Now, thanks I guess to Andrew Wiles, the competition has just about disappeared. There is no Fermat conjecture to cloud the issue. The Riemann hypothesis reigns supreme as the mathematical goal which is to be used to grab the public’s attention. How else to explain the sudden proliferation; this is at least the third¹ popular book on the subject to appear since 2003, the others being:


The reviewer has now read all three of these, in order, in each case having no dream at all that he would ever find occasion to read another popular book on this topic.

Each of these books is aimed at two audiences and, as such, faces somewhat of a dilemma. To quote Rockmore, “To distill years, even centuries, of scientific investigation for a broad and curious audience, while not raising the hackles of the experts in the field is something of an intellectual tightrope walk.”

To a certain extent this is not as serious a problem for [SAB] who is not a mathematician. He exhibits less need to try to make the mathematics understandable. Although spending some of his time on this, he concentrates more heavily on telling amusing stories about various of the players in this drama, a part which I found rather enjoyable. Unfortunately, in the latter part of the book, well, to quote from Heath-Brown’s Mathematical Review² of [SAB], “The human story is dominated by an account of Louis de Branges and his work on the Riemann hypothesis, to which the author devotes four chapters and an appendix. The reviewer found this rather depressing reading …” So did I.

The book by Rockmore is closer in spirit to the book [SAU] and, in the reviewer’s mostly positive opinion, is of roughly equal merit. Either book is enjoyable, but perhaps most people would find reading both of them to be a little too much. Like

¹I thank Allyn Jackson for drawing my attention to a fourth such book, Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem, by John Derbyshire.

²Apart from this one exception, all quotation marks refer to excerpts taken directly from the book under review.
Dirichlet could consider limits. He also introduced the notion of a real variable, whereas Euler had been in that position when he was giving “a harmonious proof for Euclid”. Specifically, he introduced, for many different purposes, the use of generating functions and, in particular, the one which was going to become the “zeta integral atoms”, move on to the fundamental theorem of arithmetic, the sieve of Eratosthenes, and Euclid’s proof of the infinitude of primes.

Soon, after some passages on how to count things asymptotically, we jump all the way forward to the “primal cartographers” Legendre and Gauss and their conjectures which would eventually become known as the prime number theorem. Here and at various places, other mathematical topics get briefly mentioned: non-Euclidean geometry, differential geometry, fast Fourier transform. These advances in the study of complex numbers, complex analysis and Fourier analysis are crucial to the Riemann hypothesis itself which is the most desirable, and also the simplest, explanation for the location of these zeros.

After some time on the ideas of Stieltjes for proving the Riemann hypothesis by showing that the Möbius function behaves like a random walk, we reach Hadamard and de la Vallée Poussin and the proof of the prime number theorem. We then turn the century. As it must, the century begins with Hilbert’s problems. But then we come to the eighth problem, and back to the Riemann hypothesis.

The first seventy years of the century saw the work of H. Bohr and E. Landau, of Hardy and Littlewood, of Cramér, of Siegel and Riemann’s Nachlass, of Weil and “a Riemann hypothesis that is true” and of Selberg, who would later be described as “the éminence grise of the Riemann hypothesis”. This summary happens in about twenty-five pages. Are my hackles showing? Not really. There are many deep and beautiful theorems from this period but perhaps not so much possibility to explain more than a few of the most striking statements to this audience.

The past thirty-five years get substantially more press. And why not? There is lots of sexy stuff happening. We start with the chance (Chowla-engineered) meeting of Dyson and Montgomery and the pair-correlation of the zeros, which ignited the revival of the Hilbert-Pólya dream of a spectral interpretation of the zeros and the consequent resolution of the Riemann hypothesis. With Odlyzko’s impressive computational evidence and further theoretical evidence provided by the higher correlations (Rudnick and Sarnak) and the function field analogues (Katz and Sarnak), it appears more and more as though the zeros are behaving like the eigenvalues of random matrices from certain fields.
families studied by the physicists! Does this help us to prove the Riemann hypothesis any more than Stieltjes was helped by the fact that the Möbius function appears to behave like the toss of a random coin? Well, at least this connection is not so straight-forward. We have “quantum chaos”, we have “life at the semi-classical limit”, we have “billiards in the Poincaré disk”.

The reviewer is not the only one to be somewhat skeptical. “Sarnak is among the first to say that in spite of all the exciting and beautiful mathematics and physics coming out of recent investigations into quantum chaos it is ‘naive’ to think that this work will culminate in the discovery of a physical system whose energies produce the zeta zeros, and thus a proof of the Riemann hypothesis.”

Still we go on, to noncommutative geometry, the central limit theorem, the Painlevé differential equations, and the shuffling of decks of cards! Why are we doing this?

“In this web of connections we truly see the stature of the Riemann hypothesis. A great problem of mathematics becomes an intellectual nexus, providing a bridge across subjects and connecting seemingly disparate ideas...And finally, with its relevance to almost all of mathematics laid bare, almost every mathematician can have a chance to dream of contributing to, and (dare we say!) even settling, this most important open problem in mathematics that is the Riemann hypothesis.”

Inside this quote is one wisp: “a chance to dream”. Now, that is something to which I can relate!

And, on the seventh day God rested. But, by afternoon she had become a little bored and decided to play a game. She set out to create the story of the primes. However, even being God, she did something that you or I would do. She used the tools with which she was familiar, the same tools she had been using all week long to create the universe.