Book Review

Science in the Looking Glass: What Do Scientists Really Know?

Reviewed by Martin Gardner

Science in the Looking Glass: What Do Scientists Really Know?
E. Brian Davies
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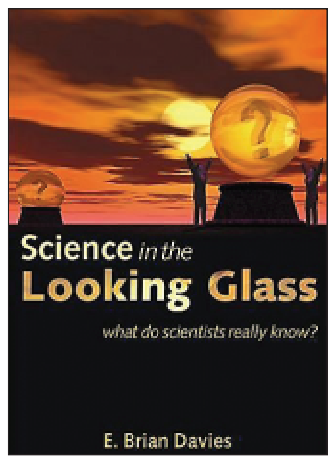
I'm not sure exactly what Brian Davies, a distinguished mathematician at King's College, London, intended the title of his fifth book to suggest. Reflect like a mirror the history and nature of science? Perhaps he also thought of his book as leading readers into a dreamlike universe as fantastic as the world Alice first entered through a rabbit hole and later through a looking glass. Whatever the intent, it is a brilliant work, beautifully written, and brimming with surprising information and stimulating philosophical speculations.

Before turning to my one caveat—unlike Davies I'm an unabashed realist who believes that mathematical objects and theorems are "out there" with a peculiar kind of reality that is independent of minds and cultures—let me go over some of the book's highlights.

Davies begins with a discussion of the uncertainties of perception. Errors of seeing are demonstrated with two amazing optical illusions. One is a ring of slash strokes that seems to rotate as the page is shifted forward and back. It is impossible, viewing the other illusion, not to be sure that one square of a checkerboard is darker than another when both are actually the same shade. Language too can be misleading. Davies does not buy Noam Chomsky's claim that there is a genetically transmitted deep "universal grammar". Interaction with environment is sufficient to account for the ability to speak. Apes failed to speak because their throats lacked the apparatus necessary for producing a great variety of sounds.



Descartes's famous effort to separate mind from body is thoroughly discredited. On the other hand, although Davies is convinced that consciousness—the awareness that one exists and has free will—is a function of a material brain, it is still a total mystery. He agrees with Roger Penrose, Oxford's mathematical physicist, that no computer made with wires and switches will ever become aware of what it is doing. Assuming that we are no more than an enormously complex pattern of molecules, Davies speculates on the possibility that our pattern could someday be scanned and transmitted to another place like the up and down beaming of characters in *Star Trek*. Can a simpler pattern, such as an apple, be so translocated?

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Physicists are hard at work trying to accomplish just such a feat and have actually succeeded in teleporting an atom. Transmitting a human, however, as Davies recognizes, raises profound questions about identity. After being "beamed down", would a translocated person be the same person or merely a replica? What if the technique produces two identical persons? Philosophers, notably Locke, have agonized over just such thought experiments. Hundreds of science-fiction tales have considered such possibilities. Penrose, by the way, has argued that if even an apple is transmitted, laws of quantum mechanics require total destruction of the original. When the captain of the Enterprise is beamed down to a planet, he cannot leave himself behind.

Davies's chapters on pure mathematics cover a wide range. He deals with imaginary and complex numbers and the difficulties that arise with rational numbers when they are enormously large or small. "Hard" problems like the four-color-map theorem have finally been proved, but by such monstrous computer printouts that the proof can be checked only by another computer. Goldbach's still unsettled conjecture that every even number greater than four is the sum of two odd primes has now been confirmed for numbers up to 10^{14}. This, Davies adds, "would be sufficient evidence for anyone except a mathematician."

A page is devoted to the notorious Collantz conjecture. Start with any number above 1. If even, halve it. If odd, replace it with $3n+1$. Continue doing this. If the procedure ends with 1, stop. The conjecture is that it will always stop. So far it has stopped for all n up to 10^{12}, but a proof remains elusive.

Davies reports the sensational discovery a few years ago by Manindra Agrawal and his two young assistants in Kampur, India, of a simple rapid method of testing whether a huge number is or isn't prime. The algorithm doesn't generate factors, but merely tests for primality, and does so in polynomial time! "Such discoveries," Davies writes, "are among the things which make it a joy to be a mathematician."

Several pages concern the innocent-seeming little problem of the three doors, which created such a stir when Marilyn vos Savant published it in her weekly *Parade* column. Modeled with playing cards it goes like this. Smith places three cards face down on a table. Only one card is an ace. You are asked to guess where the ace is by placing a finger on a card. Clearly the probability you guess right is $1/3$. Smith, who knows where the ace is, now turns face up a card that is *not* an ace. Two cards remain face down. Does not the probability your finger is on the ace go up to $1/2$? It does not! It remains $1/3$. If you now move your finger to the other card, the probability it rests on the ace rises to $2/3$! Savant gave a correct solution, but thousands of mathematicians who should have known better wrote angry letters attacking the solution. The event even made the front page of the *New York Times*.

Davies's chapters on the physical and biological sciences are as broad in scope and as illuminating as his chapters on mathematics. We learn about the mind-bending paradoxes of relativity and quantum mechanics, about chaos theory, continental drift, the ever-changing conjectures of cosmology, the anthropic principle, Thomas Kuhn's shaky views about science revolutions and paradigm shifts, and a hundred other topics on the frontiers of modern science.

A lengthy chapter on evolution rips apart the currently fashionable claim by defenders of "intelligent design" that the "irreducible complexity" of even the simplest life forms could not have evolved without the guidance of an intellgent designer, namely God. Davies ticks off a variety of facts that support the randomness of mutations. Why should a competent designer, he asks, bother to produce millions of dinosaurs only to allow them to vanish except for some small ones that turned into birds?

The world's vast amount of evil and suffering is evidence, Davies is convinced, that there is no transcendent deity supervising evolution. As Marlene Dietrich once remarked (my quote), "If there is a God, he must be crazy." Davies reproduces a lovely photograph of a snow crystal as evidence that natural laws combined with chance can produce intricate complexity.

I have touched on only a small fraction of the myriad of colorful accounts that Davies provides about today's science and mathematics. Let me now turn to my reasons for not accepting a basic theme of Davies's book. I refer to his constant bashing of mathematical realism, especially the vigorous Platonism of Penrose and Kurt Gödel.

First of all, I prefer the term realism to Platonism. Why? Because it avoids all the dismal controversies over such universals as goodness, beauty, chairness, cowness, and so on, that so agitated the minds of the medieval scholastics. No modern realist believes for a moment that numbers and theorems "exist" in the same way that stones and stars exist. Of course mathematical concepts are mental constructs and products of human culture. Everything persons think and do is part of culture. To say that numbers are mental constructs is to say something trivial—something no realist denies. The deeper question is whether these constructs have a peculiar, dimly understood kind of reality embedded in the universe in a way that is not mind-dependent. No human is needed to establish the fact that the geometrical shape of Aristotle's vase is inseparable from the vase. A spiral is inseparable from a spiral galaxy. The four corners of a cube can no more be detached from a physical model of a cube than from an ideal cube. The existence of optical illusions doesn't prevent one from seeing eight corners. You can close your eyes and feel the corners.

To a realist it is a misuse of language to say that primitive humans invented integers. What they did was invent *names*, later symbols, for properties of sets of discrete things such as fingers, pebbles, and elephants—things "out there", independent of human minds. Later they discovered the laws of arithmetic because that was how pebbles behaved when manipulated. They didn't invent the Pythagorean theorem. They found it, out there, when they measured the sides of material right triangles.

If one is a theist, believing as Paul Dirac did that God is a great mathematician, or even in the pantheistic deity of Spinoza and Einstein, then the locus of mathematical reality moves to a transcendent realm outside Plato's cave. The big debate between realism and constructivism evaporates. Paul Erdős liked to refer to God's *Book* in which all the most elegant proofs are recorded. From time to time mathematicians are permitted brief glimpses into one of the Book's infinity of pages.

In a curious way, numbers may be *more* real than pebbles. Matter first dissolved into molecules, then into atoms, then into particles, which are now dissolving into vibrating loops of string or maybe into Penrose's twistors. And what are strings and twistors made of? They are not made of anything except numbers. If so, the numbers are as much "out there" as molecules. They could be the *only* things out there. As a friend once said, the universe seems to be made of nothing, yet somehow it manages to exist. As Ron Graham remarked, mathematical structure may be the fundamental reality.

No anti-realist such as Davies, and Reuben Hersh whom he admires, thinks the moon vanishes when no one, not even a mouse, is observing it. If the moon is "out there", why not admit that the moon's circumference, divided by its diameter, is a close approximation of π even before mathematicians were around to say it and will be true if humans became as extinct as dinosaurs?

To an anti-realist, π doesn't really exist outside the minds of sentient creatures. A sequence in π's decimal expansion, such as ten sevens in a row, isn't "there" until a computer calculates it. Davies tells an amusing story about how, in his book's first draft, he wondered whether a computer would ever find the sequence 0123456789 in π. To Davies's astonishment he later discovered that this sequence actually had been found. In the unlikely case that readers would like to know, the run starts at π's $17,387,594,880^{\text{th}}$ digit.

Davies takes up the question of whether one is allowed to say that somewhere in π is a run of a thousand sevens. In talking about such things, Davies, like all anti-realists, slips into the language of realism. He writes, "we can estimate how long it would take to *find* the first occurrence" (italics mine) of a run of a thousand sevens. Again: The time it would probably take "to *find* the sequence" would be "vastly longer than the age of the universe". The word "find" of course implies that the run already exists. Davies is usually careful to avoid the word "find" because it gives the game away. "A Platonic mathematician would say that either there exists [such a run] . . . or there does not. This is certainly psychologically comfortable, but it is not necessary to accept it in order to be a mathematician." So comfortable, in fact, that anti-realists seldom hesitate to speak of "finding" (i.e., discovering) something when they really mean constructing it.

William James somewhere speaks of digits as "sleeping" in π until some mathematician wakes them up. It is a striking metaphor. A sleeping cat, however, has to sleep somewhere. To Davies and Hersh the uncalculated digits of π sleep nowhere. They just pop into reality when a computer "constructs" them.

Bertrand Russell, a firm realist, once wrote that $2+2=4$ even in the interior of the sun. As I have often said, if two dinosaurs met two other dinosaurs in a clearing there would have been four there even if no humans were around to observe them. The equation $2+2=4$ is a timeless truth, valid in all logically possible worlds because it is what philosophers since Kant have called *analytic*. Given the axioms of arithmetic $2+2=4$ can be translated into a string of symbols which, assuming the axiomatic system's formation and transformation rules, arrive at $A=A$. Two plus two is four for the same reason that there are three feet in a yard.

Like many anti-realists, Davies drifts close to a kind of social solipsism in which even the external world fades into a hazy construction of our brains. He quotes favorably from Donald Hoffman's book *Visual Intelligence: How We Create What We See*. "Why," Hoffman asks, "do we all see the same things?" Why for instance, do we all see the same moon? Everyone I know would at once answer, "Because the moon doesn't change." Not Hoffman. His reason, so help me, is "because we all have the same rules of construction." We are not seeing a moon, out there, independent of us. We are seeing our constructions of the moon!

This is far more extreme than the opinion that $2+2=4$ because we all construct numbers the same way. To suppose that people see the same cow because they have constructed the cow by the same rules boggles my mind. They see the same cow because it *is* the same cow. "Realism," I once heard Russell say in a lecture, "is not a dirty word."

Anti-realists are fond of claiming that mathematics, like science, is never certain. Morris Kline even wrote a book titled *Mathematics: The Loss of Certainty*. On the contrary, mathematics (including formal logic) is the *only* place where there is no loss of certainty. In his book *What is Mathematics, Really?*, Hersh argues that even laws of arithmetic are uncertain by considering a hotel that is missing a thirteenth floor. Take an elevator up eight floors, then go five floors more, and you reach floor fourteen. Hersh apparently thinks this violates the equation $8 + 5 = 13$. What he has done, of course, is jump from pure arithmetic to applied arithmetic, where applications are often uncertain.

Two beans plus two beans make four beans only if you assign to beans what Rudolf Carnap called a correspondence rule. In this case the rule is that each bean corresponds to 1. In the case of Hersh's elevator, if you assume that every floor corresponds to 1, then 8 floors plus 5 floors is sure to make 13 floors. Without correspondence rules, applications of mathematical truths are indeed uncertain. Two drops of water added to two drops can make a single drop. Hersh and Philip J. Davis, in their book *The Mathematical Experience*, give an even funnier example. A cup of milk, they inform us, added to a cup of popcorn doesn't make two cups of the mixture.

Euclidean geometry is not rendered uncertain because space-time is non-Euclidean. The Pythagorean Theorem is absolutely certain within the formal system of plane geometry. There is not the slightest doubt that the angles of a Euclidean triangle add to 180 degrees. Science, on the other hand, is corrigible. Decades before Karl Popper, Charles Peirce coined the term fallibilism, and awareness that science is fallible goes back to the ancient Greek skeptics. As Hume taught us, there is no *logical* reason why the sun must rise tomorrow. For all we know there might be an unknown law of inertia that would suddenly stop the earth from rotating. This is in stark contrast with mathematics, where the uncertainty of science is incapable of inflicting injuries.

But enough about the tiresome, never-ending debate between the small minority of anti-realists and the vast majority of mathematicians, including the greatest, who take realism for granted. They do their work without the slightest anxiety over the philosophical foundations of their craft.

What I admire most about Davies is his awe before the terrible mystery of time and why the universe, as Hawking recently wondered, "bothers to exist." He is aware of how little we understand the workings of Einstein's Old One. To answer his book's subtitle, scientists "really know" a great deal but what they don't know is even vaster. Here is how this book ends:

> The full complexity of reality is far beyond our ability to grasp, but our limited understanding has given us powers which we had no right to expect. There is no reason to believe that we are near the end of this road, and we may well hardly be past the beginning. The journey is what makes the enterprise fascinating. The fact that the full richness of the universe is beyond our limited comprehension makes it no less so.

The conflict between realists and their critics may come down finally to the choice of a language that is the least confusing. As President Clinton famously said, it all depends on what the meaning of *is* is.