Future High-School Math Teachers and Upper-Level Math Courses

There is considerable anecdotal evidence that high-school math teachers do not see the relevance of the upper-level math courses they took in college to the mathematics that they are teaching in high school. There are two programs at Michigan State University that have led several of us to look at this situation more closely: a Teachers for a New Era (TNE) grant whose purpose is to examine the undergraduate education of future teachers, and our senior-level capstone course for future secondary math teachers that is jointly taught by a mathematician and a math educator.

We have discovered missed opportunities for drawing connections between the math upper-level mathematics courses and high-school math. Sometimes the math is too "elementary" to be mentioned in the college course. Sometimes the topic is mentioned but not sufficiently emphasized nor connected to the high-school math, and the students forget it by time they take the capstone course. Finally some topics, such as trigonometry, have been at best briefly surveyed in post-precalculus courses. Given how important it is for us to educate future high-school teachers well, perhaps mathematics departments should encourage instructors in upper-level undergraduate math courses to include discussions about such connections.

Here are a few examples:

From complex analysis: $\sqrt{-4} \times \sqrt{-9} \neq \sqrt{36}$. This is an example of something usually considered "too elementary" to appear in a complex analysis course. But most of our future high-school teachers had not seen it before, and they were somewhat skeptical since they had grown up with $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$. The explanation is likely to involve discussing $f(z) = z^2$ as a function of the unit circle to itself that wraps the circle around itself twice. Most have not seen that kind of a description of a function before.

From multivariable calculus: lines and planes in 3-space. Each is determined by a point and a vector. In both cases the point is on the object; the vector is parallel to the line or perpendicular to the plane. Lines and hyperplanes are subspaces of n-space, so they are topics of linear algebra. Their descriptions generalize the descriptions of lines and planes in 3-space. Our capstone students had forgotten essentially all of this. We designed a project for them of taking the equations for lines and planes in 3-space and restricting them to 2-space. This yields two descriptions of lines in 2-space, which the students were asked to reconcile.

From abstract and linear algebra, several topics that are important for future teachers and hence need to be emphasized by the instructors of the courses.

1. The division algorithm. They need to know this both for $\mathbb{Z}$ and for polynomials over fields. In particular they need to know why all of the hypotheses are necessary.
2. The Fundamental Theorem of Algebra and factoring of polynomials over $\mathbb{C}$ into linear factors and over $\mathbb{R}$ into linear and quadratic polynomials.
3. Using the quotient $\mathbb{R}[x]/(x^2+1)$ to explain why it is rigorous to describe addition and multiplication in $\mathbb{C}$ as "just like polynomials except that $x^2 = -1$."
4. Matrices that induce rotations and reflections in the plane.
5. The least squares problem of fitting a regression line to points in a plane.

Some general comments:

Functions. Essentially students know a function has inputs and outputs such that every input has a unique output. Several had never had to memorize a formal definition, such as "A function consists of two sets and a rule" or "A function is a set of ordered pairs…." This means that students going off to graduate school may not have either, and I have concerns about that. In any case, future high-school teachers need to know all three definitions and understand their equivalence, as all three appear in high-school math textbooks.

Trigonometry. To our surprise, most of the students had very little trig at their fingertips. Mostly, they had not seen much trig since high school, except for graphs and a few identities in calculus. In the second capstone course, we gave these students a thorough review, emphasizing things they will need to understand when they teach. But is this something we should worry about for all math majors?

Graphs. Except for functions from the reals to the reals, students may have no idea where the graph of a function lies. They can get very creative in discussing graphs of rational functions that have factors like x^2+4 in the denominator and get confused about where the asymptotes at $\pm 2i$ go.

Many people reading this will come up with other concerns both for our future high-school math teachers and for our math majors in general. It seems like a crucial time for mathematics departments to discuss these issues.

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An expanded version of this article can be found at `http://www.math.msu.edu/~hill/`.

Letters to the Editor

No More Homework

In the letter "Homework and Google" appearing in the December 2004, *Notices*, the author expresses concern with the availability of homework solutions on the Internet and describes methods to make posted solutions invisible to search engines.

I believe trying to do this is a waste of time. In fields such as literature, history, political science, and so on, there are already literally dozens of websites which make papers available to students for small fees. This is the present, and will continue to be the future, and I believe mathematics is not far behind, if not there already.

Attempts to thwart this phenomenon are pointless. The only way to prevent the proliferation of sites selling mathematics homework solutions, and students from purveying these sites, will be to make graded homework solutions irrelevant. What do I mean by this? Let me first describe my own background.

In the country in which I was an undergraduate, the very idea of asking university undergraduates to submit mathematics "homework" for marking (grading) was so far from the norm it would have been laughable. And I mean that literally. Laughable. No one —not one student—would have carried it out. Nor would a single instructor even have attempted to do so.

"Homework", by which was meant a written assignment for turning in and marking, was totally an elementary school or high school concept, for children only. University students were supposed to be adults, not children, and were not given "homework". This is not to say we were not given problems to do in our university courses. On the contrary, we were given many typed out pages of these. But we were never required to turn them in.

Students enrolled in mathematics courses were required to attend, once a week, what were called tutorial sessions. Attendance was taken. At these approximately two-hour sessions, 30 or 40 students would sit quietly and individually working on their problems, and professors would walk around answering questions when students had them. That's it. You could also ask your instructor for help during office hours.

Problems were for us to do if and when we wanted and however we wanted. It was assumed that university students were adults, interested in the subject they studied, and would eventually (i.e., before the final exam) do their problems. The reward was not in some artificial point grading system but in learning and succeeding in courses in which students professed to be interested. In my entire undergraduate career I can only recall turning in for grading physics and chemistry laboratory reports, and even those grades were meager in the scheme of the entire course grade. Every other grade in every other course was determined by a few exams and, most importantly, a final exam.

I am not saying what I describe above was perfect. But maybe one solution to the problem of ready availability of homework solutions on the internet is to motivate students to want to learn and to find ways to take away the incentive to plagiarize and cheat. Let the students know that you will hold them accountable for the work they are supposed to do and test them in such a way as to see if they have done it. If you want to give them homework for grading, then count it for very little in the final grade, or use it to provide verbal feedback to the students. Try to give them homework unique to your course. How to do this is our modern-day challenge.

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Contacting Tulane Mathematicians

Hurricane Katrina struck the Gulf South two days before the beginning of classes at Tulane. The faculty, students, and staff evacuated to hotels, shelters, and dormitories near the region, expecting to return home within a few days. After the extent of the damage became apparent, we scattered to places all over the country and are currently in the process of setting up temporary living and working arrangements. The help that we have received in these efforts from the mathematics community has been wonderful, and on behalf of the mathematics department I would like to thank all of the departments, organizations, and individuals who have been instrumental in helping us. Many departments have offered office space to displaced Tulane faculty members; I know of no department which refused such a request. In many cases the help that we received went well beyond assisting us in our professional lives. Colleagues were instrumental in helping some of us find appropriate schools for our children, in finding apartments, and in the many other tasks involved in establishing a, more or less, normal life. These were acts of genuine kindness for which we are profoundly grateful. The mathematics community extended help for our students as well. A number of departments have admitted our graduate students for the semester. At a time of such chaos and confusion, our ability to have some sort of professional life has importance to us far beyond the actual value of whatever mathematical work we produce. As we are disconnected from familiar places and routines, we can go to talks, have discussions with friends and colleagues, while waiting to return home; all the things that we do daily at Tulane.

If anyone reading this wishes to contact a Tulane mathematician, many members of the department have registered on the Tulane Survivor Network, which can be found at `http://www.tulane.edu`, where their temporary phone numbers and email addresses can be found. Members of the department have also established a discussion group at `http://groups.google.com/group/TulaneMath` where you can learn more about the status of our department.

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