they would be in true applications. In many cases, an initial model is created, then modified along the way. Some cases are familiar, such as the evaluation of an annuity. Others are unique, such as the fascinating situation in which an engineer, armed only with a slide rule, had 24 hours to compute whether a valve would hold when a temporary rock plug was removed from a water tunnel.

Each chapter ends with a set of exercises and some suggestions for class projects. Some projects are extensive, as with the explorations of the predator-prey model; others are more modest.

The text was designed to be suitable for a one-term course for advanced undergraduates. The selection of topics and the style of exposition reflect this choice. The authors have also succeeded in demonstrating just how enjoyable the subject can be.

This is an ideal text for classes on modelling. It can also be used in seminars or as preparation for mathematical modelling competitions.

Contents: Crystallization dynamics; Will the valve hold?; How much will that annuity cost me?; Dimensional analysis; Predator-prey systems; A control problem in fishery management; Formal justice; Traffic dynamics: A microscopic model; Traffic dynamics: Macroscopic modelling; Bibliography.

Student Mathematical Library, Volume 27
January 2005, 196 pages, Softcover, ISBN 0-8218-3650-1, LC 2004046225, 2000 Mathematics Subject Classification: 00–01, 00A69; 00A71, 93A30, All AMS members $28, List $35, Order code STML/27

Discrete Mathematics and Combinatorics

Trends in Optimization

Serkan Hoşten, San Francisco State University, San Francisco, CA, Jon Lee, IBM, T.J. Watson Center, Yorktown Heights, NY, and Rekha R. Thomas, University of Washington, Seattle, Editors

This volume presents proceedings from the AMS short course, Trends in Optimization 2004, held at the Joint Mathematics Meetings in Phoenix (AZ). It focuses on seven exciting areas of discrete optimization.

In particular, Karen Aardal describes Lovasz’s fundamental algorithm for producing a short vector in a lattice by basis reduction and H.W. Lenstra’s use of this idea in the early 1980s in his polynomial-time algorithm for integer programming in fixed dimension. Aardal’s article, “Lattice basis reduction in optimization: Selected Topics”, is one of the most lucid presentations of the material. It also contains practical developments using computational tools.

Bernd Sturmfels’ article, “Algebraic recipes for integer programming”, discusses how methods of commutative algebra and algebraic combinatorics can be used successfully to attack integer programming problems. Specifically, Gröbner bases play a central role in algorithmic theory and practice. Moreover, it is shown that techniques based on short rational functions are bringing new insights, such as in computing the integer programming gap.

Overall, these articles, together with five other contributions, make this volume an impressive compilation on the state-of-the-art of optimization. It is suitable for graduate students and researchers interested in discrete optimization.

This item will also be of interest to those working in applications.

Contents: K. Aardal, Lattice basis reduction in optimization: Selected topics; A. Atamtürk, Polyhedral methods in discrete optimization; G. Cornuéjols, Graphs and combinatorial optimization; J. B. Lasserre, Integer programming duality; D. B. Shmoys, The design and analysis of approximation algorithms: Facility location as a case study; B. Sturmfels, Algebraic recipes for integer programming; S. J. Wright, Nonlinear and semidefinite programming: Index.

Proceedings of Symposia in Applied Mathematics, Volume 61
New Publications Offered by the AMS

General and Interdisciplinary

Mathematical Sciences Professional Directory, 2005

This annual directory provides a handy reference to various organizations in the mathematical sciences community. Listed in the directory are the following: officers of over thirty professional mathematical organizations; addresses of selected government agencies; academic departments in the mathematical sciences; and alphabetic listings of colleges and universities. April 2005, approximately 168 pages, Softcover, ISBN 0-8218-3621-8, List $55, Institutional member $44, Order code PRODIR/2005

Geometry and Topology

Global Calculus

S. Ramanan, Chennai
Mathematics Institute, India

The power that analysis, topology and algebra bring to geometry has revolutionized the way geometers and physicists look at conceptual problems. Some of the key ingredients in this interplay are sheaves, cohomology, Lie groups, connections and differential operators. In Global Calculus, the appropriate formalism for these topics is laid out with numerous examples and applications by one of the experts in differential and algebraic geometry.

Ramanan has chosen an uncommon but natural path through the subject. In this almost completely self-contained account, these topics are developed from scratch. The basics of Fourier transforms, Sobolev theory and interior regularity are proved at the same time as symbol calculus, culminating in beautiful results in global analysis, real and complex. Many new perspectives on traditional and modern questions of differential analysis and geometry are the hallmarks of the book.

The book is suitable for a first year graduate course on global analysis.

Contents: Sheaves and differential manifolds: Definitions and examples; Differential operators; Integration on differential manifolds; Cohomology of sheaves and applications; Connections on principal and vector bundles; Lifting of symbols; Linear connections; Manifolds with additional structures; Local analysis of elliptic operators; Vanishing theorems and applications; Appendix; Bibliography; Index.

Graduate Studies in Mathematics, Volume 65

Mathematical Physics

Solvable Models in Quantum Mechanics

Second Edition

S. Albeverio, University of Bonn, Germany, F. Gesztesy, University of Missouri, Columbia, R. Høegh-Krohn, and H. Holden (with an appendix by Pavel Exner), Norwegian University of Science & Technology, Trondheim, Norway

There is a wealth of very pretty examples of Schrödinger operators here which could be presented ... in an elementary quantum mechanics course.

—MathSciNet

The monograph presents a detailed study of a class of solvable models in quantum mechanics that describe the motion of a particle in a potential having support at the positions of a discrete (finite or infinite) set of point sources. Both situations — where the strengths of the sources and their locations are precisely known and where these are only known with a given probability distribution — are covered.

The authors present a systematic mathematical approach to these models and illustrate its connections with previous heuristic derivations and computations. Results obtained by different methods in disparate contexts are thus unified and a systematic control over approximations to the models, in which the point interactions are replaced by more regular ones, is provided.

The first edition of this monograph generated considerable interest for those learning advanced mathematical topics in quantum mechanics, especially those connected to the Schrödinger equations. This second edition includes a new appendix by Pavel Exner, who has prepared a summary of the progress made in the field since 1988. His summary, centering around two-body point interaction problems, is followed by a bibliography focusing on essential developments made since 1988.

The material is suitable for graduate students and researchers interested in quantum mechanics and Schrödinger operators.

Contents: Introduction; The one-center point interaction: The one-center point interaction in three dimensions; Coulomb plus one-center point interaction in three dimensions; The one-center δ-interaction in one dimension; The one-center δ'-interaction in one dimension; The one-center point interaction in two dimensions; Point interactions with a finite number of centers: Finitely many point interactions in three dimensions; Finitely many δ-interactions in one dimension; Finitely many...
Strings and Geometry

Michael Douglas, Rutgers University, Piscataway, NJ, Jerome Gauntlett, University of London, England, and Mark Gross, University of California San Diego, La Jolla, Editors

This volume is the proceedings of the 2002 Clay Mathematics Institute School on Geometry and String Theory. This month-long program was held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, England, and was organized by both mathematicians and physicists: A. Corti, R. Dijkgraaf, M. Douglas, J. Gauntlett, M. Gross, C. Hull, A. Jaffe and M. Reid. The early part of the school had many lectures that introduced various concepts of algebraic geometry and string theory with a focus on improving communication between these two fields. During the latter part of the program there were also a number of research level talks.

This volume contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.

The book is suitable for graduate students and research mathematicians interested in relations between mathematical physics and algebraic geometry.

This item will also be of interest to those working in algebra and algebraic geometry.

Titles in this series are published by the AMS for the Clay Mathematics Institute (Cambridge, MA).

Contents: M. R. Douglas, The geometry of string theory; B. S. Acharya, M theory, $G_2$-manifolds and four dimensional physics; S. K. Donaldson, Conjectures in Kähler geometry; J. P. Gauntlett, Branes, calibrations and supergravity; S. Gukov, M-theory on manifolds with exceptional holonomy; N. Hitchin, Special holonomy and beyond; D. Joyce, Constructing compact manifolds with exceptional holonomy; A. Kovalev, From Fano threefolds to compact $G_2$-manifolds; A. Craw, An introduction to motivic integration; A. Ishii, Representation moduli of the McKay quiver for finite Abelian groups; N. Hitchin, Moduli spaces of bundles over Riemann surfaces and the Yang-Mills stratification revisited; C. Madonna and V. V. Nikulin, On a classical correspondence between K3 surfaces II; R. Szendrői, Contractions and monodromy in homological mirror symmetry; N. Dorey, Lectures on supersymmetric gauge theory; A. Kapustin, The geometry of A-branes; R. C. Myers, Low energy D-brane actions; List of Participants.

Clay Mathematics Proceedings, Volume 3


Probability

The Theory of Probability

Fourth Edition

B. V. Gnedenko

This classic book is intended to be the first introduction to probability and statistics written with an emphasis on the analytic approach to the problems discussed. Topics include the axiomatic setup of probability theory, polynomial distribution, finite Markov chains, distribution functions and convolution, the laws of large numbers (weak and strong), characteristic functions, the central limit theorem, infinitely divisible distributions, and Markov processes.

Written in a clear and concise style, this book by Gnedenko can serve as a textbook for undergraduate and graduate courses in probability.

Contents: The concept of probability; Sequences of independent trials; Markov chains; Random variables and distribution functions; Numerical characteristics of random variables; The law of large numbers; Characteristic functions; The classical limit theorem; The theory of infinitely divisible distribution laws; The theory of stochastic processes; Elements of queueing theory; Elements of statistics; Tables; Bibliography; Index; Answers to the exercises.

AMS Chelsea Publishing

The main theme of this book is the “path integral technique” and its applications to constructive methods of quantum physics. The central topic is probabilistic foundations of the Feynman-Kac formula. Starting with main examples of Gaussian processes (the Brownian motion, the oscillatory process, and the Brownian bridge), the author presents four different proofs of the Feynman-Kac formula. Also included is a simple exposition of stochastic Itô calculus and its applications, in particular to the Hamiltonian of a particle in a magnetic field (the Feynman-Kac-Itô formula).

Among other topics discussed are the probabilistic approach to the bound of the number of ground states of correlation inequalities (the Birman-Schwinger principle, Lieb's formula, etc.), the calculation of asymptotics for functional integrals of Laplace type (the theory of Donsker-Varadhan) and applications, scattering theory, the theory of crushed ice, and the Wiener sausage.

Written with great care and containing many highly illuminating examples, this classic book is highly recommended to anyone interested in applications of functional integration to quantum physics. It can also serve as a textbook for a course in functional integration.

Contents: Introduction; The basic processes; Bound state problems; Inequalities; Magnetic fields and stochastic integrals; Asymptotics; Other topics; References; Index; Bibliographic supplement; Bibliography.

AMS Chelsea Publishing