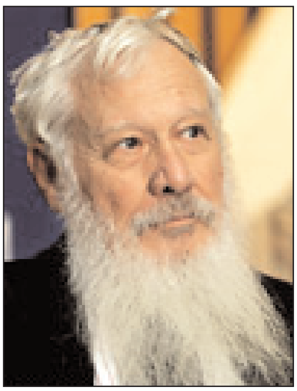
Aumann Awarded Nobel Prize

Abraham Neyman



Robert J. Aumann

Robert J. Aumann, professor of mathematics at the Hebrew University and member of the Interdisciplinary Center for Rationality there, shares (with Thomas C. Schelling) the 2005 Nobel Prize in Economics [13].

Aumann was born in Frankfurt, Germany, in 1930, and moved to New York with his family in 1938. In 1955 he completed his Ph.D. in mathematics at MIT under the supervision of George Whitehead. His thesis, in knot theory, was published in the *Annals of Mathematics* [1].

In 1955 Aumann joined the Princeton University group that worked on industrial and military applications, where he realized the importance and relevance of game theory, then in its infancy. In 1956 Aumann joined the Institute of Mathematics at the Hebrew University.

Over the past half-century, Aumann has played an essential and indispensable role in shaping game theory, and much of economic theory, to become the great success it is today. He promotes a unified view of the very wide domain of rational behavior, a domain that encompasses areas of many apparently disparate disciplines, like economics, political science, biology, psychology, mathematics, philosophy, computer science, law, and statistics. Aumann's research is characterized by an unusual combination of breadth and depth. His scientific contributions are path-breaking, innovative, comprehensive, and rigorous—from the discovery and formalization of the basic concepts and principles, through the development of the appropriate tools and methods for their study, to their application in the analysis of various specific issues. Some of his contributions require very deep and complex technical analysis; others are (as he says at times) "embarrassingly trivial" mathematically, but very profound conceptually [12, p. 1]. He has influenced and shaped the field through his pioneering work. There is hardly an area of game theory today where his footsteps are not readily apparent. Most of Aumann's research is intimately connected to central issues in economic theory: on the one hand, these issues provided the motivation and impetus for his work; on the other, his results produced novel insights and understandings in economics. In addition to his own pioneering work, Aumann's indirect impact is no less important—through his many students, collaborators, and colleagues. He inspired them, excited them with his vision, and led them to further important results [12, p. 2].

The limited space allotted to this article does not begin to allow a comprehensive account of Aumann's extensive contributions. Thus, the article must confine itself to brief commentary touching on only a small part of his work. It is important to note that the scope of each description is not indicative of the importance of the contribution. Further and more detailed accounts of Aumann's contributions may be found in [12].

We start with Aumann's study of long-term interactions, which had a most profound impact on the social sciences. The mathematical model enabling a formal analysis is a *supergame* G^*, consisting of an infinite repetition of a given one-stage game[1] G. A *pure strategy* in G^* assigns a pure strategy in G to each period/stage, as a function

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[1] *A game G in strategic form consists of a set of players N, pure strategy sets A_i for each player i, and payoff functions g_i, which describe the payoff to player i as a function of the strategy profiles $a \in A := \times_{i \in N} A_i$.*

of the history of play up to that stage. A profile of supergame strategies, one for each player, defines the play, or sequence of stage actions. The payoff associated with a play of the supergame is essentially an average of the stage payoffs.

In 1959 Aumann [2] defined the notion of a *strong equilibrium*—a strategy profile where no group of players can gain by unilaterally changing their strategies—and characterized the strong equilibrium outcomes of the supergame by showing that it coincides with the so-called *β-core* of G. When Aumann's 1959 methodology is applied to *Nash equilibrium*—a strategy profile where no single player can gain by unilaterally changing his strategy—the result is essentially the so-called *Folk Theorem* for supergames: the set of Nash equilibria of the supergame G^* coincides with the set of feasible and individual rational payoffs in the one-stage game. In 1976 Aumann and Shapley [11] (and Rubinstein[2] in independent work) proved that the equilibrium payoffs and the perfect equilibrium payoffs of the supergame G^* coincide.

Supergames are repeated games of complete information; it is assumed that all players know precisely the one-shot game that is being repeatedly played.

> The theory of repeated games of complete information is concerned with the evolution of fundamental patterns of interaction between people (or for that matter, animals; the problems it attacks are similar to those of social biology). Its aim is to account for phenomena such as cooperation, altruism, revenge, threats (self-destructive or otherwise), etc.—phenomena which may at first seem irrational—in terms of the usual 'selfish' utility-maximizing paradigm of game theory and neoclassical economics [7, p. 11].

The model of repeated games with incomplete information, introduced in 1966 by Aumann and Maschler [9], analyzes long-term interactions in which some or all of the players do not know which stage game G is being played. The game $G = G^k$ depends on a parameter k; at the start of the game a commonly known lottery $q(k)$ with outcomes in a product set $S = \times_i S_i$ is performed and player i is informed of the i-th coordinate of the outcome. The repetition enables players to infer and learn information about the other players from their behavior, and therefore there is

> a subtle interplay of concealing and revealing information: concealing, to prevent the other players from using the information to your disadvantage; revealing, to use the information yourself, and to permit the other players to use it to your advantage [8, pp. 46–47].

> The stress here is on the strategic use of information—when and how to reveal and when and how to conceal, when to believe revealed information and when not, etc. [7, p. 23].

This problem of the optimal use of information is solved in an explicit and elegant way in [9].

Another substantial line of contributions of Aumann is the introduction and study of the continuum idea in game theory and economic theory. This includes modeling *perfectly competitive* economies as economies with a continuum of traders and proving the equivalence of the core and competitive equilibrium [3] as well as the equivalence of the core (and competitive equilibrium) and the value [5], proving the existence of the competitive equilibrium [4], and introducing and extensively developing the (Aumann-Shapley) value of coalitional games with a continuum of players [10]. These models with a continuum of agents enable precise analysis of economic and political systems where groups of participants have significant influence over the outcome, but each individual's influence is negligible.

Another fundamental contribution of Aumann is "Agreeing to Disagree" [6]: it formalizes the notion of *common knowledge* and shows (the somewhat unintuitive result) that if two agents start with the same prior beliefs, and if their posterior beliefs (about a specific event), which are based on different private information, are common knowledge, then these posterior beliefs coincide. This paper had a major impact; it led to the development of the area known as *interactive epistemology* and has found many applications in different disciplines like economics and computer science.

Other fundamental contributions include the introduction and study of *correlated equilibrium*, the study of *bounded rationality*, and many important contributions to cooperative game theory: extending the theory of *transferable utility* (TU) games to general *nontransferable utility* (NTU) games, formulating a simple set of axioms that characterize the NTU-value,[3] and the "Game-theoretic analysis of a bankruptcy problem from the Talmud".[4]

Robert J. Aumann has been a member of the U.S. National Academy of Sciences since 1985, a

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[3] *Introduced in L. S. Shapley,* Utility comparison and the theory of games, *La Décision (Paris: Edition du C.N.R.S., 1969), pp. 251–263.*

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member of the Israel Academy of Sciences and Humanities since 1989, a Foreign Honorary Member of the American Academy of Arts and Sciences since 1974, and a corresponding fellow of the British Academy since 1995. He received the Harvey Prize in Science and Technology in 1983, the Israel Prize in Economics in 1994, the Lanchester Prize in Operations Research in 1995, the Nemmers Prize in Economics in 1998, the EMET prize in Economics in 2002, the von Neumann prize in Operations Research and Management Science in 2005, and the Nobel Memorial Prize in Economic Sciences in 2005. He was awarded honorary doctorates by the Universität Bonn in 1988, by the Université Catholique de Louvain in 1989, and by the University of Chicago in 1992.

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[10] R. J. Aumann and L. S. Shapley, *Values of Non-Atomic Games*, Princeton University Press, 1974, xi + 333 pp.

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[12] S. Hart and A. Neyman, Introduction, *Games and Economic Theory, Selected Contributions in Honor of Robert J. Aumann*, The University of Michigan Press, 1995, pp. 1–28, `http://ratio.huji.ac.il/dp/neyman/bookintroduction95.pdf`.

[13] Robert Aumann and Thomas Schelling, Contribution to game theory: Analyses of conflict and cooperation, `http://nobelprize.org/economics/laureates/2005/ecoadv05.pdf`.

How Many Mathematicians Have Won Nobels?

With the awarding of the 2005 Nobel in Economics to Robert J. Aumann have come remarks that he and John Nash were the only mathematicians to have received Nobel Prizes. But there have been others.

Bob Aumann was in the mathematics department at Hebrew University for many years. After writing a couple of papers in algebraic topology (his thesis area), he became one of the movers and shakers in game theory. (Remember after Aumann received his MIT Ph.D., he accepted a position at Princeton, which at that time was deeply involved in that new topic of game theory.) Indeed, I must wonder whether the award was given to Bob this year to make up for the serious mistake of not recognizing him at the same time as when Nash (also a Princeton mathematics Ph.D.) got his Nobel.

There are several other examples of mathematicians receiving the Nobel Prize. John Pople of Northwestern University (my former academic home), who was honored with the Chemistry Nobel in 1998, received his mathematics Ph.D. from Cambridge in partial differential equations. All of his research involved finding different approximations for the Navier-Stokes equation and relating it to chemistry. Until his recent death, John always was very positive in his comments about the power and value of mathematics.

Another example is Herbert Hauptman, who received the 1985 Nobel in Chemistry. Hauptman earned his mathematics Ph.D. from Maryland with a dissertation "An *N*-dimensional Euclidean algorithm".

Kenneth Arrow received the 1972 Nobel in Economics. He earned his M.A. in mathematics from Columbia, and much of his Ph.D. training was in statistics and economics. Read his work; Arrow was strongly influenced by mathematics and he uses it skillfully!

Another name is Gerard Debreu, who received the 1983 award in Economics. Debreu, who died in December 2004 and whose memory and research were recently recognized at a conference in Berkeley, received his doctorate in mathematics in France. Debreu always kept strong ties to mathematics. For instance, I was told that in the late 1970s, Debreu and Steve Smale played central roles in pulling mathematics and economics into the same building at Berkeley. During Debreu's Ph.D. training, he was strongly influenced by the Bourbaki school in France. It is easy to believe this: not only did this school fashion Debreu's mathematical tastes, but Debreu's adoption of the Bourbaki formal writing style made many of his books and papers very difficult to read.

Earlier, Leonid Kantorovich received the 1975 award in Economics. He always was a mathematician; indeed, he was chair of the mathematics group in Novosibirsk in Siberia and later a mathematics group in Moscow. His name is familiar from conformal mappings, variational methods, functional analysis, etc.

While John Bardeen (the only double Nobel winner in Physics) did not earn his Ph.D. in mathematics, he did his graduate work at Princeton in mathematical physics. Much of his work involved mathematics. Moreover, John kept close ties to the mathematics department at Urbana; e.g., he was chair of the 1979 committee to find a new chair for that department.

If one wanted to count Nobel winners who used a significant amount of fairly sophisticated mathematics in their research, one probably would have close to half of all Economics winners and several more from Chemistry and Physics (including Einstein).

There is a persistent rumor that the reason there is no Nobel in mathematics or astronomy is that, had there been one, Mittag-Leffler would have won. But, the story goes, the political problem in awarding such a prize to him was that Mittag-Leffler was having an affair with Nobel's wife.

When I was at Northwestern, one of my colleagues, Alexandra Bellow (the well known ergodic theorist), was in Sweden with her husband at the time, Saul Bellow, when he received his Nobel Prize in literature. So she checked out the story about Mittag-Leffler. When she returned she told me that there was a minor flaw in this story—Nobel never was married!

—Donald Saari, University of California, Irvine