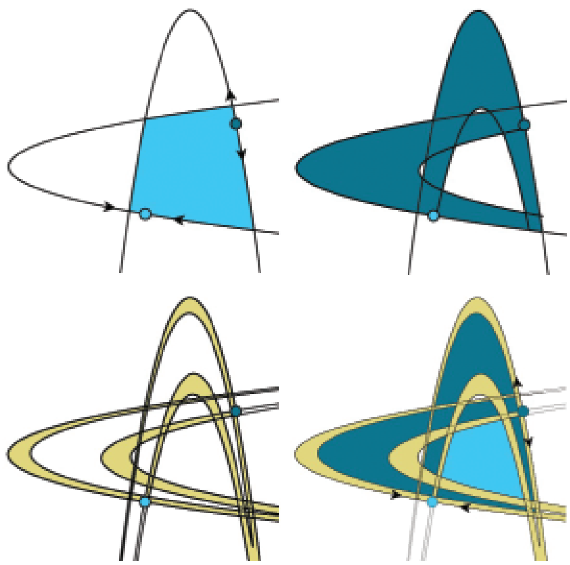
About the Cover

A Hénon horseshoe

This month's cover was suggested by Ruelle's article on strange attractors. It portrays an approximation to the non-wandering set Ω, as well as a portion of the homoclinic tangle, for the Hénon map

$$f : (x, y) \to (y, 1 - ay^2 + bx)$$

with $a = 6$, $b = 0.9$. For these values, the Hénon map does not possess an attractor, but instead offers an instance of one of Smale's horseshoes, and that is what the cover more or less illustrates. The logic is exhibited in more detail by the following sequence of pictures:



We begin with a region R bounded by parts of the stable manifold of one fixed point and the unstable manifold of the other. Any point outside R is taken off to infinity by iterates of either f or f^{-1}, so Ω lies inside it.The next pictures show $f(R)$ and $f^{-1}(R)$, then $f^2(R)$ and $f^{-2}(R)$. Each intersection $f^n(R) \cap f^{-n}(R)$ also contains Ω.

The symbolic dynamics of this example, the same as those of the horseshoe, are simple. As Ruelle mentions, the more complicated symbolic dynamics of the classical Hénon map with $a = 1.4, b = 0.3$ are specified by the *pruning front conjecture*. For more information, look at the AMS Feature Column for June 2006 (available at `http://www.ams.org/featurecolumn/`) and the book **Classical and Quantum Chaos** by Cvitanović and others, available at `http://ChaosBook.org`.

—Bill Casselman
Graphics Editor