The Millennium Grand Challenge in Mathematics

Arthur M. Jaffe

On May 24, 2000, Arthur Jaffe, then president of the Clay Mathematics Institute, announced the Millennium Grand Challenge in Mathematics towards the end of a meeting held at the Collège de France in Paris. The proof or a counterexample to seven important old mathematical conjectures would earn a US$7 million dollar reward—with US$1 million dollars for each answer. This challenge brought instant, world-wide recognition to the Institute, an organization conceived and founded by Jaffe and Landon Clay, a Boston philanthropist, just twenty months earlier. In 2006 a spotlight shines on the Poincaré conjecture, the first of these questions which may have been resolved. This essay presents a personal perspective on the background to the Challenge, as well as the founding of the Institute, a private non-profit foundation dedicated to furthering "the beauty, power, and universality of mathematical thought".

—*A.M.J.*

Evolution

The idea for the millennium grand challenge in mathematics cannot be separated from dreams of creating a new organization to support mathematical research. That vision came to fruition with the meeting of the initial three members of the Board of Directors of the Clay Mathematics Institute (CMI), just minutes after their election by the three members of CMI, on the morning of 10 November 1998.[1] The setting lent a dignified and uplifting feeling to the occasion. We met in a small, private dining room on the second floor of the Harvard Faculty Club, aptly named the "Presidents' Room" for its decoration with pictures of past Harvard presidents on the walls.

Two significant outcomes at that meeting were the election of the officers of CMI as well as constituting the Scientific Advisory Board (SAB). Attending were the three original members of the Board of Directors: Landon T. Clay, his wife Lavinia D. Clay, and the author—along with recordkeeper, Barbara Drauschke. The directors elected the author as president and as chair of the SAB, and then elected Alain Connes, Andrew Wiles, and Edward Witten as further SAB members, all without limit of time.

The agenda that morning included discussion of the first ten scientific projects to be pursued by CMI. To the best of my recollection, number eight in the list of projects read:

> **"Problems for the millennium, initial project:** Select 50 problems for publication in a book volume for the millennium, with the award of US$1,000 to the author of each manuscript. Select afterward a small number of special problems (maximum 12)."

This item received approval after minimal discussion, along with the other nine projects.

About one month afterward, I returned to the prize problem project while working in another inspirational setting—the loft in a vacation house located in New Hampshire, north of Cambridge. There the mood flowed from the view past the cathedral ceiling and through a picture window to the rolling

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[1] *See the "Epilogue" toward the end of this essay for a few further details on the formation of CMI, and how its history intertwines with that of the millennium challenge.*

Ossipee hills in the west. Was it possible to transmit this uplifting spirit to a text soliciting potential problems? I began to prepare a single page of text that CMI could circulate by mail and email. Perhaps it would be posted on mathematics department bulletin boards and on internet web sites. It might also be spread by word-of-mouth at scientific meetings. I had also just been invited to attend one of many conferences the following summer that were scheduled to review the progress of mathematics at the turn of the century and millennium. This and other similar meetings could be excellent forums for input.

Setting out the mechanism and procedures for the solicitation of fifty problems and the subsequent selections was a priority. I drafted a process to select the fifty manuscripts for publication, a second one to narrow the focus to a small number of special problems, and even went as far as drafting a letter that might be used by individual SAB members to solicit input. After several revisions, the plans appeared to be on track. I was in contact with Connes about other matters, so I asked him to critique the texts.

I then discussed the proposed details with Wiles, and this led to another point of view. He convinced me that the original approach might generate difficulties I had not anticipated; basically he questioned whether a completely open process would be best. Was it possible that powerful mathematicians who felt that their opinions were not sufficiently heeded would object and attempt to undermine the project? Wiles urged me to revise the plan in order to avoid "mathematical politics", by focusing immediately on the selection of the smaller number of special problems and omit the competition as an initial step. This also meant that the process would be more secretive than open. I went back to discuss this with Connes, who on reconsideration agreed that it was wise to modify the original plan.

But I realized that whatever plan we pursued ran the risk of controversy, and enough did arise later. So I decided that the SAB should scrap its plan to announce an open competition for fifty questions. The discussions would go outside the SAB only through individual members seeking advice from their trusted colleagues in the mathematics community.

Getting to Seven

The SAB now had to choose the problems, and this began in earnest only during the fall of 1999. No preconceived number of questions had been fixed. The upper limit of twelve seemed a reasonable bound—small enough to focus attention onto the project, yet large enough to be fairly broad. But the exact number of questions would depend upon the process, and we had no idea where the selection would lead us.

As a first step, I requested that each SAB member submit a personal list of top questions. Each of these questions should be difficult and important—a time-tested challenge on which mathematicians had worked without success. This exercise indicated some initial direction and set the background for further discussion. As I recall, everyone's list included the Riemann hypothesis and the Poincaré conjecture. So it seemed assured that these questions had common approval and would appear on the SAB's final list.

However, even in terms of these common questions we still needed to decide in what form they should be posed as challenges. Should the Riemann hypothesis be linked with some form of the Langlands' program? Should the Poincaré conjecture be linked with Thurston's more general geometrization program? This precipitated discussion of whether the millennium questions should be posed in their simplest form, or in general form. After some discussion by telephone, we arrived at a rule-of-thumb: we would prefer the simplest form of a question, at least whenever that choice seemed sensible on mathematical and general scientific grounds.[2]

From then on, the process of choice evolved through a series of telephone discussions separated by consultation and reflection. We added questions to the list one by one. With each new question we asked whether the list should be expanded or whether it might be improved by substituting a new question.

Here is one concrete example of the process: how we approached the P versus NP question. This problem arose on several lists, and the question also seemed to be "in the air" at the time. Nevertheless the SAB felt that it needed guidance from outside experts both about the relevance and difficulty of this question. And did this question represent the central thinking of the experts? After some external consultation, the SAB came to the conclusion that computer scientists regarded the P versus NP question as the most important open question in their field. At the same time, consulting some experts in mathematical logic led us to the conclusion that they regarded the same question as the outstanding open problem in logic. These opinions assured P versus NP a place on our list.

While each problem on the list was central and important, I want to stress that the SAB did not envisage making a definitive list, nor even a

[2] *I planned that the uniformity of the seven final manuscripts from this point of view, each written by different authors, would be reviewed and discussed by the SAB. For nonscientific reasons too lengthy to elaborate here, such a discussion by the SAB became impractical, and in fact it never took place.*

One can give many answers to the question, "Why pose a grand challenge in mathematics?" Three themes dominated my own thought. Focusing attention on difficult, significant, and time-tested mathematical questions emphasizes a lofty goal: strive for major, long-term satisfaction in mathematics rather than for immediate gratification. Communicating awareness to the public that important—yet unresolved—problems permeate mathematics illustrates the message that mathematics—like science—is a dynamic, complicated, and living organism. Possibly one can also inspire young students—opening new mathematical vistas for a few—while motivating them to attack major challenges in the future.

—A.M.J.

representative set of famous unsolved problems. Rather, personal taste entered our choices; a different scientific advisory board undoubtedly would have come up with a different list. The persons we consulted were experts, but they were chosen under pressure of time. However, the spirit of the selection transcends these decisions: the resulting list represents an honest attempt to convey some excitement about mathematics. We do not wish to address the question, "Why is Problem A not on your list?" Rather we say that the list highlights seven historic, important, and difficult open questions in mathematics.

The list grew after further conversations. As each problem was added the SAB began to have greater and greater difficulty—either to add a new problem, or to substitute a different one. By the end of 1999 seven questions had been chosen. At this point the SAB declared the list tentatively closed, but left open the possibility of later changes.

The report from the president to the directors' meeting on 6 January 2000 included a progress report on the project. It stated that CMI plans to offer a major financial award for the solution of particular mathematics problems, and to announce the plan publicly only after the selection of prize problems. The directors reaffirmed the project in principle, but had little new information at that point, except that the list included the Riemann hypothesis.

Even two months later—with the May 2000 announcement of the prize problems close at hand—the SAB had a further discussion about whether it might expand the list of problems. But the SAB decided to keep the list intact with the seven chosen problems. The members voted on 10 April 2000 to recommend to the directors that these seven questions be approved, as well as the US$7 million prize.

A Monetary Prize

The intention to offer some monetary prize for the solution of one of the millennium problems was always part of the picture. The reasoning behind this contained several components, each of which stands on its own, and all of which taken together remain very persuasive. But the idea to attach a fixed, US$7 million sum to the challenge came as an afterthought. It occurred during April 2000.

The original plan involved creating a prize fund within the CMI endowment. I expected that a substantial sum would be allocated each year for use of this fund in the event of an award. At the time that CMI recognized a solution to one of its problems, one would divide the amount of money in the fund by the number of remaining problems to determine the size of the award. With this plan, a solution that came shortly after the announcement of the competition would yield a modest prize. But a problem that remained unsolved over a long period after the announcement would bring a very substantial award—potentially much larger than the present US$1 million offer. A large award for an entire life's work would be fitting, and the size of the award would certainly raise public interest in mathematics.

The change in thinking on this question resulted from a couple of factors; one important event was an article in *The Times* of London detailing an offer made by the publisher Faber & Faber in an attempt to raise interest in a new book. Faber offered a US$1 million prize for the solution to the Goldbach conjecture, a question in number theory formulated in 1742. Details of the Faber offer contrasted markedly with those of the planned millennium prize. The key difference was that the CMI plan had no time limit for solving the prize problems, while the Faber offer imposed an unrealistic, two-year time limit for solving the Goldbach conjecture. This time limit allowed Faber to back its prize by insurance from Lloyd's, rather than by cash. And of course, the insurance turned out to be completely unnecessary.

But the Faber offer had already captured attention both in the printed press and on the Internet. I worried that if CMI proceeded as had been planned, the Faber news would surface and it could undermine attention to the CMI millennium challenge. The general public does not make fine distinctions. Mathematicians themselves are also unpredictable.

After reflection, my reaction was to suggest that CMI offer a US$7 million challenge from the start. This was bound to attract attention. On the other hand, the millennium challenge problems were sufficiently difficult that there was little worry that many of them would be solved in the near (or even the foreseeable) future. The Clays accepted my formulation, so I began to discuss it with the other directors and of course with the SAB.

I also believed that the US$7 million should be segregated so the prize would grow over time along with the endowment (and with inflation). The

Boston attorney for CMI argued that such a decision could be made in the future, so it would be best to wait and see the reaction. We did not segregate the prize.

A paraphrase of my letter of 12 April 2000 to the SAB and the Board of Directors illustrates the time-line. This letter requested a written vote on the problems, the rules, and the financial award; this would leave little to be reviewed by the directors at their 23 May 2000 meeting in Paris.

> The SAB has selected seven problems, following numerous conversations within and outside the SAB over the course of the past months. The SAB also considered possible experts to make a precise statement and to give background for each problem. We expect to have these descriptions available at the Paris meeting, and to publish a pamphlet with these problem write-ups and the enclosed rules….
>
> Below you find a popular name for each problem (alphabetically), and the experts who have agreed to prepare the descriptions. Presently we have three preliminary versions of these write-ups in hand, and for your interest I enclose copies:
>
> 1. The Birch-Swinnerton-Dyer conjecture (Andrew Wiles)
>
> 2. The Hodge conjecture (Pierre Deligne)
>
> 3. The Navier-Stokes equation has smooth solutions (Charles Fefferman)
>
> 4. P is not NP (Stephen Cook)
>
> 5. The Poincaré conjecture (John Milnor)
>
> 6. Quantum Yang-Mills theory exists with a mass gap (Arthur Jaffe and Edward Witten)
>
> 7. The Riemann hypothesis (Enrico Bombieri)

The SAB voted unanimously in a telephone meeting on 10 April 2000:

- To accept these seven problems as the list of Millennium Prize Problems and to recommend to the directors their approval of this selection.
- To recommend to the directors that the attached rules would be announced on 24 May 2000 (with minor refinement between now and 23 May).
- To recommend to the directors that the prize be announced as "a US$7 million prize for mathematics", with details to be given in a list of problems and in rules that would be made public on 24 May 2000.
- To request that the directors authorize to encumber US$7 million of the CMI endowment … to back this prize.

The directors confirmed this request by written ballot; it later had reconfirmation with minor changes to the proposed rules during a telephone conference on 15 May 2000.

The rules for the prize resulted from a fair amount of thought. Some features evolved from the original 1998 proposal described in the "Epilogue" below. One major safeguard involved the importance of publication of a solution. Implied is an initial review of correctness of the work by expert colleagues. In addition the rules specify a two-year waiting period after publication to ensure acceptance of the work by the mathematics community, before the CMI will even solicit expert opinions about the validity or attribution of a presumed solution to a problem.

Of course there can always be unforeseen circumstances. For example, an author of a solution may not write it down completely or might opt for self-publication, rather than for publication in an established journal. Traditional publication could change in the future, with relaxed standards of review. These and other circumstances are left for recommendation from a future scientific advisory board.

Paris

Sometime during November 1999, during the course of selection of the problems, another fact dawned upon the members of the SAB. Of course we should have been well aware of this from the start. But sometimes one needs to reflect before understanding the obvious.

Most mathematicians know that the famous set of twenty-three Hilbert problems were announced in a lecture at the 1900 Congress of Mathematicians, in Paris. So it was only natural that our list of millennium problems should be made public during the year 2000, and in Paris!

This meant that we needed to speed up everything. We had not even finished selecting the problems. But we also had to organize a meeting, to make specific preparations for the announcement, and to plan for the reaction! Fortunately we realized these time pressures only toward the end of 1999, really too late for us to worry about their consequences. In fact, at that point I was thankful that we had abandoned the original plan to proceed through fifty questions. Following that path would have made an announcement during the year 2000 totally impractical.

Alain Connes represented our Paris connection, and he graciously offered to host the CMI meeting at the Collège de France. Unfortunately some other mathematicians in Paris did not realize how positively the public would react to this challenge, and

Alain Connes (left) and Arthur Jaffe at the ceremony announcing the Millennium Prizes at the Collège de France in Paris, May 2000.

in the beginning several minor obstacles had to be overcome. In the first half of 2000, Alain made a substantial investment of time and energy, and his dedication and enthusiasm became an essential factor in the formula for the success of the meeting in Paris. Ultimately most of the organizational burden fell on the two of us. We had the indispensable assistance of a large number of enthusiastic and dedicated staff and supporters on both sides of the Atlantic.

Because of various constraints, both on the use of the Amphithéâtre Marguerite de Navarre at the Collège de France, as well as the schedule of other meetings in the Paris region, we chose 24–25 May 2000 for the millennium meeting, one day later than originally anticipated. And certainly any date seemed to pose conflicts. For example, we hoped to have both R. Bott and J.-P. Serre attend the meeting; but in February we learned that these two mathematicians had been chosen to receive the Wolf Prize. That award ceremony overlapped our meeting and affected the plans of other mathematicians as well.

CMI was fortunate that Gilbert Dagron, the distinguished Byzantine historian and administrator of the Collège (the equivalent of the president in other organizations), played another key role. Dagron became captivated by the idea of the millennium meeting and not only lent his personal support to CMI, using many resources of the Collège, but also gave a great deal of personal assistance. His deputy, Jacques Glowinski, had overseen the construction of the new amphitheater where we met; he too helped and enjoyed seeing the splendid site put to good use.

Through the good graces of Dagron, CMI also had access to the indispensable assistance of Véronique Lemaître. This extraordinary woman had the responsibility at the time for external relations at the Collège. Not only was she expert in her work, but she was also both dedicated and enthusiastic, spending long evening hours outside normal working time to make plans or telephone calls. Véronique knew and was respected by a substantial fraction of the scientific journalists representing both national and international publications and media in the Paris area. So when Véronique organized a briefing for the press on the morning of the meeting, over thirty Paris journalists appeared in person for discussion and lunch.

The Meeting

Expectations for the meeting at the Collège had mounted over the days leading up to it. The auditorium was vast—on the scale of mathematics meetings—although we had no idea how many persons would want to attend. As Paris represents a substantial mathematics community, and we had little advance idea of interest from the general scientific community or from the public, Alain Connes decided not long before the meeting that he needed to assign a ticket with a specific seat number for each attendee.

This plan created the enormous new burden of communicating with as many individuals as possible, as well as attempting to ensure that some elder mathematical statesmen (including Henri Cartan and Laurent Schwartz) would not have difficulty attending. While this caused a logistical nightmare and incredible pressure on Connes and his helpers, the plan succeeded. The audience flowed in smoothly, and every seat in the vast auditorium was filled at the start of the meeting. And for the overflow we had arranged closed-circuit video in a nearby room.

The meeting itself went splendidly, with the one exception—the newly appointed Minister of Research arrived late, causing an unintended wait and rearrangement in the schedule. The research awards to L. Lafforgue and A. Connes proceeded smoothly. The general talk on the "Importance of Mathematics" by Timothy Gowers presented an inspirational story of interaction between different disciplines and ideas within mathematics. The presentations of the seven problems by Michael Atiyah and John Tate presented a vast interwoven tapestry of mathematics.

One can review and reconsider this day, as four videos bring the proceedings to life. The French documentary filmmaker François Tisseyre worked tirelessly to present an interesting and comprehensible account of the meeting. The first video frames the day with interviews with participants as well as excerpts from various lectures. The other three videos present the main talks of Gowers, Atiyah, and Tate in their entirety, enhanced by graphical

effects that make them approachable and appealing. The publisher Springer Verlag distributes these videos.

Immediate Reaction

The immediate reaction came swiftly, as if it were the spirit of the times. Undoubtedly this was also linked to announcing the challenge in Paris, for in France the image of mathematics remains strong in the populace. In Paris it is easy to discover street names, public portraits, and busts of historic figures with mathematical significance.

Leading up to the meeting, Alain Connes had two interviews that captured the imagination of science reporter Jean-François Augereau, who eventually wrote four different articles in the 25 May issue of *Le Monde*. As is customary, the paper appeared on the previous afternoon, just as the participants were leaving the millennium meeting for dinner on 24 May. We bought a number of copies to hand out at that occasion; the paper carried a front-page photo of the SAB and CMI directors that the newspaper had discovered on the Internet.

Véronique Lemaître also had arranged for Jocelyn Gecker, a new, young, Paris-based science reporter for the Associated Press, to interview me two days before the meeting. The extensive article that she ultimately wrote appeared on 25 May in several hundred U.S. newspapers. Many of the other reporters present at the Collège also wrote stories.

The British magazine *Nature* even published an editorial on 25 May entitled: *Values of the Abstract: A new set of prizes is an apt celebration of the significance and wonder to be found in pure mathematics*, and reflecting: "It's an excellent way for a private foundation to recognize the eternal fascination that mathematics holds for people such as Hardy, and for the rest of us." This widespread positive reaction eventually led to thousands of articles appearing in other papers and magazines around the world, as well as interest by radio and television programs and on the Internet.

Just months before the meeting, the CMI had launched a website. We carefully prepared material about the millennium challenge, and made it possible for someone in Cambridge (USA) to push one button that resulted in posting the material on the Web exactly at the time the actual announcement took place in Paris.

All this precipitated a deluge of reaction far beyond what had been expected. Once the announcement became public, reaching the CMI website became totally impossible. Demand swamped the capacity of the server of the web hosting company. We had not anticipated that problem!

At this point I telephoned from Paris to John Ewing, the executive director of the American Mathematical Society based in Providence. John had served as a trusted advisor in designing the administrative structure of CMI, and he immediately offered his assistance. He was extremely happy to see so much attention being paid to mathematics and disappointed that a technical glitch frustrated so much curiosity.

Timeline: A Few Significant Dates

April 15, 1998. Over lunch at the Harvard Faculty Club, LTC asks AMJ his opinion about previously expressed ideas for a software foundation.

May 9, 1998. Alain Connes agrees while visiting Harvard that if a mathematics-oriented foundation is formed, he would be willing to become involved.

June 4, 1998. AMJ and LTC meet at the Harvard Club in Boston, and LTC mentions that his prior ideas have evolved and that he would now like to create an independent foundation devoted to fundamental mathematics. At this point, the formation of such an entity appears likely.

June 24, 1998. AMJ faxes to LTC an outline of several proposed mathematics projects, including "Prize 2000".

June 28, 1998. AMJ returns from travel four days earlier than planned in order to continue the discussions of a possible foundation with LTC.

August 19, 1998. During the International Congress of Mathematicians in Berlin, AMJ and Andrew Wiles dine together. They discuss the probable creation of a new foundation for mathematics, and in that event AMJ invited Wiles to serve on an advisory board.

September 25, 1998. The CMI becomes a corporate entity, registered in the state of Delaware.

October 27, 1998. LTC transfers shares of stock to a CMI account in Boston, creating the CMI endowment.

October 28, 1998. Edward Witten, while visiting Harvard, agrees to serve on an advisory board for CMI.

November 10, 1998. The members of CMI meet to elect the CMI directors. The directors meet to elect officers of CMI and the historic members of the Scientific Advisory Board.

May 10, 1999. A set of public lectures at MIT marks the formal opening of CMI.

January 6, 2000. The CMI directors approve the initial plans for a millennium challenge and for the meeting in Paris, tentatively set for 23–24 May 2000.[3]

April 10, 2000. The SAB formally approves the seven problems and the US$7 million millennium challenge; two days later the details are mailed to the directors with a request to confirm these decisions.

May 15, 2000. The plan is reconfirmed during a telephone conference of the CMI Board of Directors.

May 24, 2000. CMI announces the millennium challenge problems at the Collège de France. Simultaneously the website of CMI posts news of the challenge, and *Le Monde* publishes a front-page photograph and story. Worldwide reaction follows immediately.

—*A.M.J.*

[3] *Scheduling difficulties resulted in shifting the Paris meeting to one day later than the original plan.*

John's plan was to mirror the CMI website on the AMS web server, and to redirect requests for the CMI web address to the Society. The AMS server not only hosts mathematical news, but provides many electronic journals and other services to a worldwide community of mathematicians; its capacity and bandwidth was far greater than the server run by the web hosting company that CMI used. This solution would be temporary, until the CMI could make arrangements for a more robust host; but it would solve the problem. We implemented this plan immediately, with several phone calls between Cambridge and Providence to assist in transferring the files to mirror.

Not long afterward I returned from Paris to Cambridge, where I was greeted by a telephone call from John Ewing. There was a new problem: the volume of requests to view the CMI web pages threatened to crash the entire AMS website—including the AMS journals and the bookstore! This was unacceptable, and it looked like John would have to disconnect the CMI.

We discussed the numbers. Although the traffic redirected to the AMS address was still increasing, one saw a bit of leeway for it to stabilize before disaster hit. So we agreed to wait a day or two before John made his decision. We expected that the traffic would die down to a more manageable level after the initial reaction, and one week had already passed. Luckily the internet traffic did quickly come to equilibrium, and eventually CMI found a web host that offered more substantial bandwidth. It may be a while before internet activity related to mathematics again reaches the fever pitch of May 2000; hopefully that will arise from a mathematical discovery that fascinates the world.

Many amateurs who learned of the challenge did not realize the difficulty or subtlety of the challenge problems. Less than a year after the announcement, the CMI had received over six hundred letters, emails, and manuscripts from persons claiming that they could understand and solve one (and sometimes all) of the problems. A few of these individuals even sent their manuscripts to established journals in the hope of publication. While amateurs have always found an attraction in famous open problems, the publicity of the millennium challenge seemed to focus their attention.

The *Boston Globe* ran an interesting account by David Appell on 27 March 2001. For background the reporter interviewed some mathematicians who edit professional journals. David Goss of the *Journal of Number Theory* recounted, "They're really coming out of the woodwork. At times I am almost getting more crank stuff than legitimate stuff." Some of these amateur authors even complain their work received unfair treatment because editors summarily rejected their submissions, without explicitly pointing to flaws in their logic. In fact frequent submissions do pose undue burden on the editor and reviewers for a journal. However, to my knowledge the short-term anomaly of many amateur submissions has declined over time.

Reflection

Can one give an assessment of the millennium challenge five years after its launch? Cause and effect in life cannot easily be quantified into a mathematical law. But clearly the existence of the challenge has had a resounding impact on the number of papers, lectures, courses, conferences, manuscripts, and summer schools devoted to important, fundamental questions in mathematics. Within the community of research mathematicians the challenge has had profound impact.

It also catalyzed an enormous peak in public awareness of mathematics outside the research community. It affected the Internet, radio, television, as well as newspapers, magazines, and books. In fact the number of popular books about mathematics has increased substantially in the past five years; some recent books describe individual challenge problems, others discuss the challenge more broadly. Again this may not all be attributed to a single cause, but the overall effect is striking. Clearly the level of popular interest in recent mathematical work on the Riemann hypothesis and the Poincaré conjecture has been much greater than what one might expect without the climate generated by the challenge.

Some anecdotal evidence gathered from conversations with undergraduates suggests that the millennium problems have already had substantial impact within the student world —although limited experience can only suggest such effectiveness.

Presumably the most profound consequences of the millennium challenge project lie in the future. I hope that it will inspire mathematics and encourage potential mathematicians in a positive way for years to come.

Epilogue: Brief Background on CMI

Although I was dogged for some years by early-morning thoughts about forming an entity like CMI, it was only during 1997 that these dreams began to crystallize into something concrete, and about a year later they became a reality. In order to understand how this happened, let's backtrack.

After George Mackey retired from the Harvard mathematics department in 1985, the dean designated me the successor to his named chair. As a result, I began to lunch on a regular basis with the donor, a Boston businessman named Landon T. Clay (LTC) whom I had met casually some fifteen years earlier. He had been a generous benefactor to Harvard in the past, including the endowment of two chairs in different departments, as well as

the donation of a substantial fund to assist the dean in recruiting new faculty.

These lunches were generally quite interesting; we often discussed the activity in the mathematics department. A fundamental boost in activity resulted from the opening up of travel between Eastern Europe and the West. Those events began in 1988, during my term as department chair, and served as a precursor to the dramatic political changes soon to take place in that part of the world.

With the blessing, a small amount of money, and a great deal of encouragement from Harvard president Derek Bok, as well as a grant to the Harvard mathematics department from the Sloan Foundation, we invited a handful of young Russians to visit the following year as "Harvard Prize Fellows". In addition, I. Gelfand and A. Schwarz visited jointly between Harvard and MIT. Ultimately many of the friends of these fellows visited as well, producing a virtual invasion. During the academic year 1989–90, approximately twenty-five Russians spent time at Harvard!

LTC liked this activity; he also expressed his opinion based on his experience on various Harvard committees that the university administration did not appreciate the department's value. As a result of our interaction and discussion, he offered to establish a fund to invite visitors and to enable research projects in the department. In 1990 he directed over US$4 million income (over twenty years) from a trust in his name into the mathematics department. He also helped me establish a group of "Friends" of the mathematics department who ultimately assisted in many other ways.

Seven years later during 1997 he related at one of our lunches that many factors led him to contemplate establishing an "operating foundation". Sometime afterward he advised me that he had formulated a plan to create such a foundation devoted to software. I made no comments on these plans; at the time, my opinion had not been solicited.

Eventually LTC did seek my views; again it happened over lunch during the following year on 15 April 1998 at the Harvard Faculty Club. I recall answering this query as best I could, and shortly afterward writing a letter to him. I suggested that a foundation devoted to software would have difficulty competing with large existing corporate entities which had enormous financial resources at their disposal. In my mind it made scientific sense, and would be cost-effective, to consider creating a foundation devoted to mathematics. I also offered my counsel and assistance, in case he decided to follow that alternative path. Without making any commitments, that topic recurred on at least two other occasions over the next six weeks, including once during a scientific meeting held at Harvard.

After this lunch, and anticipating the possible founding of a mathematics organization, I began to turn over in my mind what persons might be suited to work together in a friendly and compatible atmosphere, were relatively accessible for consultation, and would have impeccable judgment and reputation. Among those I met with privately over the next six months were Alain Connes (at a conference in Cambridge on 9 May), Andrew Wiles (over dinner on 19 August preceding his lecture to the International Congress of Mathematicians in Berlin), and Edward Witten (during his visit on 28 October to lecture at Harvard). Each agreed in principle to participate.

Let's return to the chronology. On 4 June 1998, the morning of the graduation ceremony at Harvard, I met LTC for several hours in the Boston Harvard Club. He had invited me for breakfast that we ate in the dining room, after which we retired to a small upstairs meeting room for an extended discussion. That day LTC projected the attitude that he had made up his mind to start a mathematics organization, although the details were up in the air. He requested some written guidelines from me about what I might propose to do, both in the way of structure and the purpose of such an organization. He wanted it to be independent from Harvard, but possibly located on Harvard land. I responded by letter on 12 June 1998, summarizing our conversation. This letter included a brief outline of a possible plan for the structure of such an organization and reiterated that from 15 June to 2 July I planned to travel abroad. Later I would formulate and communicate further ideas.

Twelve days afterward on 24 June, I followed up this letter with a fax from a scientific meeting in Les Houches, France. During that meeting I also had the opportunity to consult with A. Connes, L. Faddeev, and J. Fröhlich. This fax of 24 June included a list of potential initial projects for the mathematics foundation. Among these, the millennium problem project appeared as the following proposal:

> "**Prize 2000:** In association with the millennium, I recommend a monetary prize for the solution of one of a small number of outstanding, long-range mathematics problems. These problems will be formulated by the Scientific Board and published in the year 2000. The problems will be published in a special article that also outlines procedures to determine a winner. In order to be eligible for consideration for this prize, the solution of the prize problem must be published in a refereed mathematics journal …. The correctness of the solution must be accepted by the leaders of

the mathematics community. For establishing questions of priority, members of the Scientific Board will investigate or have experts investigate. In case of lack of agreement by the mathematics community about the correctness or completeness of a published solution or about the proper attribution of a solution, the Scientific Board has discretion not to award a prize. An author may bring his or her own published work to the attention of the Foundation for consideration. Only an individual or individuals (as distinct from an organization, department or other group of persons) may receive this prize.

"Since the prize will be awarded only on rare occasion, a substantial prize fund may accumulate; this would focus great importance on solving these problems, and give substantial publicity to the prize. Both the principal and income to this principal will accumulate with other annual increments until the solution of all prize problems.

"However, in case the Foundation ceases to exist at some time in the Future, the prize fund will be transferred to another entity that agrees to administer the prize under the same conditions as if it had been under the auspices of the Foundation…."

I had planned to give a mathematics lecture at the University of Geneva at the beginning of July. However, communications with Boston led me to cut short the trip and return to Cambridge four days early on 28 June, in order to continue discussions about the foundation. While on the airplane, I began to prepare a document that summarized further thoughts about the scientific goals of the organization, and even proposed some twenty alternative names. It detailed the purpose as:

"to provide conditions to stimulate outstanding original research; to educate mathematicians or scientists about new discoveries; to encourage gifted students to pursue mathematical or scientific careers; and to recognize and reward unusual achievements in mathematical research."

These words ultimately became the first draft of the statement of purpose that would appear in the Bylaws of the CMI.

The official organization of CMI waited until September, when LTC outlined his intention to create a foundation to W. Warren and J. Olivieri of the firm Dewey Ballantine in New York. At that meeting he also chose the name CMI, and four days later on 25 September 1998, the CMI became a nonprofit Delaware corporation. Many details evolved over the next few months, and while they are central to CMI, they are peripheral to the millennium challenge.

The public recognition of CMI as an organization took place at a series of lectures at MIT on 10 May 1999, followed by a dinner to honor the donor. Speakers included M. Atiyah, LTC, H. Ferguson, D. Gergen, D. Herschbach, AMJ, B. Mazur, W. Odom, C. Vest, and E. Witten, with A. Wiles as keynote speaker. In addition, J.-P. Bourguignon, W. Browder, F. Caspersen, R. Colwell, L. Faddeev, D. Mumford, and K. Osterwalder made remarks after dinner.