

# Carleson Receives 2006 Abel Prize

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2006, worth 6,000,000 Norwegian kroner (US\$920,000) to LENNART CARLESON, professor emeritus at the Royal Institute of Technology, Sweden, and at the University of California, Los Angeles, “for his profound and seminal contributions to harmonic analysis and the theory of smooth dynamical systems.”

In 1807 the versatile mathematician, engineer, and Egyptologist Jean Baptiste Joseph Fourier made the revolutionary discovery that many phenomena, ranging from the typical profiles describing the propagation of heat through a metal bar to the vibrations of violin strings, can be viewed as sums of simple wave patterns called sines and cosines. Such summations are now called Fourier series. Harmonic analysis is the branch of mathematics that studies these series and similar objects.

For more than 150 years after Fourier’s discovery, no adequate formulation and justification was found of his claim that every function equals the sum of its Fourier series. In hindsight this loose statement should be interpreted as regarding every function for which “it is possible to draw the graph”, or more precisely, every continuous function. Despite contributions by several mathematicians, the problem remained open.

In 1913 it was formalized by the Russian mathematician Lusin in the form of what became known as Lusin’s conjecture. A famous negative result of Kolmogorov in 1926, together with the lack of any progress, made experts believe that it would only be a matter of time before someone constructed a continuous function for which the sum of its

Fourier series failed to give the function value anywhere. In 1966, to the surprise of the mathematical community, Carleson broke the decades-long impasse by proving Lusin’s conjecture that every square-integrable function, and thus in particular every continuous function, equals the sum of its Fourier series “almost everywhere”.

The proof of this result is so difficult that for over thirty years it stood mostly isolated from the rest of harmonic analysis. It is only within the past decade that mathematicians have understood the general theory of operators into which this theorem fits and have started to use Carleson’s powerful ideas in their own work.

Carleson has made many other fundamental contributions to harmonic analysis, complex analysis, quasi-conformal mappings, and dynamical systems. Standing out among them is his solution of the famous corona problem, so called because it examines structures that become apparent “around” a disk when the disk itself is “obscured”, poetically analogous to the corona of the sun seen during an eclipse. In this work he introduced what has become known as Carleson measures, now a fundamental tool of both complex and harmonic analysis.

The influence of Carleson’s original work in complex and harmonic analysis does not limit



Lennart Carleson

itself to this. For example, the Carleson-Sjölin theorem on Fourier multipliers has become a standard tool in the study of the “Kakeya problem”, the prototype of which is the “turning needle problem”: how can we turn a needle 180 degrees in a plane, while sweeping as little area as possible? Although the Kakeya problem originated as a toy, the description of the volume swept in the general case turns out to contain important and deep clues about the structure of Euclidean space.

Dynamical systems are mathematical models that seek to describe the behavior in time of large classes of phenomena, such as those observed in meteorology, financial markets, and many biological systems, from fluctuations in fish populations to epidemiology. Even the simplest dynamical systems can be mathematically surprisingly complex. With Benedicks, Carleson studied the Hénon map, a dynamical system first proposed in 1976 by the astronomer Michel Hénon, a simple system exhibiting the intricacies of weather dynamics and turbulence. This system was generally believed to have a so-called strange attractor, drawn in beautiful detail by computer graphics tools, but poorly understood mathematically. In a great tour de force, Benedicks and Carleson provided the first proof of the existence of this strange attractor in 1991; this development opened the way to a systematic study of this class of dynamical systems.

Carleson’s work has forever altered our view of analysis. Not only did he prove extremely hard theorems, but the methods he introduced to prove them have turned out to be as important as the theorems themselves. His unique style is characterized by geometric insight combined with amazing control of the branching complexities of the proofs.

Carleson is always far ahead of the crowd. He concentrates on only the most difficult and deep problems. Once these are solved, he lets others invade the kingdom he has discovered, and he moves on to even wilder and more remote domains of science.

The impact of the ideas and actions of Lennart Carleson is not restricted to his mathematical work.

He has played an important role in the popularization of mathematics in Sweden. He wrote the popular book *Matematik för vår tid (Mathematics for Our Time)*, and he has always been interested in mathematical education.

Carleson has had twenty-six Ph.D. students, many of whom became professors at universities in Sweden and elsewhere. As director of the Mittag-Leffler Institute near Stockholm from 1968 to 1984, he realized the original vision of Mittag-Leffler, building the Institute as we now know it, a foremost international research center in mathematics. He also placed special emphasis on the role of the Institute in the mentoring of young

mathematicians, a tradition that continues to this day.

As president from 1978 to 1982 of the International Mathematical Union (IMU), Carleson worked hard to have the People’s Republic of China represented. He also convinced the IMU to take the contributions of computer science to mathematics into account and was instrumental in the creation of the Nevanlinna Prize, rewarding young theoretical computer scientists. As president of the Scientific Committee of the fourth European Congress in Mathematics, in 2004, he started the initiative of the Science Lectures, where distinguished scientists discuss the most relevant aspects of mathematics to science and technology.

Lennart Carleson is an outstanding scientist with a broad vision of mathematics and its role in the world.

Lennart Carleson was born on March 18, 1928, in Stockholm. He received his Ph.D. from Uppsala University in 1950 and did postdoctoral work at Harvard University, 1950–1951. He has held positions at Uppsala University; the University of Stockholm; the University of California, Los Angeles; and the Royal Institute of Technology in Sweden. In addition to serving as director of the Mittag-Leffler Institute from 1968 to 1984, he has served on the scientific committee for the Institut des Hautes Études Scientifiques since 1983. He is a member of the Royal Swedish Academy of Sciences and a foreign member of several scholarly academies around the world. He received the AMS Steele Prize for a Seminal Contribution to Research (1984), the Wolf Prize (1992), the Lomonosov Gold Medal of the Russian Academy of Sciences (2002), and the Sylvester Medal of the Royal Society, London (2003).

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