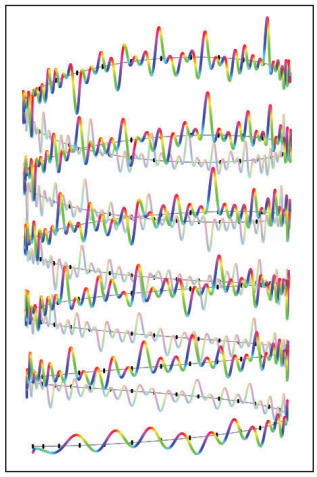
About the Cover

Z(t) on the critical line

Two articles in this issue are concerned with ζ-functions, the review of Dan Rockmore's book and the article by Jeffrey Stopple. This month's cover displays the behavior of $\zeta(s)$ on the critical line $\Re(s) = 1/2$. The function

$$\xi(s) = \Gamma(s/2)\pi^{-s/2}\zeta(s)$$

satisfies the functional equation $\xi(s) = \xi(1-s)$ and therefore takes real values on the critical line. If $\vartheta(t)$ is the argument of $\Gamma(1/4 + it/2)\pi^{-it/2}$ then $Z(t) = e^{i\vartheta(t)}\zeta(1/2 + it)$ also takes real values, and this is what is graphed along the helix. Lengths of the natural unit 2π are marked. The colors display the angle $\vartheta(t)$.



The behavior of $Z(t)$ encodes, in principle, the mysterious distribution of prime numbers, and it is hard to look at its graph without trying to read a message from it. But then humans are always trying to read meaning into random patterns.

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