The Mathematics Autodidact's Aid

Kristine K. Fowler

> The Universe (which others call the Library) is composed of an indefinite, perhaps infinite number of hexagonal galleries…. Let it suffice for the moment that I repeat the classic dictum: *The Library is a sphere whose exact center is any hexagon and whose circumference is unattainable.*
>
> —Jorge Luis Borges, The Library of Babel

The image of the library as the mathematician's laboratory suggests that of Borges's infinite library universe: so much is available it can be difficult to navigate effectively through it. A person trying to learn a new area of mathematics benefits from pointers shared by a knowledgeable guide, perhaps an adviser or colleague or librarian. Recording a few such signposts, which sketch a possible route through the basic topics, is the aim of this article. The suggestions of resources come from various mathematicians and a few librarians, identified in each section; they wrote full treatments of these subjects in Fowler (2004). The present brief guide doesn't attempt to identify the "best" books, but rather to give the independent mathematics learner recommendations of reasonable starting points; therefore resources that are particularly suitable for self-study, mostly at the graduate level, have been preferred where possible.

Finding Definitions and Formulas

To look up an unfamiliar term or topic, first see the standard *Encyclopaedia of Mathematics* edited by Hazewinkel; the subscribed Internet version is searchable. A shorter, very handy resource is *Eric Weisstein's World of Mathematics*, freely available online (`http://mathworld.wolfram.com/`); print versions appear as the *CRC Concise Encyclopedia of Mathematics*. Check for a specific integral, polynomial, or transform in the *Handbook of Mathematical Functions* by Abramowitz and Stegun; this classic will be updated by 2006 as the "Digital Library of Mathematical Functions" (`http://dlmf.nist.gov`). Other useful facts, from the formula for volume of an icosahedron to random number generators, may be found in the more general *Mathematics Handbook for Science and Engineering* by Råde and Westergren.

Writing and Researching

Good writing involves much more than text coding, useful as LaTeX manuals may be. For advice on structuring and composing papers, read Higham's *Handbook of Writing for the Mathematical Sciences*, which also discusses preparing and delivering talks. When compiling a bibliography or searching for literature on a topic, crucial issues include the choice of database (MathSciNet is standard for many searches, but not for a 1918 article or a biomathematics textbook) and the type of search (for example, using the Mathematics Subject Classification can generate a more comprehensive search than keywords). For helpful context and tips, see Molly T. White's "Tools and strategies for searching the research literature" in Fowler (2004).

History of Mathematics

For an overview, read either Struik's *Concise History of Mathematics*, which assumes you know most of the mathematics discussed, or the first section of *Math through the Ages* by Berlinghoff and Gouvêa, which assumes you don't. To delve deeper, look at Katz's very good textbook, *History of Mathematics: An Introduction*, and Grattan-Guinness's *Norton History of the Mathematical Sciences: The Rainbow of Mathematics*, which emphasizes more recent work and applied topics. Then take the doorway into the historical literature provided by *The Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, edited by Grattan-Guinness. The bibliographies in these books will guide further study in more specific areas. Key general reference sources include the *Biographical Dictionary of Mathematicians*, a subset of Gillispie's *Dictionary of Scientific Biography*;

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and the "MacTutor History of Mathematics Archive" (`http://www-history.mcs.st-andrews.ac.uk/history/`), edited by O'Connor and Robertson. Finally, *Mathematics across Cultures: The History of Non-Western Mathematics*, edited by Selin and D'Ambrosio, is a good way to broaden one's historical horizon. (advice from Fernando Gouvêa)

Number Theory

The following five books together provide an orientation to the whole field, starting with the least technical: Davenport's classic *The Higher Arithmetic*; *Introduction to the Theory of Numbers* by Hardy and Wright, a masterly exposition from an analytic viewpoint; Goldman's *The Queen of Mathematics: A Historically Motivated Guide to Number Theory*; *A Classical Introduction to Modern Number Theory* by Ireland and Rosen, with an outstanding presentation of the famous Weil conjectures; and finally Hua's extensive *Introduction to Number Theory*, with a strong emphasis on analytic number theory. For the global theory of algebraic numbers, see Marcus's *Number Fields*, a systematic introduction with a good set of exercises; for the local theory, the acknowledged standard is *Number Theory* by Borevich and Shafarevich. The elementary connections between number theory and algebraic geometry are discussed in the excellent *Rational Points on Elliptic Curves* by Silverman and Tate; a more advanced treatment of arithmetic algebraic geometry follows in Silverman's *Arithmetic of Elliptic Curves*. Apostol's *Introduction to Analytic Number Theory* effectively presents the standard topics, such as the distribution of prime numbers and the zeta and L-functions. (advice from Jay R. Goldman)

Combinatorics

For a broad range of topics at the beginning graduate level, see *A Course in Combinatorics* by van Lint and Wilson. Marcus's elementary *Combinatorics: A Problem Oriented Approach* is appropriate for self-study; more advanced is Lovász's *Combinatorial Problems and Exercises*. The standard comprehensive text is Aigner's *Combinatorial Theory*. The *Handbook of Combinatorics*, edited by Graham, Grötschel and Lovász, and the World Combinatorics Exchange (`http://www.combinatorics.org`) are useful reference sources. Good graph theory options include *Introduction to Graph Theory* either by West or by Wilson; Diestel's *Graph Theory*, with its many exercises; or Bollobás's thorough *Modern Graph Theory*. Recommended resources for other major topics: Stanley's *Enumerative Combinatorics*; the discussion of designs and codes in Hall's *Combinatorial Theory*; Spencer's *Ten Lectures on the Probabilistic Method* as applied to combinatorics; *Introduction to Lattices and Order* by Davey and Priestley, followed by Grätzer's authoritative *General Lattice Theory*; three articles in the *Handbook of Combinatorics* on the interaction of algebra and topology with combinatorics, by Alon, Björner, and Lovász et al.; Fejes-Tóth's *Regular Figures* as preliminary to a higher-level discussion of packing, covering, and tiling; connections with geometry in Coxeter's *Regular Polytopes* and *Combinatorial Geometry* by Pach and Agarwal; and Lawler's *Combinatorial Optimization: Networks and Matroids*, which also serves as an introduction to linear programming. (advice from Victor Reiner)

Abstract Algebra

Exercises and a leisurely manner make Jacobson's *Basic Algebra* more accessible than van der Waerden's important but dense *Modern Algebra*. Bourbaki's *Algebra* is abstract but very clear; for a more elementary treatment, try *Algebra* by Birkhoff and Mac Lane. The *Handbook of Algebra*, edited by Hazewinkel, should be available as a reference. Lam's *Ring* trilogy (*First Course, Exercises*, and *Lectures*) is a recommended entry to noncommutative algebra. Kaplansky's *Commutative Rings* admirably introduces its subject; then see the exhaustive *Commutative Algebra* by Samuel and Zariski. Next steps after elementary linear algebra include Gantmakher's *Theory of Matrices* (both text and reference book) and Smirnov's *Linear Algebra and Group Theory*, with solved exercises. Suzuki covers the classification of finite simple groups in *Group Theory*. Chapters 4 and 5 of Bourbaki's *Algebra* introduce field and Galois theory; *Galois Theory* by Edwards provides exposition with a historical perspective. Follow Osborne's overview of *Basic Homological Algebra* with Mac Lane's *Categories for the Working Mathematician*, an essential subtopic. The *Introduction to Lie Algebras and Representation Theory* by Humphreys is suitable for self-study. Montgomery's *Hopf Algebras and Their Actions on Rings* and Bredon's *Sheaf Theory* cover other key topics. (advice from Edgar Enochs)

Algebraic and Differential Geometry

If you are starting from scratch in algebraic geometry, begin with Reid's *Undergraduate Algebraic Geometry* or *An Invitation to Algebraic Geometry* by Smith, Kahanpää, Kekäläinen and Traves. Next, for inspiration, read the sections on curves and their Jacobians in Mumford's *The Red Book of Varieties and Schemes*; here Mumford explains the AMAZING SYNTHESIS, that there are three totally distinct ways to think about complex curves. Then look at Mumford's *Algebraic Geometry I: Complex Projective Varieties*. You are now prepared to tackle Hartshorne's *Algebraic Geometry* and *Principles of Algebraic Geometry* by Griffiths and Harris. To test your understanding, go through the many examples in Harris's *Algebraic Geometry: A First Course*. You would now easily be ready to start real work.

To get started in differential geometry, carefully study an undergraduate text; choose one by O'Neill, Henderson, Thorpe, do Carmo, or Millman and Parker. Next, turn to Morgan's *Riemannian Geometry: A Beginner's Guide*. By now you are ready for one of the following four books: Aubin's *A Course in Differential Geometry*, Chavel's *Riemannian Geometry: A Modern Introduction*, Lee's *Riemannian Manifolds: An Introduction to Curvature*, or Petersen's *Riemannian Geometry*. At the same time, go through Milnor's *Morse Theory*. Of course, you should also be picking and choosing theorems, examples, and techniques through various other books. (advice from Thomas Garrity)

Real and Complex Analysis

Serious study of analysis begins with a rigorous course in advanced calculus, based on a text such as Buck's *Advanced Calculus* or Apostol's *Mathematical Analysis*. Next is analysis in metric and topological spaces; see Simmons's *Introduction to Topology and Modern Analysis*. The standard graduate-level real analysis texts are Folland's *Real Analysis: Modern Techniques and Their Applications* and Royden's *Real Analysis*. To acquire true mastery, work through the challenging *Selected Problems in Real Analysis* by Makarov et al. The standard complex analysis texts are Ahlfors's *Complex Analysis* and Conway's *Functions of One Complex Variable I* (the latter particularly suitable for self-instruction). Useful reference sources include Krantz's *Handbook of Complex Variables* and the monumental *Applied and Computational Complex Analysis* by Henrici. Rudin's challenging *Real and Complex Analysis* insightfully integrates the two branches. Recommended sources for further study are Conway's *Course in Functional Analysis*; Körner's *Fourier Analysis*, followed by Stein's *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*; and Daubechies's *Ten Lectures on Wavelets*, the standard introduction to this hot topic. Important works in classical analysis are *Special Functions* by Andrews, Askey and Roy; and Szegő's *Orthogonal Polynomials*. (advice from John N. McDonald)

Ordinary and Partial Differential Equations

The standard undergraduate text, with lots of examples and applications, is *Elementary Differential Equations and Boundary Value Problems* by Boyce and DiPrima. Recommended graduate-level ODE texts include the standard *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, by Hirsch, Smale and Devaney; *Theory of Ordinary Differential Equations* by Coddington and Levinson, known for its clarity and challenging problems; and Chicone's recent *Ordinary Differential Equations with Applications*. Other ODE texts include the classics by Hurewicz and Arnold, which both emphasize geometric methods and ideas; the text by Amann provides an introduction to nonlinear analysis. The basic PDE text is Weinberger's *First Course in Partial Differential Equations with Complex Variables and Transform Methods*; Strauss's *Partial Differential Equations: An Introduction* is more elementary. The other two commonly used graduate PDE texts are by Evans and Folland. Temam's *Navier-Stokes Equations: Theory and Numerical Analysis* covers an essential special topic. Zwillinger's *Handbook of Differential Equations* serves as a convenient reference tool for solution techniques of ODEs as well as PDEs. Further study in dynamical systems could start with Devaney's *First Course in Chaotic Dynamical Systems* or *Nonlinear Differential Equations and Dynamical Systems* by Verhulst. (advice from John N. McDonald and Jan Figa)

Topology

An undergraduate point-set topology course might use Armstrong's *Basic Topology* or *Introduction to Topology* by Gamelin and Greene. Follow with Hatcher's *Algebraic Topology* (in print as well as free online at `http://www.math.cornell.edu/~hatcher`); Bredon's *Topology and Geometry*; or *Differential Forms in Algebraic Topology* by Bott and Tu, which includes an essential introduction to spectral sequences. For vector bundles, consult the standard *Characteristic Classes* by Milnor and Stasheff. Atiyah's *K-Theory* is the accepted reference in its area. Graduate-level differential topology study should include the excellent expositions in Milnor's various lecture notes, beginning with *Topology from the Differentiable Viewpoint*, as well as Hirsch's foundational *Differential Topology*. Kosinski's *Differential Manifolds* continues into surgery theory. For low-dimensional topology read *Three-Dimensional Geometry and Topology* by the master, Thurston. Livingston's *Knot Theory* attractively introduces this intuitive and visually appealing subject; alternatively, read Lickorish's *Introduction to Knot Theory* or the broader treatment in *Knots, Links, Braids and 3-Manifolds* by Prasolov and Sossinsky. Dieudonné's *A History of Algebraic and Differential Topology 1900–1960* provides a historical survey of the whole field as well as expositions of the mathematical ideas following the original sources. (advice from Allen Hatcher)

Probability Theory and Stochastic Processes

After an undergraduate probability course, read Feller's classic *Introduction to Probability Theory and Its Applications* or Gut's student-friendly *Intermediate Course in Probability*. Further study requires the underlying measure theory; the presentation in Ash's *Probability and Measure Theory* is recommended. Rao's excellent and thorough *Probability Theory with Applications* is at a higher level. Study of distributional limit theorems, a substantial part of the soul of probability theory, could continue with *Random Summation: Limit Theorems and Applications* by Gnedenko and Korolev, and den Hollander's *Large Deviations*. Useful reference sources include *Statistical Distributions* by Evans, Hastings and Peacock; and the Probability Web (`http://www.mathcs.carleton.edu/probweb/probweb.html`). Ross's *Stochastic Processes* or *Introduction to Stochastic Processes* by Hoel, Port and Stone are preliminary sources in this field; after acquiring the necessary background in measure theory and probability, go to Durrett's *Essentials of Stochastic Processes* and then Rao's more general *Foundations of Stochastic Analysis*. Important specific areas are covered in *Introduction to Stochastic Integration* by Chung and Williams, Øksendal's *Stochastic Differential Equations*, Kallianpur's *Stochastic Filtering Theory*, *Gaussian Processes* by Hida and Hitsuda, and *Continuous Martingales and Brownian Motion* by Revuz and Yor. (advice from Randall J. Swift)

Numerical Analysis

These four texts together provide an overview, from beginning graduate level to more specialized: *Numerical Mathematics* by Quarteroni, Sacco and Saleri; *Matrix Computations* by Golub and Van Loan, a broad reference book for numerical linear algebra; Iserles's *First Course in the Numerical Analysis of Differential Equations*; and *Theoretical Numerical Analysis* by Atkinson and Han, which introduces a functional analysis framework for studying numerical analysis. A classic text covering many topics not available elsewhere is *Analysis of Numerical Methods* by Isaacson and Keller. The *Handbook of Numerical Analysis*, edited by Ciarlet and Lions, gives advanced introductions to major topics, and *Acta Numerica* annually surveys current research. Noteworthy texts in specific areas include Kelley's *Iterative Methods for Linear and Nonlinear Equations, Numerical Optimization* by Nocedal and Wright, and Davis's classic *Interpolation and Approximation*. Standard resources for the two major types of ODE problems are Gear's *Numerical Initial Value Problems in Ordinary Differential Equations* and *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations* by Ascher, Mattheij and Russell. In the truly enormous area of PDEs, recommended introductions include Thomas's two-volume *Numerical Partial Differential Equations*, Braess's *Finite Elements*, and *Numerical Approximation of Partial Differential Equations* by Quarteroni and Valli. For integral equations see Atkinson's *Numerical Solution of Integral Equations of the Second Kind.* (advice from Kendall E. Atkinson)

Mathematical Biology

Good general sources aimed at mathematicians rather than biologists are Murray's standard *Mathematical Biology* and Haefner's *Modeling Biological Systems: Principles and Applications*. Allen's *Stochastic Processes with Biology Applications* covers a neglected subject. Recent research is accessibly presented in *Case Studies in Mathematical Modeling—Ecology, Physiology, and Cell Biology,* edited by Othmer et al. In *Population Biology: Concepts and Models*, Hastings's exposition is suitable for self-study; then see Freedman's *Deterministic Mathematical Models in Population Ecology* and Roughgarden's *Theory of Population Genetics and Evolutionary Ecology*. Consult *Dynamic State Variable Models in Ecology* by Clark and Mangel for applications varying from conservation biology to human behavior. The mathematical framework for epidemiology is provided in *Infectious Diseases of Humans: Dynamics and Control* by Anderson and May. See Lesk's *Introduction to Bioinformatics* or Waterman's *Introduction to Computational Biology: Maps, Sequences and Genomes* to begin studying these rapidly growing areas. *Statistical Methods in Bioinformatics* by Ewens and Grant teaches the basics of probability and statistics in the same context. Percus's *Mathematics of Genome Analysis* is more advanced and specialized. For cell biology and movement, read Segel's *Modeling Dynamic Phenomena in Molecular and Cellular Biology*. Follow the general, advanced *Mathematical Physiology* by Keener and Sneyd with Hoppensteadt's more focused *Introduction to the Mathematics of Neurons: Modeling in the Frequency Domain*. Gardner's *Notices* article on "Geometric tomography" introduces the relevant inverse problems. (advice from Claudia Neuhauser)

Mathematics Education

Research and theory development are the most active areas for graduate study. Several handbooks introduce the range of issues, especially the first and second *International Handbook of Mathematics Education*, both edited by Bishop and others. See the *Handbook of International Research in Mathematics Education*, edited by English; and *Handbook of Research on Mathematics Teaching and Learning*, edited by Grouws, for an international and American focus, respectively. Methodologies are discussed in the *Handbook of Research Design in Science and Mathematics Education,* edited by Kelly and Lesh. A standard work on development of number concepts and operations is *Adding It Up: Helping Children Learn Mathematics,* edited by Kilpatrick et al. (freely available at `http://www.nap.edu/books/0309069955/html/`). The highly influential *Principles and Standards for School Mathematics*, produced by the National Council of Teachers of Mathematics (`http://standards.nctm.org/`), sets the current vision for the U.S. educational system; see also *A Research Companion to Principles and Standards for School Mathematics,* edited by Kilpatrick et al. *Ethnomathematics: Challenging Eurocentrism in Mathematics Education*, edited by Powell and Frankenstein, presents now classic social justice papers. *Teaching, Multimedia, and Mathematics: Investigations of Real Practice* by Lampert and Ball examines technology's role in mathematics teaching. Ernest's *Philosophy of Mathematics Education* thoroughly explores the issues in its field. (advice from Kelly Gaddis, Jane-Jane Lo, and Jinfa Cai)

Mathematics Culture and Recreations

The preceding recommendations promote learning mathematical ideas, but it is interesting to consider also the nature and context of the mathematics enterprise. Essays written by mathematicians about their field give insights into its culture, discussing themes such as the art and beauty of mathematics, the intellectual characteristics of mathematicians, and how they collaborate and compete. Hardy's *A Mathematician's Apology* is deservedly the most famous of these. Gowers covers the issues concisely at the end of *Mathematics: A Very Short Introduction*. A more leisurely exploration appears in *Mathematics: People, Problems, Results*, classic articles compiled by Campbell and Higgins, complemented by the profiles and interviews in *More Mathematical People,* edited by Albers et al. Two entertaining resources address popular ideas about mathematics and mathematicians: Reinhold's "Math in the Movies" (`http://world.std.com/~reinhold/mathmovies.html`) and Kasman's "Mathematical Fiction" (`http://math.cofc.edu/faculty/kasman/MATHFICT/`). To return to doing mathematics but still on the lighter side, try the "Macalester College Problem of the Week", edited by Wagon (`http://mathforum.org/wagon/`), or get the ultimate CD of *Martin Gardner's Mathematical Games*. An important component of mathematics is having fun!

For Further Reading:

KRISTINE K. FOWLER, editor, *Using the Mathematics Literature*, Marcel Dekker, New York, 2004.

DAVE RUSIN, "Mathematical Atlas", `http://www.math.niu.edu/~rusin/known-math/welcome.html`.

MARTHA A. TUCKER and NANCY D. ANDERSON, *Guide to Information Sources in Mathematics and Statistics*, Libraries Unlimited, Westport, CT, 2004.