

Book Review

Incompleteness: The Proof and Paradox of Kurt Gödel

Reviewed by Juliette Kennedy

Incompleteness: The Proof and Paradox of Kurt Gödel

Rebecca Goldstein

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Popular books on mathematics play an important role in the lay public's education. But as is known to anyone who has given a popular mathematics lecture or written about a famous theorem for an audience of nonmathematicians, doing justice to the mathematics in question is almost impossible in those circumstances. Rebecca Goldstein, the MacArthur Foundation fellow and author of *The Mind-Body Problem* (a novel which seems to be quite popular among mathematicians) attempts an even more difficult task in her short new book *Incompleteness: The Proof and Paradox of Kurt Gödel*, namely, to place a significant piece of mathematics—Gödel's Incompleteness Theorems—in the context of the wider intellectual currents of the twentieth century, both within the mathematical logic and the philosophy of mathematics communities, as well as within the intellectual culture at large.

The theorems are presented in the context of a somewhat detailed personal and intellectual biography of Gödel as well as in that of the various schools in foundations of mathematics in existence at the time. A vast amount of material is covered: everything from the history of the Vienna Circle, to Gödel's philosophical differences with Wittgenstein, to the Hilbert Program, to Gödel's views on appointments at the Institute for Advanced Study, to many aspects of his personal biography, in addition to the account of the two

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Incompleteness Theorems. This means that some of these areas are covered more comprehensively than others.

In places the author succeeds creditably; for example, her portrayals of behind-the-scenes academic life will likely be of interest to readers who enjoy such material—indeed, such portrayals seem to be her forte.

As is often the case with books about mathematics written by nonmathematicians though, shortfalls of precision occurring here and there will leave mathematicians unsatisfied; and the misstatement of the Fixed Point Theorem on page 180, the heart of the matter technically, makes it, unfortunately, quite impossible for anyone to reconstruct the proof of the First Incompleteness Theorem from Goldstein's account.

In this review I will comment on the three main aspects of Goldstein's book, apart from her technical account of the theorems: first, her portrayal of the life and personality of Gödel and the social surroundings in which he worked; second, her main claim, which is that Gödel's work has been misunderstood and misused by postmodernists and other intellectuals; and third, her rather substantial discussion of foundational issues, which unfortunately is the weakest of these three aspects of the book.

The author's novelistic skills are at their most conspicuous in the section of the book devoted to her colorful portrait of Gödel. From p. 59:

I think it is fair to say...that like so many of us Gödel fell in love while an undergraduate. He underwent love's ecstatic transfiguration, its radical re-ordering of priorities, giving life new focus and meaning. One is not quite the same person as before.

Kurt Gödel fell in love with Platonism, and he was not quite the same person as he was before.

Another example of this theme occurs on p. 110, where Gödel is described as “a man whose soul had been blasted by the Platonic vision of truth.”

Goldstein includes all of the standard anecdotes about Gödel, as reported by the occasional, usually confounded, eyewitness. Some of these are quite amusing, for example, the story of Gödel’s citizenship hearings (pp. 232–233), in which a logical inconsistency Gödel discovered in the American Constitution threatened to upset the proceedings.

The psychological analysis the author sprinkles here and there into the biographical material is nothing if not enterprising. For example, Goldstein has a theory about the source of Gödel’s psychological difficulties (pp. 48–49):

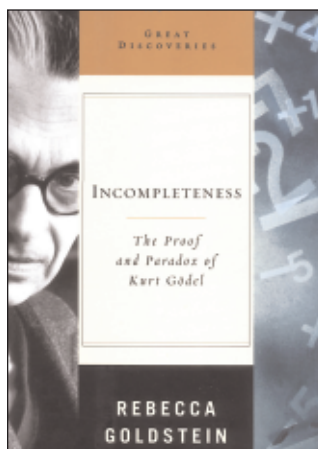
as I hope will become ever clearer in the chapters to come, the internal paradoxes in Gödel’s personality were at least partially provoked by the world’s paradoxical responses to his famous work.

See also p. 57:

...the precocious Gödel grasped the limits of parental omniscience at about the age of five. It would be comforting, in the presence of such a shattering conclusion, especially when it’s reinforced by serious illness a few years later, to derive the following additional conclusion...the grownups around me may be a sorry lot, but luckily I don’t need to depend on them. I can figure out everything for myself. The world is thoroughly logical and so is my mind—a perfect fit.

Quite possibly the young Gödel had some such thoughts to quell the terror of discovering at too young an age that he was far more intelligent than his parents. It would explain much about the man he would become.¹

¹ Some of the important memoirs about Gödel include those by his brother Rudolf Gödel, his classmate Olga Tausky-Todd [18], the obituary of Gödel for the Royal Society by Georg Kreisel [11], as well as other writings of Kreisel on Gödel, Hao Wang’s three books based on his conversations with Gödel [14], [15], [16], as well as Stephen Kleene’s memoir [10], to name just a few. The reader is also referred to the biography of Gödel by John Dawson entitled *Logical Dilemmas* [4], as well as to Palle Yourgrau’s portrayal of Gödel in *A World without Time* [17], which focuses mostly on Gödel’s friendship with Einstein and the scientific work which grew out of it.



Those familiar with Gödel’s life may find all this somewhat reductive; although it is clear from the author’s portrayal of him that she very much sympathizes with Gödel. Some occasional wrong notes include the discussion on pp. 226–227, in which some key background facts are omitted, as well as

the discussion on p. 223, in which the author ventures to describe Gödel’s marriage to his wife Adele, a lively and witty woman who seems to have been somewhat out of place in Princeton, as “weird”, “according to just about everyone”. One wonders about this characterization of their marriage, when many of the Gödels’ friends seem to give a different impression in their reports of it. In fact, on the whole the portrait of Adele is a bit ungenerous, with the author making very heavy weather, for example, about such things as Mrs. Gödel’s appalling—to Goldstein—taste in home decoration.

Does the real person come through in this account? Gödel was an extremely private person who at the same time suffered, as many creative people do, from disabling episodes of anxiety and depression. The episodes became worse with age. It is not a very pretty story; but his productivity, given the circumstances, makes it a very moving one—albeit one that in the end, at least in this book, may remain to be told.

That said, interviews the author has conducted with the principals, for example Armand Borel, have yielded valuable new information about Gödel’s relationship with his colleagues at the Institute for Advanced Study, answering the question why Gödel was so isolated from his colleagues during his later years there. Goldstein also draws upon her experience as a graduate student in philosophy at Princeton, which has put her in the position of being able to speak firsthand about the Princeton academic culture of the time—even if her perspective is very much that of an awestruck student.

The book is centered around the claim that, in an ironic twist of events, the “intellectual community”, as Goldstein refers to it, used Gödel’s own incompleteness theorems to discredit his philosophical Platonism; that the Incompleteness Theorems became “grist for the postmodern mill”, if not the main weapon in the contemporary “revolt against objectivity”; that consequently Gödel, to whom the notion of mathematical truth was an absolute and objective one, had to battle to the end

of his days against postmodern misconceptions and misconstructions of his theorems, which were misinterpreted to show that there is no such thing as truth.

Platonism in the context of foundations of mathematics is essentially the view that mathematics is a descriptive science, although, unlike the empirical sciences, the domain described is thought to consist of abstract objects. Another tenet of Platonism is that the concept of mathematical truth is a meaningful one. Gödel held this view from about 1925 onwards (though he wavered a bit between then and about 1940). Gödel also thought infinitistic methods were fully acceptable, something which, in his case, had to do with his Platonism.

The author's description of Gödel's mature Platonism is essentially correct. But her interpretation of the "intellectual community's" reaction to the Incompleteness Theorems, although making for a dramatic story, and probably correct in some particulars, is an oversimplification of the facts. One problem is that there seem to be at least two communities described as misconstruing Gödel's theorems: his colleagues in the logic and foundations of mathematics communities, and secondly the culture at large together with the postmodern philosophy community—quite a different audience for those theorems. Although it is the second of these that is referred to more often, the claim is a global one. Unfortunately this leads to some confusion, especially since it is often unclear to which community the author is referring in particular instances; and the abundance of vague references to, for example, "eminent thinkers", (p. 198) or "intellectual gurus", (p. 40), or, simply, "they", (pp. 135-6) is no help:

The irony of course is that while his theorems were accepted as of paramount importance, others did not always hear what he was attempting to say in them. They heard—and continue to hear—the voice of the Vienna Circle or of existentialism or postmodernism or of any other of the various fashionable outlooks of the twentieth century. They heard everything except what Gödel was trying to say.

The text is full of such assertions, without their ever being pursued. The reader may find this vexing: Who are "they"?

At least within the foundational community, a critical attitude toward semantic notions, such as the concept of mathematical truth, together with a bias against infinitistic methods, had been entrenched for decades before the Incompleteness Theorems. It was the driving force behind most of the foundational schools of the first half of the twentieth century and was simply the dominant

view of Gödel's time. For example, the Hilbert Program aimed to show that all that was required to formalize mathematics were finitary axioms stated in a precise syntax together with finitary rules of proof (proving consistency was the second desideratum of it). In showing that mathematics can be construed as a "formal game of symbols", a slogan which came into use at the time, mathematics would be put on a firm foundation by eliminating reference to infinite objects, as well as the use of unstated assumptions or proof procedures that might lead to paradoxes. The part of mathematics which remains, and to which it is possible to reduce the infinitary part, was called the *inhaltlich*, or contentual, part. Formalism was Brouwer's term for the school associated with the Hilbert Program.

Indeed the influence of the Hilbert Program on Gödel can be seen from the fact that Gödel stated and proved the Incompleteness Theorems "syntactically", i.e., so as to avoid any reference to the notion of truth in the standard interpretation—that is, truth in the domain of the natural numbers. He himself viewed the informal argument, which involves the concept of truth in the standard interpretation, as sufficient.

As for whether the Incompleteness Theorems served as a further stimulus for the anti-semantic or anti-truth view within the mathematical logic and foundations community, to some extent the influence flowed in the opposite direction, as the author notes, upsetting the entrenched view. This is because, with respect to carrying out the envisaged formal reconstruction of mathematics, eliminating semantic notions, such as meaning and truth, depends on showing that formal provability is all that is needed.

The First Incompleteness Theorem refutes this by showing that the concept of mathematical truth (if one accepts the concept at all) *properly contains*, in a sense, the concept of formal provability.² Therefore the latter cannot bear the burden of the formalization.

Gödel demonstrated, then, if not the indispensability of semantic notions altogether, at least some grounds for their necessity. Many accepted the view that semantics were ineliminable, at least those, unlike strict finitists, who were not predisposed against the idea. And on the other hand, many did not: the fact that the Incompleteness Theorems can be formulated and proved completely syntactically led some to conclude that the theorems say nothing at all about the validity—or lack thereof—of semantic methods. As for vindicating the use of infinitistic methods, Gödel had already achieved this, to some extent, with his

² Gödel's way of putting it was to say that the activity of the mathematician cannot be mechanized. See p. 164, [8].

completeness theorem, the main result of his doctoral thesis (submitted October 1929).

In sum, differing views as to what constitutes contentual mathematics led to the establishment of various foundational schools long before the Incompleteness Theorems were published. But the complicated story of how those foundational schools absorbed the impact of those theorems is touched upon so briefly here, and is blended in so indistinguishably with references to the second community's absorption of those theorems, that it is difficult to support the central claim about the misuse and misinterpretation of Gödel's works.

The response of experts to the theorem aside, the author's claim that postmodern philosophers or thinkers have used Gödel's results to show, for example, that truth is relative, or nonexistent, seems plausible.³ How Gödel's theorems were ultimately stirred into the mix, though, seems to be a very large topic, deserving, possibly, a book of its own.⁴

On the side of scholarship, it is to be regretted that a book which has and will continue to gather such a wide and enthusiastic readership (at least judging from the satisfied customer reviews on Amazon) should contain at the same time such a truly alarming preponderance of factual and conceptual errors regarding other matters.⁵ Names of major figures, such as Georg Kreisel's, are misspelt. Many crucial dates are incorrect, including, twice in the book, the date of Gödel's death (off by two years). Hilbert's famous list of problems numbered 23 and not 10, as Goldstein has it, and Tarski's original surname was Tajtelbaum (usually written Teitelbaum), not Tannenbaum.⁶ The diagram on page 125 is incorrect. The existence of non-standard models of arithmetic follows already from Gödel's Completeness Theorem—the First Incompleteness Theorem is not needed. The continuum

hypothesis, namely the question whether there are any infinite cardinals strictly between \aleph_0 , the cardinality of the natural numbers, and 2^{\aleph_0} , that of the reals, is misstated—the author confuses ordinals with cardinals. The First Incompleteness Theorem itself is misstated on p. 191 (although the author gets it right elsewhere).

Also on the continuum hypothesis, Goldstein claims that Gödel's landmark paper "What is Cantor's Continuum Hypothesis?", "...explains how Cantor's continuum hypothesis has been shown to be independent of the axioms of set theory...." But as the paper was written in 1947, well before the independence had been shown, it cannot possibly explain how the independence of the continuum hypothesis has been shown.⁷

The author's lengthy discussion of the Hilbert Program, which she characterizes as being dedicated to "eliminating intuitions", is not exactly erroneous, but does violence to the spirit of that program, in the opinion of this reviewer, and will justifiably perplex mathematicians.

From p. 129 and p. 133:

The drive for limiting our intuitions went even further. The aim became to eliminate intuitions altogether.

If it could be shown that logically consistent formal systems are adequate for proving all the truths of mathematics, then we would have successfully banished intuitions from mathematics.

She also remarks (p. 131) that

We don't have to appeal to our intuitions about numbers or sets or space in laying down the givens of a formal system.

Giving a finitary formal reconstruction of mathematics of the kind the Hilbert school envisaged in no way eliminates intuitions from mathematics. Formal systems are to be set up exactly on the basis of our intuitions. Of course one can arbitrarily set up a great variety of formal systems; but only those which are conceived on the basis of our

³ The author seems to use "postmodernism" to refer to the views associated with a much wider group of philosophers, writers, sociologists, and so on, than those associated with the French school around, for example, Lacan and Derrida. This may confuse European readers.

⁴ There is a growing literature in the area of postmodern commentaries of Gödel's theorems. For example, Régis Debray has used Gödel's theorems to demonstrate the logical inconsistency of self-government. For a critical view of this and related developments, see Bricmont and Sokal's *Fashionable Nonsense* [13]. For a more positive view see Michael Harris's review of the latter, "I know what you mean!" [9]. See also the recently published [6] by Torkel Franzén as well as Franzén's "The popular impact of Gödel's incompleteness theorem", this issue of the Notices.

⁵ Apart from the error which has been noted earlier, in the treatment of the First Incompleteness Theorem itself.

⁶ Though Stanley Tennenbaum was an important logician and Princeton figure.

⁷ The revised 1964 version of Gödel's paper contains a very brief appendix commenting on P. J. Cohen's proof of this result, published while Gödel's revised paper was in proofs; perhaps this explains Goldstein's error. What is striking about Gödel's paper is that in it he predicts that the independence of the continuum hypothesis will be shown. That is, he predicts the 1963 result due to P. J. Cohen, that the negation of the continuum hypothesis is consistent with the Zermelo-Fraenkel axioms for set theory. This in spite of the fact that Gödel himself proved in 1937 that the continuum hypothesis is itself consistent with the axioms of set theory, a result that is in some sense opposite to what Cohen proved.

intuitions about number, set, and so on, can possibly be of interest. Again, what the Hilbert school wanted was to capture mathematical content in the kind of formal system in which reference to infinite entities does not occur. It is only in that precise sense one can say those entities have been “eliminated”. For this to be possible one needs to define what it is that is to be eliminated, for a start, and infinite entities such as \aleph_1 are sufficiently clearly defined—whereas the concept of intuition is not.

On the philosophical side of things, connections between the notion of pure intuition and the Hilbert Program are deep and important. Getting the concept of intuition right, whether it be the Kantian notion or otherwise, was a project of considerable importance to the adherents of the Hilbert Program, throughout the 1920s.⁸ Simply put, the Kantian notion of pure intuition was seen by them to be the very *basis* of what Hilbert called “the finite mode of thought”. As Bernays put it in 1928:

The “finitistic attitude” required by Hilbert as a methodological basis must be characterized epistemologically as a form of pure intuition. ([12], p. 170)

And indeed Gödel’s so-called *Dialectica* paper of 1958 (see [8]) contains an extensive discussion of the topic.

For pellucid discussions of these matters by the principals, mathematicians are referred to *Philosophy of Mathematics* [1], a collection of landmark papers by Poincaré, von Neumann, Gödel, and others involved in these developments. For the Incompleteness Theorems themselves, mathematicians are referred to Gödel’s original 1931 paper, “Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I,”⁹ a powerful piece of mathematics written in majestic prose.¹⁰

As an aside, on the topic of different communities’ response to the Incompleteness Theorems, the readers of this review may see themselves as comprising a third community—that of working mathematicians. What was the impact of the Incompleteness Theorems on this community? This is an interesting question.

One might think of the average mathematician as taking the view that mathematics is complete—

⁸ Consequently they wrote about this extensively. See for example Hilbert’s “Foundations of Elementary Number Theory”, reprinted in [12].

⁹ Reprinted with a facing English translation in [7]. The excellent introductory note to it by Kleene is also strongly recommended.

¹⁰ There are many modern accounts of the theorem. See, for example “The Incompleteness Theorem”, by Martin Davis, this issue of the Notices.

in some sense of the term. There are different ways to interpret this, but for the sake of the present discussion let us take the assertion that mathematics is complete to mean that any mathematical statement, suitably precisely stated, is going to be decided one way or the other by mathematical means—as following from some *suitable* set of axioms for example (at least theoretically, that is, leaving aside questions such as those involving resources). In fact this very question, namely whether mathematics is complete in a more general sense than the technical one with respect to which he answered the question negatively, occupied Gödel himself to a very great degree—and long after he proved the Incompleteness Theorems. For example, in the late 1930s he pondered the existence of absolutely undecidable sentences, by which he meant precisely stated mathematical assertions “undecidable, not just within some particular axiomatic system, but by *any* mathematical proof the human mind can conceive.” ([8], p. 310) Interestingly, after a short period during which he entertained the possibility that there might be absolutely undecidable mathematical sentences, Gödel came to the conclusion around the early 1940s—and held to it for the remainder of his life—that this would turn out not to be the case. There would be no absolutely undecidable sentences in mathematics. What he called “Hilbert’s original rationalistic conception”—that “for any precisely formulated mathematical question a unique answer can be found” ([8], p. 163)—was, in his view, the correct one.

How is this possible? Didn’t Gödel prove that mathematics is incomplete (if consistent)?

The answer, of course, is no. What Gödel proved is that certain formal systems, including canonical ones like Peano Arithmetic or Zermelo-Fraenkel set theory, are incomplete. He did not prove that mathematics is incomplete in the sense we have defined completeness above; and therefore in some sense the question is still an open one.

An issue which bears on the question being open is the fact that, as it turns out, there are different classes of undecidable sentences. The so-called Gödel sentences published by Gödel in 1931, do not seem to have much to do with mathematical practice as such. The “I am unprovable” sentences arising from the First Incompleteness theorem are somewhat ad hoc. (Indeed Gödel himself referred to the theorems occasionally, in conversations with Kreisel, as the result of a “parlor trick”, see [11].) In any case they, as well as the other Gödel sentences, namely those involving consistency, can be decided simply by passing in a natural way to systems of so-called “higher type”.¹¹

¹¹ See [3], as well as “The impact of the incompleteness theorem on mathematics”, by Solomon Feferman, this issue.

In Conversation with Rebecca Goldstein

Rebecca Goldstein's 1983 novel *The Mind-Body Problem* has been widely admired among mathematicians for its authentic depiction of academic life, as well as for its exploration of how philosophical issues impinge on everyday life. Her new book, *Incompleteness: The Proof and Paradox of Kurt Gödel*, is a volume in the "Great Discoveries" series published by W. W. Norton. The series aims to take a fresh look at great stories in science and mathematics. Another mathematical book in this series is *Infinity and More* by novelist David Foster Wallace (reviewed in the June/July 2004 issue of the *Notices*).

In March 2005 the Mathematical Sciences Research Institute (MSRI) in Berkeley held a public event in which its special projects director, Robert Osserman, talked with Goldstein about her work. The conversation, which took place before an audience of about fifty people at the Commonwealth Club in San Francisco, was taped and later broadcast on radio.

A member of the audience posed a question that has been on the minds of many of Goldstein's readers: Is *The Mind-Body Problem* based on her own life? She did indeed study philosophy at Princeton, finishing her Ph.D. in 1976 with a thesis titled "Reduction, Realism, and the Mind". She said that while there are correlations between her life and the novel, the book is not autobiographical. Many have speculated that the model for one of the main characters, Noam Himmel, is Princeton philosopher Saul Kripke. Not so, said Goldstein: All of the characters are fictional composites, except for the protagonist's father, who is based on Goldstein's own father.

One of the attractions of writing about Gödel's work for the general public, she said, is that his main theorems seem to say something about the nature of mathematics itself and even to reach beyond boundaries of the field. "To have a result that has the rigor of mathematics and the reach of philosophy is beautiful," Goldstein remarked.

She also talked about the relationship between Gödel and his colleague at the Institute for Advanced Study, Albert Einstein. The two were very different: As Goldstein put it, "Einstein was a real mensch, and Gödel was very neurotic." Nevertheless, a friendship sprang up between the two. It was based in part, Goldstein speculated, on their both being exiles—exiles from Europe and intellectual exiles. Gödel's work was sometimes taken to mean that even mathematical truth is uncertain, she noted, while Einstein's theories of relativity were seen as implying the sweeping view that "everything is relative." These misinterpretations irked both men, said Goldstein. "Einstein and Gödel were realists and did not like it when their work was put to the opposite purpose."

—Allyn Jackson

In that spirit some (see [5]) refer to this type of incompleteness as "residual incompleteness", a phrase meant to capture the idea that this kind of incompleteness arises only as an artifact of formalization—after all, such sentences simply do not arise in ordinary unformalized mathematics—and is an exception to completeness of an entirely inessential nature, more or less on a par with including zero as a counterexample to the axiom that every element of a field has a multiplicative inverse. The basic question whether mathematics is complete in the sense we defined it is therefore not affected by Gödel's examples. As Gödel put it in referring to the undecidable sentences of 1931: "As to problems with the answer Yes or No, the conviction that they are always decidable remains untouched by these results."¹²

The situation regarding statements emerging from set theory, however, such as the continuum hypothesis, is much more complicated, much more

¹² See [8], pp. 174–5. There is a point of view that emphasizes more the importance of what we have called "residual incompleteness". But this would not have been Gödel's view.

interesting—and much more threatening to the Platonist viewpoint.

As was noted above, the continuum hypothesis is independent of the (highly canonical) Zermelo-Fraenkel axioms. But the continuum hypothesis is an elementary statement from the multiplication table of cardinal numbers, as Gödel put it in his 1947 "What is Cantor's Continuum Problem?" (reprinted in [1]); it has a clear and unambiguous meaning. Therefore it ought to have a definite truth value. As Gödel wrote in that paper:

Only someone who...denies that the concepts and axioms of classical set theory have any meaning (or any well-defined meaning) could be satisfied with such a solution,¹³ not someone who believes them to describe some well-determined reality. For in this reality Cantor's conjecture must be either true or false, and its undecidability from the axioms as known today can only mean

¹³ That there is no way to settle the continuum problem definitively. (Footnote the author's.)

that these axioms do not contain a complete description of this reality...

But there is a problem with deciding whether the continuum hypothesis is true or false in this sense—that is, with deciding the continuum hypothesis by finding a natural extension of the Zermelo-Fraenkel axioms that decides it (not to mention the problem with deciding other set-theoretical statements such as that asserting the existence of measurable cardinals). The reason is that there seem to be a number of natural extensions of the Zermelo-Fraenkel axioms that decide the continuum hypothesis in different ways—natural, at least, at first glance. But then, just as in the case of the parallel axiom in geometry, the question of the truth value of the continuum hypothesis takes on a different meaning. Is the parallel axiom true or not? To most mathematicians this is a meaningless question. The answer depends upon which geometry one is referring to. Similarly with the continuum hypothesis, some argue that its truth value depends upon which model of set theory one is in, so to speak, among a spectrum of natural models to choose from. This is a kind of formalism to which, for example, Cohen has subscribed (see his *Set Theory and the Continuum Hypothesis* [2]).

Of course many argue against the notion that the continuum hypothesis is analogous to the parallel axiom, and indeed Gödel was among the first (if not the very first) to argue against the analogy in his 1947 paper mentioned above. Among other arguments given there, Gödel observes that, “as against the numerous plausible propositions which imply the negation of the continuum hypothesis, not one plausible proposition is known which would imply the continuum hypothesis.” (See [8], p. 264.)

This counts in favor of the continuum hypothesis being false.

The arguments he gave in that paper have grown into the so-called “large cardinal program”, which is the program of finding a natural extension of the Zermelo-Fraenkel axioms that decides the mathematical statements one is interested in, meaning the “natural” ones, or more generally, those not arising from residual incompleteness.

As of today, the technical developments have not settled this issue in a definitive way. On the one hand the large cardinal program of Gödel is alive and well. Accordingly, for an important faction of set theorists, the continuum hypothesis is simply a problem to be solved—granted a very difficult one—just like any other mathematical problem (see [5]). This means, interestingly enough, that it is still possible for the discoverer of incompleteness to be vindicated in his view that mathematics is, for all practical purposes, complete. On the other hand it

is easy enough to find set theorists who disagree with Gödel; in fact many, perhaps even a majority, of set theorists see themselves as standing in—or near—the formalist camp.¹⁴

Returning to the book under review, of its three facets—Gödel’s incompleteness theorems, Gödel’s theorems set against the background of the intellectual currents of his time, and finally Gödel the man as well as the behind-the-scenes look at the academic life of his contemporaries: as noted earlier, the author’s account of the incompleteness theorems is not sufficient to reconstruct them, while some defects in the treatment of the second aspect of the book have also been indicated. As for the third aspect of the book, as mentioned, a rather colorful portrait of Gödel is to be found in it.

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¹⁴There is of course much more to be said about which independent statements need to be taken seriously, at least by Platonists. The present discussion has omitted the natural statements independent of Peano arithmetic of the Paris-Harrington type, as well as those arising from the work of Harvey Friedman. For discussion of these and related matters see the above cited “The impact of the incompleteness theorem on mathematics”, by Solomon Feferman, this issue of the Notices. For a discussion of Gödel’s results in set theory together with their impact, see “How Gödel transformed set theory”, by Juliet Floyd and Aki Kanamori, also this issue.

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2006 Events Celebrating the Gödel Centenary

Several events are taking place around the world to mark the one hundredth anniversary of the birth of Kurt Gödel. Below is a partial list.

March 25–26, University of Edinburgh: Conference on “Truth and Proof: Kurt Gödel and the Foundations of Mathematics”, organized by Jeffrey Ketland.

April 25–29, Technical University of Brno: Centennial conference on Gödel’s life and work, organized by Masaryk University, Brno, and the Institute of Philosophy, Czech Academy of Sciences, Prague.

April 27–29, University of Vienna: “Horizons of Truth: Logics, Foundations of Mathematics, and the Quest for Understanding the Nature of Knowledge”. This conference celebrating the Gödel centenary is organized by the Kurt Gödel Society with the support of the John Templeton Foundation. Website: <http://www.logic.at/goedel2006/>.

May 17–21, Montreal: The 2006 annual meeting of the Association for Symbolic Logic will include several sessions devoted to Gödel, organized by a subcommittee chaired by Charles Parsons.

May 18–21, University of Lille: Workshop on “Kurt Gödel: The writings”, organized by Pierre Cassou-Nogués and Mark van Atten.

June 30–July 5, University of Wales, Swansea: The conference “Computability in Europe 2006: Logical approaches to computational barriers”, organized by Arnold Beckman and John Tucker, will include a special session “Gödel Centenary: His Legacy for Computability”, organized by John Dawson and Matthias Baaz, and lectures on Gödel by Martin Davis and John Dawson. Website: <http://www.cs.swan.ac.uk/cie06/give-page.php?1>.

August 12–14, Seattle: The IEEE Symposium on Logic in Computer Science, part of the 2006 Federated Logic Conference, will include a special session on Gödel, organized by Moshe Vardi. Website: <http://www.informatik.hu-berlin.de/lics/lics06/>.

In addition, a major indoor and outdoor exhibition on Gödel will open in Vienna in April (curated by Karl Sigmund and John Dawson; see the article by Karl Sigmund in this issue of the *Notices*).

A panel session on “Gödel’s Contributions to the Foundations of Mathematics”, has also been proposed (by Linda Bercerra and Ron Barnes of the University of Houston Downtown) for MathFest 2006, August 10–12, in Knoxville.