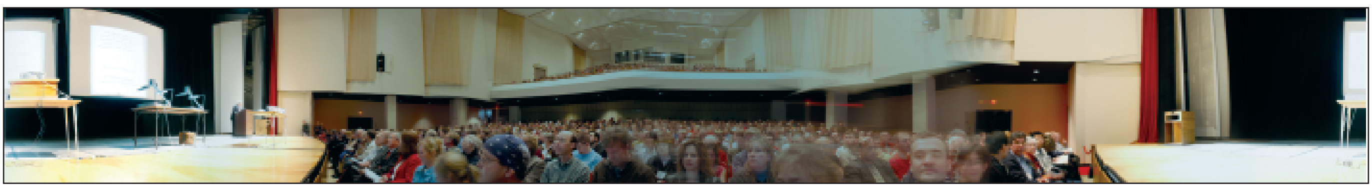
Sir Michael Atiyah's Einstein Lecture: "The Nature of Space"

G. W. Johnson and Mark E. Walker



Sir Michael Atiyah, winner of both a Fields Medal and an Abel Prize, delivered the first annual Einstein Public Lecture at the University of Nebraska-Lincoln.[1] The smashing success of Atiyah's talk inspired the local student newspaper, *The Daily Nebraskan*, to quip "Usually Mick Jagger is the only petite Brit who can entertain a sold-out, adoring American audience. But on Friday afternoon, the renowned English mathematician Sir Michael Atiyah showcased both his uncanny sense of humor and genius while delivering a lecture on 'The Nature of Space' to a full-capacity crowd at the University of Nebraska-Lincoln's Kimball Recital Hall."

Sir Michael's lecture was intended for the general public. Indeed, the general public came: Over 850 people filled the lecture hall and many others had to be turned away at the door. Probably well over 400 people in the audience were not part of the conference itself, but rather consisted of a mix of students, from high-school on up, faculty from physics, philosophy, and other disciplines, and other members of the community. Sir Michael offered something for everyone in this diverse crowd. He discussed the major themes of 20th century science while hinting at the technical details. His lecture touched on issues in mathematics, physics, philosophy, and even evolution and neurophysiology. One part of his lecture concerned recent research on the human brain and how it might affect our understanding of mathematics and physics as well as long-standing philosophical issues.

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[1] *Atiyah's talk was delivered on October 21 as part of the 2005 AMS Fall Central Sectional Meeting, hosted by the Department of Mathematics at the University of Nebraska-Lincoln.*

Einstein's *Annus Mirabilis*

The year 2005 is well suited to begin the Einstein Lectures as it marks both the 100th anniversary of Einstein's *annus mirabilis* (miraculous year) and the 50th anniversary of Einstein's death. We expand somewhat upon Atiyah's remarks concerning Einstein.

Einstein submitted four articles to *Annalen der Physik* in the year 1905, three of which are regarded as masterpieces. One of these concerned Brownian motion (the first of five papers Einstein wrote on this topic), and it represented an important contribution to the molecular-kinetic theory of heat, providing support for the atomic theory at a time when it was still in doubt. Einstein's 1905 paper on the photo-electric effect was an early and major contribution to quantum theory. Einstein was never satisfied, however, with the way that probability theory enters into quantum mechanics; this was the source of his famous assertion that "God does not play dice." These two contributions alone would be enough to make Einstein an important figure in the history of physics, but his work on Special Relativity, which was also written in 1905 and which was followed by General Relativity (in 1916), certainly place him at or near the top of anyone's list of the creative geniuses of physics. It has been claimed that each of these three 1905 papers was worthy of a Nobel Prize in physics, although only his work on the photo-electric effect was so honored, in 1921.

Here are some easily stated consequences of relativity theory:

1. The velocity of an object may appear different to different observers, but the velocity of light, c, is the same for all observers.
2. Energy and mass are related by the equation $E = mc^2$.
3. Space and time are not independent of one another—rather, motion through space influences an observer's measurement of time.
4. The geometry of space—in particular, its relationship with mass—is drastically different from what was believed prior to general relativity.

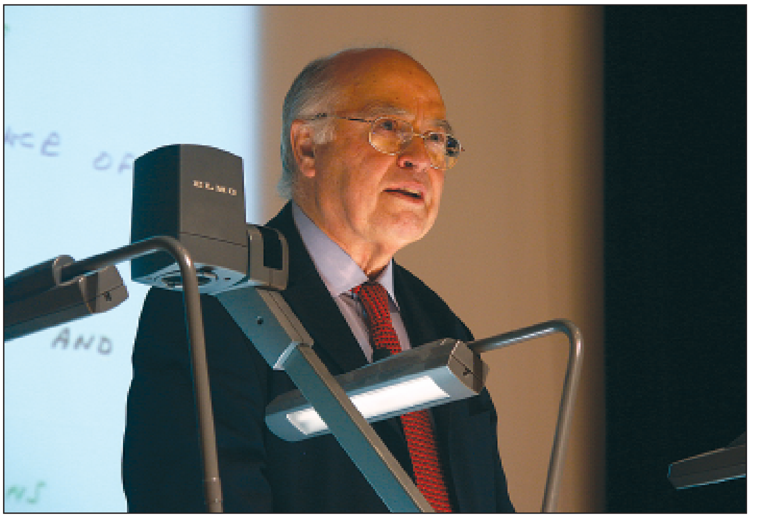
While being a creative genius in physics, Einstein was a consumer of mathematics, but his work, especially in relativity theory, has had a tremendous impact on mathematics.

It is amazing that the accomplishments of Einstein's *annus mirabilis* occurred while he was a twenty-six-year-old clerk in the patent office in Bern, Switzerland. As Atiyah pointed out, despite Einstein's excellent training in physics, he along with many other new graduates found it difficult to obtain an academic job.

Fundamental Philosophical Questions

Sir Michael touched on not only many areas of mathematics and physics, but also topics in philosophy, neuro-physiology, the nature of the human brain, and the theory of evolution. He asserted that understanding space is the fundamental problem of physics, and his talk focused extensively on the relationship between mathematics and physics, particularly with regard to the nature of space.

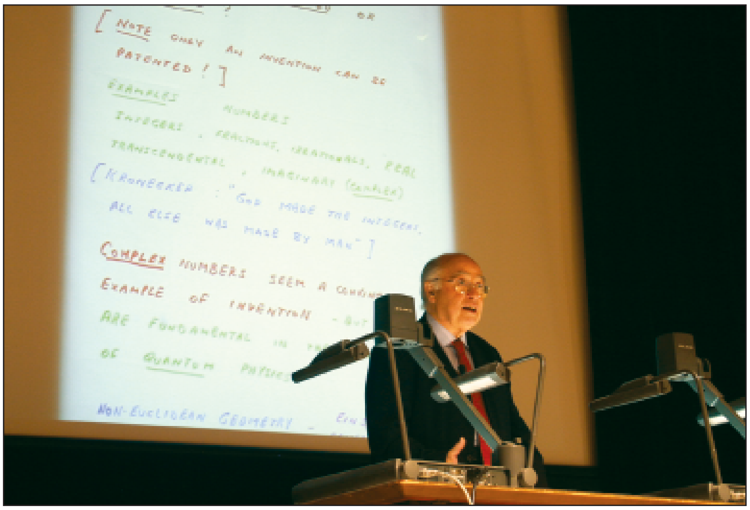
Plato believed that the world of ideal forms exists apart from the world perceived by our senses, whereas David Hume held that all knowledge is derived from sensory experience. Your point of view on this subject tends to influence your view of the role of mathematics and whether, in particular, you think mathematics is discovered or invented. Atiyah proposed that many (perhaps most) mathematicians hold the former view. Nearly everyone would agree that the integers were discovered and not invented. While Kronecker held the extreme view that "God made the integers, all else was made by man," most of us would likely accept that the rational numbers and even the real numbers were discovered. Some might point to the complex numbers, by contrast, as a convincing example of invention. Complex numbers are now known, however, to be fundamental in the real world of quantum mechanics. Similarly, although non-Euclidean geometry was "invented" before Einstein, it plays an important role in general relativity.



Sir Michael Atiyah.

The orthodox view among physicists is that mathematics was invented as a language and a tool to deal with the physical world. Eugene Wigner, however, has pointed out the "unreasonable effectiveness of mathematics in the natural sciences"—that is, if mathematics is merely invented, how is it that mathematics that was invented to explain things at the "human scale" also applies at very small scales (nuclear) and very large scales (cosmology)? Atiyah's own view is that mathematics *originates* from the physical world but is *organized* and *developed* by the human brain. Moreover, this relationship is complicated by the fact that the brain is itself a part of the physical world and is thereby affected by it and cannot be completely segregated from it. Indeed, an evolutionary point of view is that humans evolved by natural selection so that the human mind is adapted to and reflects physical reality. Mathematical thinking is thus an incidental consequence of evolution. For example, the rules of logic are deduced from experience with cause and effect. This point of view, however, still does not address Wigner's observation.

Current research in neuro-physiology is shedding light on how the brain actually works. Atiyah himself has been collaborating on research into how the brain behaves when one is thinking about mathematics and about different types of mathematics. Neuro-physiology reveals that the rules of logic and grammar (the underpinnings for mathematics and language) appear to be "hard-wired" in the brain. As a consequence of evolution, we are born with the capacity to do mathematics and to learn language. New research has also raised questions about the nature of "conscious decisions". Atiyah speculated that old philosophical questions, including those about the nature of mathematics, will be transformed by future research in neuro-physiology, in much the same way that the ancient



Atiyah, delivering the 2005 Einstein Lecture.

question of "What is life?" was transformed by the discovery of DNA.

Physics and the Nature of Space

Atiyah presented a brief history of physics as it relates to the nature of space. He began with the Earth-centered picture of the second-century astronomer and mathematician Ptolemy. Ptolemy's theory of epicycles, circles rolling on other circles, describes the motion of the sun and the planets. It agrees well with observation and lasted a thousand years. By placing the sun at the center of the solar system, Copernicus achieved a simpler mathematical description that made the same predictions as Ptolemy's theory. The view that *simplicity* ought to be the aim of all physical theory remains prevalent to this day. Kepler wanted to explain the number and positions of the planets in terms of the five Platonic solids and how they can be inscribed in each other. Remarkably, his model accounts for the orbits of the planets known to him within an accuracy of about five percent.

Newton's Law of Universal Gravitation served as a paradigm for all subsequent physical theory—it is both simple and universal, covering, for example, the motion of an apple falling from a tree as well as the motion of the planets, the comets, and the tides. Assisted by the experimental work of Faraday, James Clerk Maxwell found the equations that unite and govern electricity and magnetism via the electromagnetic field. As with the Copernican model of the solar system, the hallmark of Maxwell's theory is its simplicity. Moreover, Maxwell's laws are widely applicable, covering, for example, such down-to-earth phenomena as the behavior of lights, radios, and the telephone. The physicist Richard Feynman believed that, thousands of years in the future, Maxwell's discovery of the laws of electrodynamics will be judged as the most significant event of the 19th century.[2]

It was at this point, in the early twentieth century, that Einstein appeared. Einstein described how mass curves the space-time continuum in general relativity, and he formulated the basic geometric idea in simple mathematical equations. Gravity in general relativity is a modification of Newtonian gravity, differing only negligibly from the latter in our everyday word.

Quantum mechanics was well-developed by the end of the 1920s, and it represented a totally new, mathematically more sophisticated subject. As mentioned above, it relies on the arithmetic of the complex numbers. Whereas the observables of Newtonian mechanics consist of a finite number of position and momentum coordinates, these are replaced in quantum mechanics by position and momentum operators. These operators are self-adjoint, but typically are unbounded and fail to commute. This noncommutativity yields the Heisenberg Uncertainty Principle, which tells us *on theoretical grounds* that the more precisely we know the position of a particle, the less precisely we know its momentum. In spite of the difficulties involved, quantum mechanics is a spectacular success and is the basis of atomic physics.

Einstein tried to find a unified field theory that would include general relativity and electromagnetism. He did not believe quantum mechanics would be an element of such a final theory because he did not accept the uncertainty inherent in quantum mechanics. This view spawned a great philosophical debate, in which Einstein and Niels Bohr were regarded as the main antagonists. The orthodox view among physicists today is that Einstein was wrong, but Atiyah rejected the orthodox view, to some degree, and spent a good deal of time promoting Einstein's viewpoint in this debate.

In the mid-twentieth century, the nuclear forces were studied, geometrically interpreted, and combined with Maxwell's equations. The generality of the results obtained would have pleased Einstein, but quantum mechanics was still used, and general relativity was not. String theory entered the scene in the last quarter of the twentieth century; it aims to combine all of the fundamental forces, including gravity. For this reason, string theory is sometimes called "the theory of everything". String theory is a stunningly complicated theory of the physical world, and some hold the view that it represents a twenty-first century idea that was "accidentally" discovered in the twentieth century.

Here are some important characteristics of string theory:

1. It requires more dimensions, 10 (or 11), than the usual 3 + 1 dimensions of space-time. The

[2] *See* The Feynman Lectures on Physics, *Vol. II.*

additional 6 (or 7) dimensions are hidden from our normal experience of reality.
2. Whereas more classically, the basic objects such as electrons, protons, and quarks, could be thought of as point particles, string theory interprets such objects as being very small "strings". This allows for a "smoothing out" of the singularities that arise in the classical picture when such particles come close together. Even the singularities associated with mixing quantum mechanical and gravitational forces are resolved in this manner.
3. Very sophisticated geometry is used, involving a vast amount of mathematics, both old and new.
4. No unique model or picture has emerged out of string theory, but rather several versions exist. These different theories are now known to be different facets of the same theory. What has happened to the "real world"?
5. Quantum mechanics remains the basic framework.

String theory has had a remarkable and mysterious impact on pure mathematics, leading to many new concepts and results. In some cases, such results have been given proofs in the traditional mathematical sense. In other situations, the "results" merely fit well with known mathematical results or accepted features of string theory. In particular, string theory has had an impact in

1. algebraic geometry, by addressing enumerative questions concerned with counting algebraic curves satisfying certain conditions;
2. knot theory, by construction of new topological invariants of knots that can sometimes distinguish a knot from its mirror image;
3. four-dimensional geometry, by giving new, unexpected, and very deep results that are unique to four dimensions; and
4. various branches of algebra.

A New Paradigm?

If a "theory of everything" emerges from string theory, we will discover a universe built on *fantastically intricate mathematics*. In particular, the Calabi-Yau manifolds that make up the hidden dimensions are extremely complicated. Atiyah suggested that it is not satisfying that the true theory would be so complicated—even writing down the terms of the theory requires a vast amount of background.

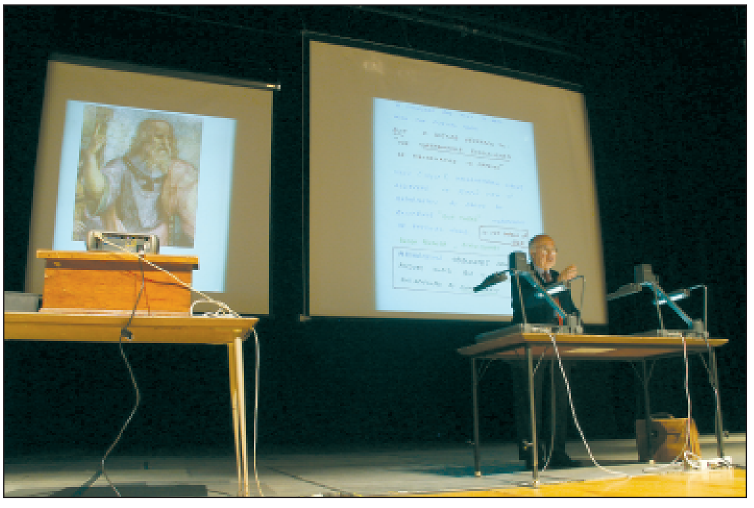
Perhaps, according to Atiyah, a new paradigm is needed; perhaps the complicated mathematics appearing in string theory is merely "in the eye of the beholder". That is, maybe we do not understand the fundamental nature of reality well enough, and this misunderstanding is leading to such exceptionally complicated mathematics. String theory, from this point of view, is only our method of approximating a simple reality. Perhaps, Atiyah suggested, we should follow Einstein and question quantum mechanics.

In order to make progress, we might need to dispense with some piece of accepted dogma. Relativity, quantum mechanics, and string theory have already dispensed with many previously held tenets, and so one might ponder whether there remains any such dogma left to throw away. Atiyah noted that all physical models since Newton, including even quantum mechanics, have assumed one basic premise—that we can *predict the future from full knowledge of the present*. Atiyah suggested an alternative to this paradigm: Perhaps we need full knowledge of the present *and the past* in order to predict the future. That is, maybe the universe has memory. As a simple example, the notion of the velocity of an object is viewed as being a property of the present, but, in reality, to measure velocity one needs to know not only where the object is now but where it was a moment earlier.

Atiyah's hypothesis possibly leads to several interesting consequences:

1. The mathematics used in physical theory would become more difficult, since all previously used mathematics in physics assumes that knowledge of the present suffices. With the new paradigm, for example, retarded (or delay) differential equations would become necessary.
2. Since we do not have complete knowledge of the past, uncertainty would arise. This might shed light on the uncertainty inherent in quantum mechanics.
3. Perhaps the complicated mathematics of string theory arises from our attempt to understand the full implications of the theory of general relativity without incorporating the knowledge of the past.

Atiyah does not promote discarding older, time-tested physical theories. Rather, such a new



paradigm ought to build on the old theories, much as relativity builds on Newtonian mechanics.

Speculations and Questions

There are various attitudes among physicists toward string theory. Some dismiss it as fancy mathematics that is unrelated to the real world, since string theory makes no testable predictions. Others believe the mathematical applications of string theory give confidence in the physical insights and indicate that the theory is on the right track. From this point of view, mathematical applications become a kind of alternative to experimental evidence. A third point of view is that we should continue to push forward with string theory in the hope that the new results and ideas that emerge will serve as a guide for finding a final unified theory.

Atiyah concluded his talk by speculating on the meaning of all this—quantum field theory, string theory, and their mathematical applications. What will the future physical theory look like? The aim is to unify quantum mechanics, the physics of the very small, with general relativity, the physics of the very large. Supersymmetry is a symmetry in which physical laws are unchanged when bosons and fermions are interchanged. Superstring theory, a supersymmetric string theory, is a perturbative approach, one that Atiyah compared with the theory of epicycles developed by Ptolemy. But what is the real theory; that is, what is being perturbed? Is it M-theory, a currently incomplete theory unifying all five versions of string theory? Is the universe really built using all this sophisticated machinery or is this an example of mathematics imposed by us? Perhaps the real physics is simpler and one should adhere to the dictate of Occam's razor—concepts should not be multiplied beyond necessity. Do we need to modify quantum mechanics? Atiyah closed by saying "This is for young people: Go away and explore it. If it works, don't forget I suggested it. If it doesn't, don't hold me responsible."

The second Einstein Public Lecture in Mathematics was delivered on April 29, 2006, in conjunction with the AMS Spring Western Sectional Meeting at San Francisco State University. Benoît Mandelbrot of Yale University spoke on "The nature of roughness in mathematics, science, and art".

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